

The EPFL Cryogenic MOSFET Model

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Abstract—This paper presents the EPFL cryogenic MOSFET compact-model framework for CMOS operation from room temperature down to 4.2 K. Its physics-based core includes temperature-dependent electrostatics, threshold voltage, subthreshold swing, and mobility, while semi-empirical extensions account for series resistance and high-field transport. The framework captures incomplete and field-assisted ionization, interface traps, band-tail-induced saturation of the subthreshold swing, and mobility changes, and provides a basis for cryogenic circuit-model implementation.

Index Terms—Cryo-CMOS, cryogenic MOSFET, compact model, quantum computing, incomplete ionization, freeze-out, interface traps, subthreshold swing, threshold voltage, mobility.

I. INTRODUCTION

Cryogenic CMOS is a strong candidate for the local control and readout of large solid-state qubit arrays, as reviewed from the qubit, front-end-electronics, and cryogenic-MOSF perspectives in [1]. Nearby multiplexers, amplifiers, and bias circuits can reduce wiring, delay, and heat load. Low-temperature operation can also improve mobility, leakage, and thermal noise for analog, RF, and qubit-readout circuits. However, MOSFET behavior changes greatly at low temperature. Circuit models must therefore include threshold-voltage shifts, mobility changes, subthreshold-swing saturation, interface traps, and dopant freeze-out.

Most cryogenic solutions are based on empirical changes to room-temperature model cards, so their ability to predict behavior outside the fitted data is limited. Earlier EPFL studies of 28-nm bulk and FDSOI CMOS showed that a physics-based model combining Poisson–Boltzmann electrostatics with drift-diffusion transport can match measurements down to 4.2 K when incomplete ionization, interface traps, and band-tail-related SS saturation are included [5], [6], [8], [10]. This paper summarizes that framework and identifies the temperature-dependent parameters needed in cryogenic model cards, thereby linking device physics to circuit simulation.

II. GENERAL CRYOGENIC MOSFET COMPACT-MODEL FRAMEWORK

The EPFL approach converts cryogenic measurements and device physics into temperature-dependent compact-model parameters. A cryogenic MOSFET model should not only adjust V_{TH} and mobility, but also explain the physical origin of each temperature dependence. Table I lists the main parameters and simple relations showing how temperature enters the model. Symbols are defined at first use, so the table serves as a roadmap. Electrostatics, interface traps, and band tails control weak inversion, whereas localized band-edge states affect

weak-to-moderate inversion. Mobility dominates strong inversion, and semi-empirical series-resistance, velocity-saturation, and short-channel terms extend the physics-based core at high current. These relations identify quantities that should be extracted and fitted consistently over the full temperature range. Fig. 1a shows their evolution from room temperature to 77 K and 4 K.

A. Incomplete Ionization and Freeze-Out

As the temperature decreases toward the cryogenic range, the bandgap $E_g(T)$ widens slightly, while the intrinsic carrier concentration $n_i(T)$ decreases sharply. These changes modify the Fermi potential and the threshold-voltage reference. The ionized dopant concentration can also become much lower than the implanted concentration. For an acceptor-doped body, the ionized dopant density can be written as [6]

$$N_A^-(T, \psi, V_{ch}) = \frac{N_A}{1 + g_A \exp \left[\frac{\psi_A - (\psi - V_{ch})}{U_T} \right]}, \quad (1)$$

where $U_T = k_B T / q$ is the thermal voltage, k_B is the Boltzmann constant, q is the elementary charge, N_A is the implanted acceptor density, ψ is the local electrostatic potential, g_A is the acceptor-state degeneracy factor, ψ_A is the acceptor potential, and V_{ch} is the channel potential. Incomplete ionization changes the ionized dopant density, depletion charge, and threshold voltage. In a compact model, it can be included by replacing the usual threshold-voltage expression with a cryogenic V_{TH} relation [9], [11]. A further correction can then account for the threshold-voltage shift caused by interface-trap charge,

$$V_{TH}^{cryo}(T) = [2\Phi_F - \Delta\Phi_F + V_{SB} + \Phi'_m - \Phi_F^* + \Gamma_b \sqrt{2\Phi_F - \Delta\Phi_F + V_{SB}}] + \Delta V_{TH}^{it}(T), \quad (2)$$

where Φ_F is the usual Fermi potential, Φ_F^* is its incomplete-ionization-corrected value, $\Delta\Phi_F = \Phi_F - \Phi_F^*$, V_{SB} is the source-to-body voltage, Φ'_m is the effective gate-to-semiconductor work-function term, and Γ_b is the body-effect coefficient. The bracketed term includes incomplete ionization through Φ_F^* and $\Delta\Phi_F$, while $\Delta V_{TH}^{it}(T)$ represents the shift caused by interface-trap charge. This corrected $V_{TH}^{cryo}(T)$ can be used in a standard CMOS compact model to include both effects in cryogenic I_D – V_G curves. Dopant freeze-out mainly changes the electrostatic reference and V_{TH} . Interface traps and band-edge states mainly degrade the subthreshold current because the gate-induced electric field partly re-ionizes dopants near the MOS interface. This distinction is important because neglecting either mechanism prevents a consistent

TABLE I
KEY TEMPERATURE-DEPENDENT PARAMETERS AND SIMPLE COMPACT RELATIONS FOR CRYOGENIC MOSFET MODELING.

Block	Simple temperature-dependent relation	Main cryogenic role
Electrostatics	$E_g(T), n_i(T) \propto T^{3/2} \exp[-E_g(T)/(2k_B T)], \Phi_F^*(T) = U_T \ln[N_A^-(T)/n_i(T)]$	Slight bandgap widening, carrier freeze-out, and incomplete ionization set the electrostatic reference, leakage, and V_{TH} .
Dopants/depletion	$N_A^-(T, \psi, V_{ch}) = N_A/[1 + g_A \exp((\psi_A - (\psi - V_{ch}))/U_T)], \Gamma_b(T) \propto \sqrt{N_A^-(T)/C_{ox}}$	Incomplete and field-assisted ionization modify depletion charge and body factor.
Threshold voltage	$V_{TH}^{cryo}(T) = V_{TH,0}(T) + \Delta V_{TH}^{it}(T), \Delta V_{TH}^{it} = -Q_{it}(T)/C_{ox}$	Turn-on shift from bandgap, Fermi-potential, depletion, and trap-charge changes.
Electrostatic slope	$m_{it}(T) = 1 + [C_{depl}(T) + C_{it}(T)]/C_{ox}, SS_{el}(T) = m_{it}(T)U_T \ln(10)$	Trap occupation weakens gate control and increases the electrostatic part of the subthreshold swing.
Band-tail statistics	$SS(T) = m_{it}U_T \ln(10) a(T, W_t), SS_{sat} = m_{it}(W_t/q) \ln(10)$	Generalized Boltzmann statistics introduce the deep-cryogenic SS floor.
Transport	$\mu_{free} = (L/W)I_D/(Q_i V_{DS}), \mu_{free}(T, V_{GS}) \simeq \mu_0(T)/[1 + \theta(T)(V_{GS} - V_{TH})]$	Phonon, Coulomb, and surface-roughness scattering control strong-inversion current.
Access/high-field	$I_{Dsat} \propto \mu_{free}(T)C_{ox}(W/L)(V_{GS} - V_{TH})^2$, with $R_s(T), R_d(T), v_{sat}(T)$, and $L_{sat}(T)$	Semi-empirical series-resistance, velocity-saturation, and short-channel extensions limit high-current operation.
Circuit-level	$I_{spec}(T) = 2n_{eff}(T)\mu_0(T)C_{ox}U_T^2(W/L), g_m/I_D \simeq 1/[m_{eq}(T)U_T]$	Charge-based scaling, transconductance efficiency, analog/RF design, noise, and mismatch.

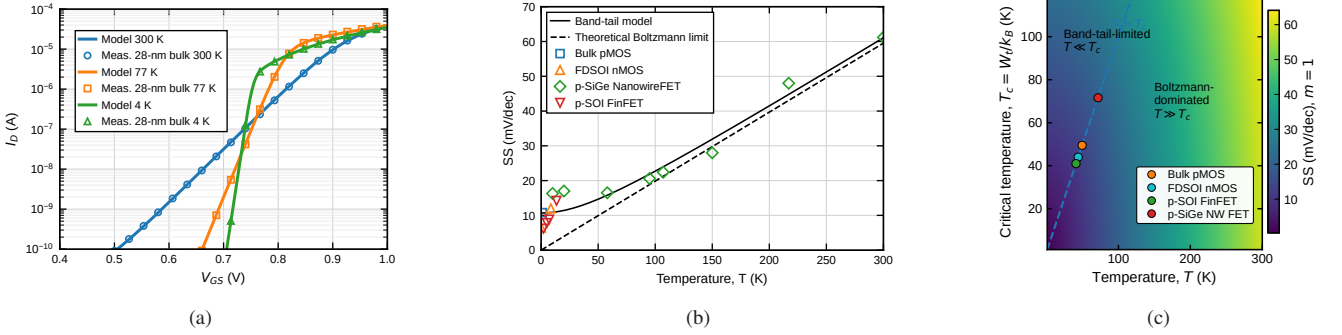


Fig. 1. Representative cryogenic trends: (a) transfer characteristics of 28-nm bulk CMOS at 300, 77, and 4 K; (b) subthreshold swing versus temperature for several FET technologies, with the Boltzmann limit and a band-tail model; and (c) a schematic regime map in which $T = T_c$ separates Boltzmann-dominated and band-tail-limited operation.

description of both the threshold-voltage shift and the subthreshold degradation.

B. Interface Traps, Band Tails, and Subthreshold Swing

The measured subthreshold swing at 4.2 K is much larger than the ideal Boltzmann limit $U_T \ln(10)$. The EPFL model attributes this increase to two distinct physical mechanisms. The charge-based treatment of interface traps is described in [15], [16]. Interface traps weaken the gate control of the channel, while band-tail states change the carrier statistics and set the deep-cryogenic floor of SS .

For a discrete interface-trap level $E_{t,j}$, the trap occupation is expressed as

$$f_s(E_{t,j}, T) = \frac{1}{1 + g_t \exp\left[\frac{\psi_{t,j} - (\psi_s - V_{ch})}{U_T}\right]}, \quad (3)$$

where $E_{t,j}$ is the trap energy, g_t its degeneracy factor, ψ_s the surface potential, and $\psi_{t,j}$ the corresponding trap potential. The corresponding interface-trap capacitance contributes to the weak-inversion slope factor,

$$m_{it}(T) = 1 + \frac{C_{depl}(T) + C_{it}(T)}{C_{ox}}, \quad (4)$$

where C_{depl} , C_{it} , and C_{ox} are the depletion, interface-trap, and oxide capacitances per unit area, respectively. Interface

traps increase SS by weakening the link between gate voltage and surface potential. For a nearly uniform trap contribution in weak inversion, the additional interface-trap contribution can be approximated by [6]

$$\Delta SS_{it} \simeq \frac{qN_{it}}{C_{ox}} \frac{g_t}{(1 + g_t)^2} \ln(10). \quad (5)$$

Here, N_{it} is the areal interface-trap density. This contribution is important, but it cannot by itself account for the observed saturation of SS at deep cryogenic temperature.

The saturation is captured by including shallow band-tail states near the band edges and replacing the usual Boltzmann carrier-density relation with generalized Boltzmann relations [10], [12]. For the conduction-band tail, the electron density contains a thermal term and a band-tail term,

$$n \simeq (n_{i,bt1} + n_{i,b}) \exp\left(\frac{E_{F,n} - E_i}{k_B T}\right) + n_{i,bt2} \exp\left(\frac{E_{F,n} - E_i}{W_{t,c}}\right), \quad (6)$$

where $E_{F,n}$ is the electron quasi-Fermi level, E_i is the intrinsic energy level, $n_{i,b}$ is the bulk thermal prefactor, $n_{i,bt1}$ and $n_{i,bt2}$ are band-tail density prefactors, and $W_{t,c} = k_B T_c$ is the conduction-band-tail energy width. It is an energy

scale, whereas $U_T = k_B T/q$ is a voltage; therefore, the two quantities are not the same. This leads to a compact expression for the cryogenic subthreshold swing,

$$SS(T) \simeq m_{it} \frac{k_B T}{q} \ln(10) \frac{1+r}{1+r\theta_c}, \quad (7)$$

$$\theta_c = \frac{k_B T}{W_{t,c}}, \quad r = \frac{n_{i,bt2}}{n_{i,bt1}}.$$

Equation (7) explicitly separates the two contributions. The factor m_{it} accounts for gate electrostatics and interface traps, while $(1+r)/(1+r\theta_c)$ accounts for semiconductor carrier statistics modified by the band tail. At high temperature, or for sharp band edges, this correction approaches unity and the usual Boltzmann expression is recovered. In the deep-cryogenic limit, its asymptotic value is

$$SS_{\text{sat}} = m_{it} \frac{W_{t,c}}{q} \ln(10) = m_{it} \frac{k_B T_c}{q} \ln(10), \quad (8)$$

Importantly, the low-temperature floor of SS is set by the band-tail width rather than by the thermal voltage. The often observed $1/T$ increase of the extracted slope factor should therefore be treated as an equivalent slope factor, not as a true $1/T$ dependence of $m_{it} = 1 + (C_{\text{depl}} + C_{\text{it}})/C_{\text{ox}}$.

For compact-model implementation, the two mechanisms should enter separately. Interface traps can be included through $C_{\text{it}}(T)$ or a fitted $m_{it}(T)$, while the band tail can be included by replacing U_T with $U_{\text{SS}} = a(T, W_t)U_T$, with W_t denoting the relevant band-tail energy width, where $a = (1+r)/(1+r\theta_c)$. A simulator may instead use $m_{\text{eq}} = m_{it}a(T, W_t)$ or $T_{\text{eq}} = a(T, W_t)T$. From measured $SS(T)$, m_{it} identifies the gate-electrostatic contribution and W_t the carrier-statistical contribution. This avoids unrealistically large fitted trap densities and improves comparison across technologies.

C. Threshold Voltage and Interface-Trap-Induced Shift

Threshold voltage is a key parameter in cryogenic circuit design. In this model, it is described by (2). The bracketed term includes incomplete ionization through the corrected Fermi potential, while the interface-trap contribution is written as the electrostatic shift [6], [11]

$$\Delta V_{\text{TH}}^{\text{it}}(T) = -\frac{Q_{\text{it}}(T)}{C_{\text{ox}}}, \quad (9)$$

where $Q_{\text{it}}(T)$ is the temperature-dependent areal interface-trap charge density. Interface traps shift V_{TH} through this additional charge density, while incomplete ionization acts through dopant ionization and depletion charge. The corrected relation can then be used as the $V_{\text{TH}}(T)$ input of a standard CMOS compact model.

D. Mobility and Saturation Current

Low-field mobility usually increases as temperature falls because phonon scattering becomes weaker. The mobility extraction follows the free-carrier method of Jazaeri *et al.*, which is derived from the drift-diffusion current and does not

assume a fixed gate-voltage dependence [14]. In the linear region, the free-carrier mobility is

$$\mu_{\text{free}}(T, V_{\text{GS}}) = \frac{L}{W} \frac{I_D}{Q_i(T, V_{\text{GS}})V_{\text{DS}}}, \quad (10)$$

where L and W are the channel length and width, I_D is the drain current, V_{DS} is the drain-to-source voltage, and Q_i is the mobile inversion-charge density per unit area calculated from the charge-based electrostatic model. The extracted mobility can then be represented by the low-field mobility $\mu_0(T)$ and mobility-degradation coefficient $\theta(T)$; V_{GS} denotes the gate-to-source voltage. For circuit design, the saturation-current trend can be estimated as

$$I_{\text{Dsat}}(T) \propto \mu_{\text{free}}(T)C_{\text{ox}} \frac{W}{L} [V_{\text{GS}} - V_{\text{TH}}(T)]^2. \quad (11)$$

This relation shows the competition between higher mobility and a higher threshold voltage at low temperature. Depending on the gate bias and device dimensions, either effect may dominate, so the saturation current does not always increase during cooling. For short-channel devices, the physics-based transport core can be combined with semi-empirical terms for saturation velocity $v_{\text{sat}}(T)$, characteristic saturation length $L_{\text{sat}}(T)$, source/drain resistances $R_s(T)$ and $R_d(T)$, DIBL, and channel-length modulation.

III. TEMPERATURE-DEPENDENT PARAMETER EXTRACTION

The parameters in Table I should be continuous functions of temperature rather than fitted independently at each point. This enables reliable interpolation and improves predictivity.

A typical sequence first extracts $V_{\text{TH}}(T)$ from the transfer characteristics and then obtains $SS(T)$ and the effective subthreshold slope factor. Next, $\mu_0(T)$ and the mobility-degradation parameter are extracted in the linear region. Finally, the saturation curves are fitted using the temperature-dependent threshold voltage, mobility, and charge-based current relation. This follows the simplified-EKV method used in earlier EPFL cryogenic CMOS studies [5], [8].

Fig. 1b compares reported subthreshold-swing values across transistor technologies. Unlike the ideal Boltzmann limit, the measurements approach a finite, technology-dependent floor at deep-cryogenic temperature, illustrating the widespread saturation of $SS(T)$ caused mainly by band-tail broadening and related nonidealities [10], [17], [18].

Figure 1c gives a schematic regime map of this behavior. The critical temperature $T_c = W_t/k_B$ is the temperature scale associated with the band-tail energy width W_t , and the line $T = T_c$ separates Boltzmann-dominated from band-tail-limited operation. Representative device markers, slightly offset for clarity, illustrate why different technologies can show distinct low-temperature SS floors.

IV. REPRESENTATIVE TRENDS AND VALIDATION

Fig. 2a shows the threshold-voltage shift, $\Delta V_{\text{TH}} = V_{\text{TH}} - V_{\text{TH},300\text{K}}$, for 28-nm bulk CMOS devices at 300 K, 77 K, and 4.2 K. Cooling produces a positive shift of about 0.1–0.2 V,

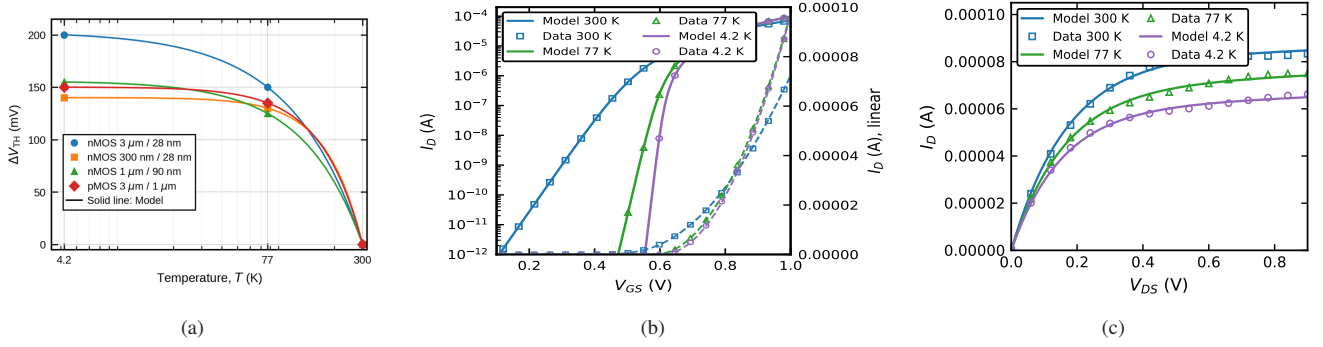


Fig. 2. Representative trends: (a) threshold-voltage shift in 28-nm bulk CMOS; (b) I_D - V_{GS} of a long-channel 28-nm FDSOI nMOS at 300, 77, and 4.2 K on logarithmic and linear scales; and (c) I_D - V_{DS} of a short-channel 28-nm bulk nMOS at the same temperatures. Symbols mark representative data points and lines show compact-model trends.

showing that $V_{TH}(T)$ must be included in the model card. The main contributions are slight bandgap widening, the sharp decrease in intrinsic carrier density, incomplete ionization, and interface-trap charge; their relative importance depends on device type, bias, and temperature [13].

Figs. 2(b) and 2(c) illustrate transfer and output trends after adding the temperature-dependent parameters. The physics-based core covers weak-to-strong inversion, while semi-empirical high-field and short-channel terms extend the output regime. Localized states or nonuniform traps can also cause a transfer-curve inflection and reduce weak-to-moderate inversion current. Together, these trends capture the main cryogenic electrostatics and transport.

V. IMPLEMENTATION IN STANDARD COMPACT MODELS

Cryogenic modeling requires more than rescaling room-temperature parameters. Incomplete ionization shifts V_{TH} , interface traps increase the slope factor, and band tails set the deep-cryogenic SS floor. Mobility rises during cooling but remains limited by non-phonon scattering and vertical field. Thus, $I_{Dsat}(T)$ must use consistent $V_{TH}(T)$ and $\mu_{eff}(T)$, with semi-empirical extensions for high-field and short-channel effects.

A compact implementation can then be proposed in the form

$$P(T) = P_{300} + a_1 \ln\left(\frac{300}{T}\right) + a_2 \left(1 - \frac{T}{300}\right) + a_3 F_{cryo}(T), \quad (12)$$

where P_{300} is the value of parameter P at 300 K, P denotes V_{TH} , the nominal slope factor n_0 , μ_0 , or θ , a_1 - a_3 are fitting coefficients, and $F_{cryo}(T)$ activates the low-temperature transition. The $V_{TH}(T)$ function should remain physically grounded. In subthreshold, the electrostatic term $m_{it}(T)$ and statistical term $U_{SS} = a(T, W_t)U_T$ should be implemented independently. Mobility can follow a Matthiessen-like combination, while $I_{Dsat}(T)$ is derived from the fitted $V_{TH}(T)$ and $\mu_{eff}(T)$. This gives consistent model cards from 300 K to 4.2 K.

VI. CONCLUSION

To the best of our knowledge, the Cryogenic EPFL model is the first physics-based compact model developed specifically

for cryogenic MOSFETs. It describes incomplete ionization, interface traps, band-tail statistics, threshold-voltage shift, and mobility through physical temperature-dependent parameters, providing a basis for cryogenic quantum-control and readout models.

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