

A Fokker-Planck Approach for Modeling Temperature-Dependent Two-Phase Switching Dynamics in Ferroelectrics

Wei Zhang
Department of ECE
National University of Singapore
Singapore 117583
wzhang@u.nus.edu

Leming Jiao
Department of ECE
National University of Singapore
Singapore 117576
leming@u.nus.edu

Jianze Wang
Department of ECE
National University of Singapore
Singapore 117583
jianze.wang@u.nus.edu

Xiao Gong
Department of ECE
National University of Singapore
Singapore 117576
elegong@nus.edu.sg

Xuanyao Fong
Department of ECE
National University of Singapore
Singapore 117583
kelvin.xy.fong@nus.edu.sg

Abstract—We propose a Fokker–Planck (FP) approach to model the evolution of the polarization-state probability distribution under field-driven drift and thermal diffusion. The proposed model provides a continuous formulation that unifies direct and thermally activated switching within the same framework. We demonstrate that our framework reproduces the temperature-dependent two-phase switching behavior such as experimentally measured $\Delta P_{FE}-t_{PW}$ characteristics from 10 K to 300 K. The simulated distribution evolution reveals the impact of thermal fluctuation on the switching pathway and long-time kinetics, demonstrating our framework to be an efficient and physically interpretable tool for stochastic FE switching modeling.

Index Terms—FE switching, two phase dynamics, Fokker-Planck approach, field-driven drift, thermal diffusion

I. INTRODUCTION

Ferroelectric (FE) switching in emerging non-volatile memories (NVMs) often exhibits a two-stage transient response rather than a single time-constant deterministic transition [1]. As illustrated in Fig. 1(a), the polarization change ΔP_{FE} consists of a fast short-pulse component followed by a slower long-time evolution. This behavior can be understood from the energy-landscape in Fig. 1(b), where the applied electrical field induces a drift force toward the switched state, while thermal fluctuations introduce stochastic diffusion across local barriers. Therefore, a distribution-level dynamic model is required to capture the combined effects of field-driven direct switching and thermally activated stochastic switching. Fig. 2(a) shows the pulse measurement scheme used to extract the write-pulse-width-dependent polarization change, including reset/pre-write initialization, a write pulse with varied amplitude and pulse width t_{PW} , and a subsequent negative-down read operation. The measured $\Delta P_{FE}-t_{PW}$ characteristics at room temperature (RT, $T = 300$ K) and cryogenic temperature (CT, $T = 10$ K) are shown in Fig. 2(b) and Fig. 2(c), respectively, confirm the

coexistence of fast direct switching and slow thermally activated switching [2]. The pronounced temperature dependence of the long-time response further indicates that thermal-noise-induced stochasticity plays an essential role in FE switching dynamics. Although Monte Carlo (MC) simulations can reproduce stochastic switching transients, they require a number of samples to accurately reconstruct the time evolution of the switching-state distribution. The Fokker–Planck (FP) formalism provides a more efficient alternative by directly solving the probability-density evolution under deterministic drift and stochastic diffusion. FP-based approaches have been successfully applied to stochastic modeling in other NVMs, including voltage-controlled magnetic anisotropy (VCMA)-based magnetic tunnel junctions (MTJs) for thermal-noise-induced magnetization dynamics [3], as well as spin-transfer-torque magnetoresistive random-access memory (STT-MRAM) and resistive random-access memory (RRAM) devices for stochastic write-process modeling [4].

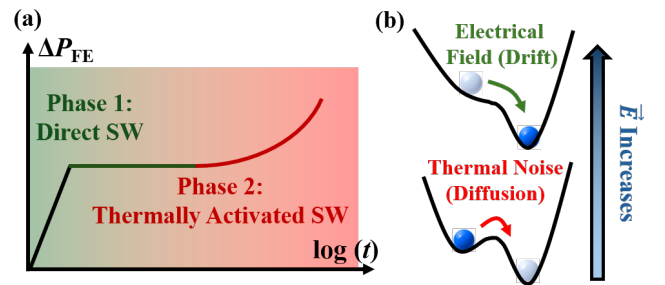


Fig. 1. (a) Illustration of the proposed dynamic two-phase FE switching mechanism. (b) Energy landscape indicating electrical-field-induced drift and thermal-noise-induced diffusion.

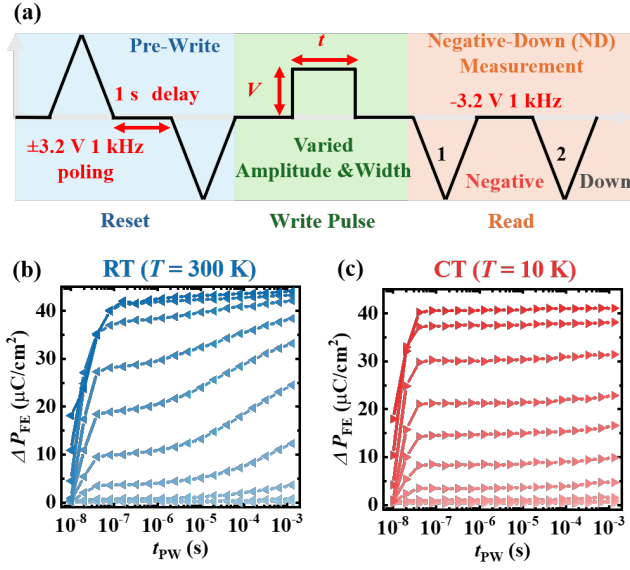


Fig. 2. (a) Experimental measurement for extracting ΔP_{FE} , and ΔP_{FE} as a function of pulse width t_{PW} under different applied voltages at (b) RT ($T = 300$ K) and (c) CT ($T = 10$ K).

In this work, we propose an FP framework to model FE switching as the evolution of a polarization probability distribution governed by field-driven drift and temperature-dependent thermal diffusion. Our proposed approach provides a unified description of the experimentally observed two-phase switching behavior and its temperature dependence.

II. FOKKER-PLANCK MODELING FRAMEWORK

FE switching is modeled as the time evolution of a polarization probability density function (PDF), $\rho(P, t)$, in a field-dependent FE energy landscape [5]–[7]. The FE free-energy density is described by the Landau-Khalatnikov (LK) equation:

$$F(P, E) = \frac{1}{2}\alpha P^2 + \frac{1}{4}\beta P^4 + \frac{1}{6}\gamma P^6 - EP, \quad (1)$$

where P is the FE polarization, E is the applied electric field, and α , β , and γ are the Landau coefficients. The term $-EP$ represents the electrical work from the applied field, which tilts the double-well energy landscape and reduces the switching barrier toward the field-driven polarization state.

The deterministic component of the switching dynamics is obtained from the LK description of FE polarization relaxation [6], [7]. Accordingly, the field-driven drift velocity in polarization space is written as:

$$v(P, E) = -\frac{1}{\eta} \frac{\partial F(P, E)}{\partial P} \quad (2)$$

where η is the viscosity-like relaxation coefficient. A diffusion process is introduced in polarization space to model thermal fluctuations. The resulting one-dimensional FP equation is:

$$\frac{\partial \rho(P, t)}{\partial t} = -\frac{\partial}{\partial P} [v(P, E)\rho(P, t)] + D(T) \frac{\partial^2 \rho(P, t)}{\partial P^2} \quad (3)$$

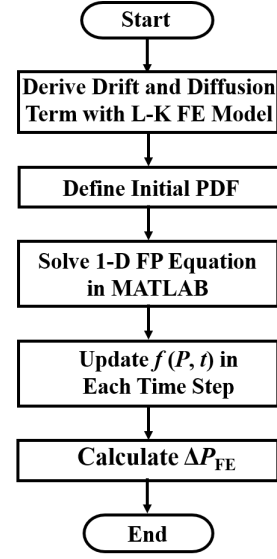


Fig. 3. Flow chart of the proposed FP simulation framework.

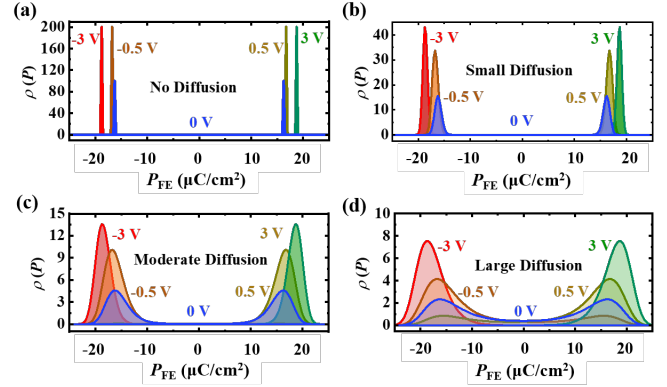


Fig. 4. Simulated static polarization probability density $\rho(P)$ under different diffusion strengths and applied voltages (a) without diffusion; (b) small diffusion slightly broadens the distribution near each polarization state. (c) Moderate and (d) large diffusion further broadens $\rho(P)$ and increases thermally-assisted transition.

The first term describes the drift of $\rho(P, t)$ toward the field-favored energy minimum, corresponding to direct field-driven switching. The second term describes thermal-noise-induced broadening of the polarization distribution, which enables a finite probability for polarization states to cross the energy barrier at longer pulse widths.

The temperature-dependent diffusion coefficient is defined through the fluctuation-dissipation relation as

$$D(T) = \frac{k_B T}{\eta}, \quad (4)$$

where k_B is the Boltzmann constant and T is the temperature. Thus, increasing T enhances diffusion and strengthens the thermally activated switching component. In contrast, at reduced temperature, $D(T)$ becomes smaller and the transient response is mainly governed by the deterministic drift term.

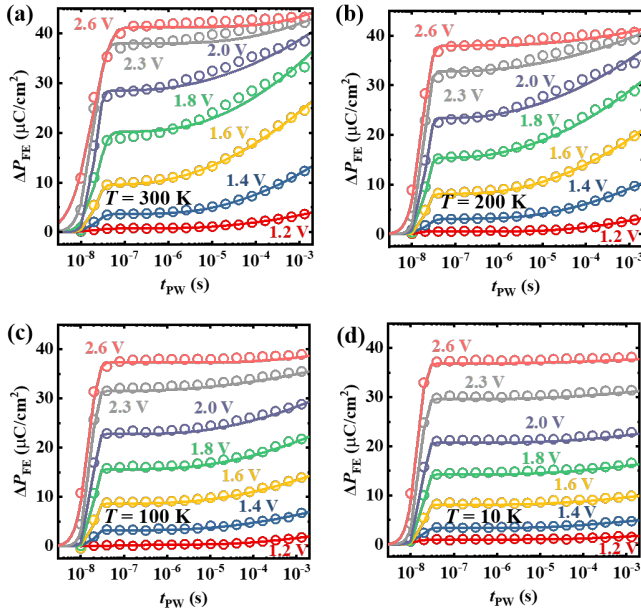


Fig. 5. Model calibration and validation against measured polarization-transient characteristics. (a),(b) Measured and simulated ΔP_{FE} - t_{PW} characteristics at RT ($T = 300$ K) and CT ($T = 10$ K), respectively. Symbols denote experimental data, and solid lines denote FP simulation results. The proposed model reproduces the two-phase FE switching behavior and its temperature dependence.

During the transient evolution, the polarization PDF satisfies the normalization condition

$$\int_{-\infty}^{+\infty} \rho(P, t) dP = 1 \quad (5)$$

The average FE polarization is obtained from the first moment of the distribution,

$$\bar{P}_{FE}(t) = \int_{-\infty}^{+\infty} \rho(P, t) P dP \quad (6)$$

and the simulated polarization change is calculated as

$$\Delta P_{FE}(t) = \bar{P}_{FE}(t) - \bar{P}_{FE}(0) \quad (7)$$

The overall simulation process is summarized in Fig. 3. The LK model first determines the field-dependent drift term, while the thermal diffusion coefficient is set by the temperature. Given an initial polarization PDF after the reset/pre-write condition, the FP equation is solved in the time domain to update $\rho(P, t)$ and calculate ΔP_{FE} . The role of diffusion is further illustrated in Fig. 4(a)-(d). When D is small, $\rho(P)$ remains narrow and the switching dynamics is dominated by field-driven drift. As D increases, the distribution spreads over a wider polarization range, increasing the probability of thermally assisted barrier crossing. This drift-diffusion picture explains the coexistence of short-time direct switching and long-time thermally activated switching, as well as their strong temperature dependence.

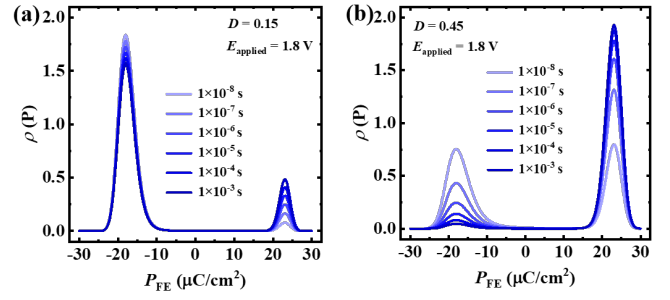


Fig. 6. Simulated dynamic evolution of the polarization probability density $\rho(P)$ under a relatively low applied field for (a) small and (b) large diffusion coefficients. A larger diffusion coefficient broadens the polarization distribution and increases the probability of thermally assisted barrier crossing.

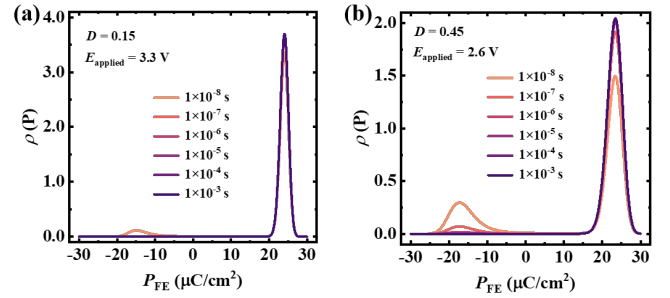


Fig. 7. Simulated dynamic evolution of the polarization probability density $\rho(P)$ under a relatively high applied field for (a) small and (b) large diffusion coefficients. The high field accelerates polarization switching through drift, while diffusion still affects the transient distribution width and long-time relaxation.

III. RESULTS AND DISCUSSION

A. Model Validation and Calibration

The proposed FP framework is calibrated and validated against the measured polarization-transient characteristics over a wide temperature range. As shown in Fig. 5(a)-(d), the simulated ΔP_{FE} - t_{PW} characteristics reproduce the experimentally observed two-phase switching behavior from room temperature to cryogenic temperature. In particular, the model captures the rapid polarization change at short pulse widths, the slow long-time switching tail, and the suppression of the thermally activated component at reduced temperature. This agreement confirms that the FP formulation provides a unified description of both field-driven direct switching and thermally assisted stochastic switching.

B. Dynamic Evolution of Polarization Distribution

The dynamic evolution of the simulated polarization distribution further reveals the stochastic nature of FE switching. Fig. 6(a) and (b) show the transient probability density $\rho(P)$ under a relatively low applied field. For a small diffusion coefficient, the distribution remains narrow and only a limited portion of the probability density transfers to the switched polarization state. As the diffusion coefficient increases, thermal fluctuation broadens $\rho(P)$ and enhances the probability

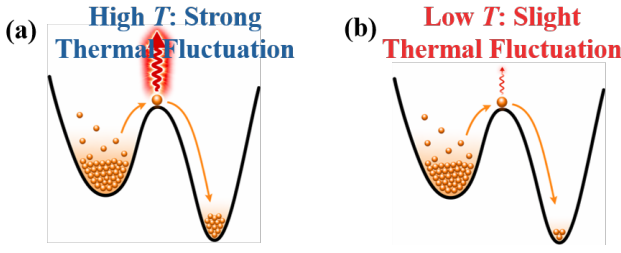


Fig. 8. FE energy landscape within temperature-dependent thermal fluctuation at (a) high temperature with strong thermal fluctuation, and at (b) low temperature with reduced thermal fluctuation.

TABLE I
COMPARISON AMONG STATE-OF-THE-ART FE SWITCHING MODELS
FROM DIFFERENT PERSPECTIVES

Spec.	Preisach	TDGL	NLS	MC	FP
ST SW	No	Yes	No	Yes	Yes
LT SW	No	No	Yes	Yes	Yes
Multi-domain	Yes	No	Yes	Yes	Yes
Continuity	Continuous	N/A	Discrete	Discrete	Continuous

of barrier crossing, leading to a stronger long-time thermally assisted switching component. Fig. 7 (a) and (b) show the corresponding evolution under a higher applied field. In this regime, the field-induced drift strongly tilts the FE energy landscape and rapidly drives the polarization distribution toward the final switched state. Therefore, the switching transient is mainly accelerated by the applied field, while diffusion still modulates the distribution width and the residual long-time relaxation. These results indicate that the applied field primarily controls the drift rate, whereas temperature-dependent diffusion controls the stochastic broadening and delayed barrier-crossing probability. The physical insight is summarized in Fig. 8 (a) and (b). At high temperature, strong thermal fluctuation broadens the polarization-state distribution and assists polarization transition across the energy barrier. At low temperature, the reduced diffusion suppresses thermally assisted barrier crossing, making the switching dynamics more drift dominated. Therefore, the interplay between field-driven drift and thermal diffusion governs the dynamic distribution evolution and gives rise to the experimentally observed two-phase FE switching behavior.

C. Benchmark

Table I benchmarks the proposed FP framework against representative FE switching models [8]–[12]. Compared to other classical FE switching models, our proposed FP framework directly evolves a continuous polarization probability density and therefore simultaneously captures short-time direct switching, long-time thermally activated switching, and multi-domain distribution evolution.

IV. CONCLUSION

An FP framework was developed to model two-phase FE switching dynamics through the time evolution of the PDF.

By combining field-driven drift and temperature-dependent thermal diffusion, our model captures both short-time direct switching and long-time thermally activated switching within a unified formulation. We showed our framework can reproduce experimentally measured polarization-transient characteristics from 10 K to 300 K and reveals the role of thermal fluctuation in delayed barrier crossing. Our FP-based approach provides a continuous distribution-level description for deeper stochastic FE switching analysis and physical understanding.

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REFERENCES

- [1] L. Jiao, Z. Zhou, Z. Zheng, X. Wang, J. Xie, D. Zhang, Q. Kong, G. Liang, and X. Gong, "First observation of the temperature-dependent two-phase switching of the HfO₂-based ferroelectric polarization: New insights and modeling of the switching dynamics," in *Proc. IEEE Int. Electron Devices Meeting (IEDM)*, San Francisco, CA, USA, Dec. 2024, pp. 1–4.
- [2] L. Jiao, Z. Zhou, Z. Zheng, X. Wang, J. Xie, D. Zhang, Q. Kong, Y. Feng, C. Sun, G. Liang, and X. Gong, "Temperature-dependent two-phase switching of the HfO₂-based ferroelectric polarization," *IEEE Trans. Electron Devices*, vol. 72, no. 5, pp. 2716–2724, May 2025.
- [3] V. P. K. Miriyala, X. Fong, and G.-C. Liang, "Influence of size and shape on the performance of VCMA-based MTJs," *IEEE Trans. Electron Devices*, vol. 66, no. 2, pp. 944–949, Feb. 2019.
- [4] D. Das and X. Fong, "A Fokker-Planck approach for modeling the stochastic phenomena in magnetic and resistive random access memory devices," *IEEE Trans. Electron Devices*, vol. 68, no. 12, pp. 6124–6131, Dec. 2021.
- [5] W. Zhang, Y. Xie, J. Wang, and X. Fong, "3D Phase Field Simulation and Variability Analysis of Polycrystalline Ferroelectric Memories," in *Proc. 2024 IEEE Int. Conf. IC Design and Technology (ICICDT)*, Singapore, Sep. 2024, pp. 1–4.
- [6] A. K. Saha, P. Sharma, I. Dabo, S. Datta, and S. K. Gupta, "Ferroelectric transistor model based on self-consistent solution of 2D Poisson's, non-equilibrium Green's function and multi-domain Landau Khalatnikov equations," in *Proc. IEEE Int. Electron Devices Meeting (IEDM)*, San Francisco, CA, USA, Dec. 2017, pp. 13.5.1–13.5.4.
- [7] W. Zhang, J. Wang, and X. Fong, "Polar-Axis Orientation Fluctuations and the Impact on the Intrinsic Variability in Ferroelectric Capacitors," *IEEE J. Explor. Solid-State Comput. Devices Circuits*, vol. 11, no. 1, pp. 74–80, Jul. 2025.
- [8] N. Gong, X. Sun, H. Jiang, K. S. Chang-Liao, Q. Xia, and T. P. Ma, "Nucleation limited switching (NLS) model for HfO₂-based metal-ferroelectric-metal (MFM) capacitors: Switching kinetics and retention characteristics," *Appl. Phys. Lett.*, vol. 112, no. 26, Art. no. 262903, Jun. 2018.
- [9] A. T. Bartic, D. J. Wouters, H. E. Maes, J. T. Rickes, and R. M. Waser, "Preisach model for the simulation of ferroelectric capacitors," *J. Appl. Phys.*, vol. 89, no. 6, pp. 3420–3425, Mar. 2001.
- [10] Z. Zhou, L. Jiao, J. Zhou, Q. Kong, S. Luo, C. Sun, Z. Zheng, X. Wang, D. Zhang, G. Liu, G.-C. Liang, and X. Gong, "Time-dependent Landau-Ginzburg equation-based ferroelectric tunnel junction modeling with dynamic response and multi-domain characteristics," *IEEE Electron Device Lett.*, vol. 43, no. 1, pp. 158–161, Jan. 2022.
- [11] C. Alessandri, P. Pandey, A. Abusleme, and A. Seabaugh, "Monte Carlo simulation of switching dynamics in polycrystalline ferroelectric capacitors," *IEEE Trans. Electron Devices*, vol. 66, no. 8, pp. 3527–3534, Aug. 2019.
- [12] M. Thesberg, M. N. K. Alam, B. Truijen, B. Kaczer, P. J. Roussel, Z. Stanojević, O. Baumgartner, F. Schanovsky, M. Karner, and H. Kosina, "On the modeling of polycrystalline ferroelectric thin films: Landau-based models versus Monte Carlo-based models versus experiment," *IEEE Trans. Electron Devices*, vol. 69, no. 6, pp. 3105–3112, Jun. 2022.