

# Analytical SOT MRAM Modeling: From Deterministic to Fokker-Planck based Stochastic Switching

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**Abstract**—Spin-orbit-torque (SOT)-driven switching of the free layer in magnetic tunnel junctions (MTJs) is a promising mechanism for cache-oriented MRAM due to its fast switching speed. In addition to the damping-like torque (DLT), the field-like torque (FLT) can assist the switching by reducing the threshold current. The objective of this work is to provide a reliable foundation for circuit compatible compact model for SOT-MRAM by improving the existing numerical modeling approach. We present an analytical framework for deterministic switching to describe the threshold current as a function of pulse width and introduce a two-dimensional Fokker-Planck (2D FP) solver to investigate the SOT switching variability to explicitly capture the effect of FLT on dynamics along with DLT. The deterministic model is validated against experimental data, while the stochastic model is compared with results from a stochastic Landau-Lifshitz-Gilbert (s-LLG) solver which provide complementary insight into system behavior.

**Index Terms**—MTJ, SOT, Field-like, Fokker-Planck, s-LLG

## I. INTRODUCTION

Magnetic random-access memory (MRAM) has emerged as a promising embedded memory technology owing to its nonvolatility, high speed, high endurance, and competitive density [1]. Various aspects of switching in MTJ has been extensively studied whereas spin polarized current driven switching is one of the most promising technologies [2]. One of the approaches is to apply current through MTJ which is polarized by a fixed layer that induces spin transfer torque on the free layer, known as STT MRAM [3]. It has disadvantage of breakdown of MgO in the MTJ from high applied currents. In particular, SOT-MRAM offers superior endurance because it separates the write and read paths, minimizing dielectric stress mechanisms. Here current is driven through a heavy metal layer with strong spin orbit coupling (SOC), generating a transverse current by accumulating up spin and down spin in opposite boundaries, known as spin hall effect (SHE) [4]. Thus the damping like SOT torque (DLT)  $\mathbf{m} \times \mathbf{m} \times \boldsymbol{\sigma}_y$  acts on magnetic moments of the adjacent free layer to induce motion towards in-plane direction and then full switching is realized in presence of in-plane external field in order to break the SOT mirror symmetry [5]. However, in most experiments it was observed that SOT torque composed of field like torque (FLT) along with DLT one [6] can promote or hinder switching. FLT behaves as an effective transverse field along the y-direction, thereby promoting magnetization ( $m$ ) precession, as shown in

Fig. 1 which is described by Landau-Lifshitz-Gilbert (LLG) equation

$$\frac{dm}{dt} = -\gamma\mu_0(m \times H_{\text{eff}}) + \alpha(m \times \frac{dm}{dt}) - \gamma\mu_0 H_{\text{DL}}(m \times m \times \sigma_y) + \gamma\mu_0 H_{\text{FL}}(m \times \sigma_y), \quad (1)$$

where  $H_{\text{eff}}$  includes the perpendicular magnetic anisotropy (PMA) field and the applied in-plane field along  $x$ , and  $\sigma_y$  denotes the spin-polarization direction from a current flow in the  $x$ -axis. The effective SOT fields are  $H_{\text{DL}} = \frac{\hbar\theta_{\text{SH}}}{2eM_s t_{\text{FM}}} J$  and  $H_{\text{FL}} = \beta H_{\text{DL}}$ , where  $M_s$ ,  $\theta_{\text{SH}}$ ,  $J$ ,  $t_{\text{FM}}$  and  $\beta$  are the saturated magnetization, spin Hall angle, current density, free-layer thickness and field like parameter respectively.

Under our sign convention in Eq.(1), a positive field-like parameter enables FLT to assist DLT in counteracting the Gilbert damping torque, leading to a reduction in the threshold current [7]. However, most prior studies have focused mainly on DLT-induced switching and have often neglected the contribution of FLT [8]. In addition, to extend our model in stochastic regime, conventional sLLG solver is employed. However, a key drawback is the increased computational cost, particularly due to the large number of trajectories to achieve low write error rates (WER).

In this paper, we present two main contributions including the effect of FLT on switching dynamics in deterministic regime and stochastic regime to define error rate metrics. First, we modify the existing DLT-based analytical model to include FLT and capture the threshold current density  $J_{th}$  as a function of pulse width. Secondly, we develop a 2D-FP solver using the Finite Volume Method (FVM) by extending earlier one-dimensional STT (spin-transfer-torque)-PMA formulations to SOT dynamics and validated against stochastic LLG solver. This flux-conserving formulation enables reliable estimation of very small WER at room temperature. It provides a compact-model-friendly framework for SOT error-rate prediction.

## II. METHODOLOGY

### A. Deterministic Model : Threshold Current Formulation

The LLG equations in spherical coordinates  $(\theta, \phi)$  can be derived from Eq.(1):

$$\frac{d\theta}{dt} = \frac{1}{\tau_d} [-\cos\theta \sin\theta + h_x(\cos\theta \cos\phi - \frac{\sin\phi}{\alpha}) - h_s(\frac{\cos\theta \sin\phi}{\alpha} - \cos\phi) + \beta h_s(\cos\theta \sin\phi + \frac{\cos\phi}{\alpha})], \quad (2)$$

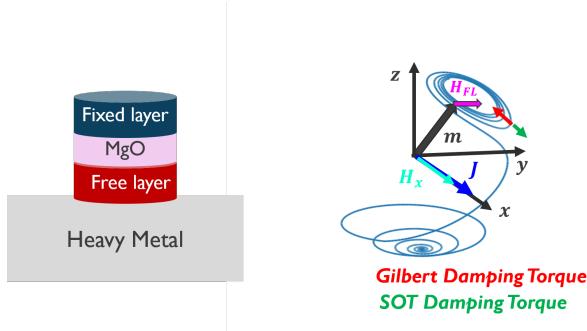


Fig. 1. Illustration of a magnetic tunnel junction device with a heavy metal layer beneath, serving as the SOT track. The current ( $J$ ) and in-plane field ( $H_x$ ) are applied along the positive  $x$  direction. The field-like torque induced transverse field ( $H_{FL}$ ) aids the damping-like SOT to overcome Gilbert damping by increasing precessional amplitude.

and

$$\sin \theta \frac{d\phi}{dt} = \frac{1}{\alpha \tau_d} [\cos \theta \sin \theta - h_x (\cos \theta \cos \phi + \alpha \sin \phi) - h_s (\alpha \cos \theta \sin \phi + \cos \phi) + \beta h_s (-\cos \theta \sin \phi + \alpha \cos \phi)]. \quad (3)$$

Here Eq.(2) represents the switching dynamics of  $m_z(\cos \theta)$  and Eq.(3) characterizes the precession of magnetic moments around  $H_{eff}$ . Integration over  $(0, \pi/2)$  of Eq.(2) gives the threshold current ( $J_{th}$ ) as a function of pulse width where condition on  $\phi$  plays a very important role which is derived from stability condition [9]. For the DLT-only case,  $m_y$  component is approximately zero ( $\phi \sim 0$ ) as switching in  $m_z$  is defined by an abrupt change in  $m_y$  [10] shown in Fig. 2 (star mark). But, FLT induced switching includes the contribution of non-zero  $m_y$  ( $\phi = \tan^{-1}(\beta)$ ) due to the effective field-like transverse field along the  $y$  direction.

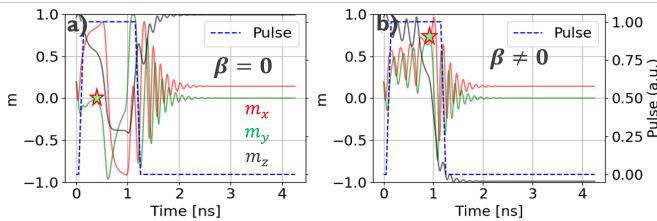


Fig. 2. Comparison between switching dynamics in a) absence and b) presence of SOT field like torque. Condition on  $\phi$  is marked as star in  $m_y$  component.

## B. Stochastic Framework

1) *sLLG Model Description*: Conventional stochastic LLG formalism is used to quantify the stochastic nature of switching and its corresponding error rate by adding thermal field along with  $H_{eff}$  in Eq.(1) :

$$H'_{eff} = H_{eff} + H_{th}(t). \quad (4)$$

The thermal field is modeled as gaussian white noise which satisfies the following conditions :

$$\langle H_{th,i}(t) \rangle = 0, \quad (5)$$

$$\langle H_{th,i}(t) H_{th,j}(t') \rangle = \frac{2\alpha K_B T}{\gamma \mu_0 M_s V} \delta_{ij} \delta(t - t'). \quad (6)$$

Here,  $K_B$ ,  $T$  and  $V$  are Boltzmann constant, temperature and volume of magnetic free layer of MTJ respectively.

2) *2D Fokker Planck Model Description*: This Model is based on evolution of probability density function ( $\rho$ ) on the unit sphere which satisfies the normalization condition.

$$\int_0^{2\pi} \int_0^\pi \rho(\theta, \phi, t) \sin \theta d\theta d\phi = 1. \quad (7)$$

The FP equation is defined in conservative form:

$$\frac{\partial \rho}{\partial t} + \nabla_s \mathbf{J} = 0, \quad (8)$$

where,  $\nabla_s$  is the spherical divergence operator over surface and  $\mathbf{J}$  is the probabilistic flux.

$$\mathbf{J} = \mathbf{v}\rho - D\nabla_s \rho. \quad (9)$$

Here,  $\mathbf{v}(\theta, \phi)$  represents the drift equations defined in Eq.(2) and Eq.(3) and  $D$  is diffusion parameter.

$$D = \frac{\alpha \gamma k_B T}{(1 + \alpha^2) \mu_0 M_s V}. \quad (10)$$

The necessity of 2D FP model compared to existing 1D model is to capture the two stage switching mechanism, that follows an out-of-plane  $\rightarrow$  in-plane  $\rightarrow$  out-of-plane path where the intermediate in-plane state is governed by the alignment of moments along  $\sigma_y$  which maximizes the  $m_y$  component during pulse application.

We solve the FP equation using the FVM method instead of FDM or FEM schemes [11]. Its flux-conserving nature and good long-time stability makes it suitable for achieving more reliable values for WER in SOT devices. It is estimated using the following formulation :

$$WER = 1 - P_{sw}(t), \quad (11)$$

where,  $P_{sw}(t)$  is the switching probability defined as :

$$P_{sw}(t) = \int_0^{2\pi} \int_{\pi/2}^\pi \rho(\theta, \phi, t) \sin \theta d\theta d\phi \quad (12)$$

Here, switching happens from  $m_z = +1$  to  $m_z = -1$ . This model represents the mapping of large number of trajectories into probability density profile and validated against conventional sLLG solver.

TABLE I  
DEFAULT SOT PARAMETERS.

| Parameter                                         | Value                            |
|---------------------------------------------------|----------------------------------|
| Saturated magnetization( $M_s$ )                  | $1 \times 10^6$ A/m              |
| Gilbert damping const.( $\alpha$ )                | 0.1                              |
| Magnetic anisotropy field ( $H_{k,eff}$ )         | 0.26 T                           |
| Diameter                                          | 80 nm                            |
| Temperature (T)                                   | 300K                             |
| Free layer thickness( $t_{FM}$ )                  | 1 nm                             |
| SOT channel dimension( $W_{SOT} \times t_{SOT}$ ) | $110 \times 3.5$ nm <sup>2</sup> |

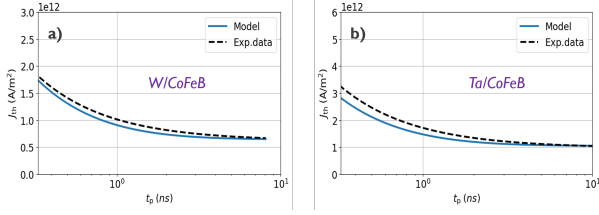


Fig. 3. Analytical Model of threshold current density ( $J_{th}$ ) versus pulse width ( $t_p$ ) is validated against experimental data [12] for two different heavy metals a) tungsten (W) and tantalum (Ta). Model shows good agreement for both cases.

### III. RESULT

The analytical threshold-current model is validated against experimental data using the parameters mentioned in table I [12] for two SOT tracks, W ( $\beta = 0.3$ ,  $\theta_{SH} = -0.3$ ) and Ta ( $\beta = 1.05$ ,  $\theta_{SH} = -0.11$ ). The model accurately reproduces the dependence of threshold current on pulse width for both material systems Fig. 3. This model demonstrates the significant role which FLT can play in the switching of real SOT-MRAM devices.

The 2D FP model is benchmarked against the s-LLG solver simulating for  $10^5$  trajectories through three complementary comparisons. First, the evolution of the PDF is examined to confirm that the stochastic switching dynamics are correctly captured. Here, peak of PDF is shifted from  $\theta = 0$  to  $\theta = \pi$  with the passage of time, implying the switching of  $m_z$  component from +1 to -1 (Fig. 4).

Second, the WER is evaluated as a function of time at a fixed applied current and in-plane field to assess the dynamic error behavior depicting both pulse on and off regime (Fig. 4). All the trajectories are tend to align in-plane direction showing a prominent peak of PDF near  $\theta = \pi/2$  as long as pulse is on. In relaxed regime, diffusion becomes more dominant where some of the trajectories switch back to the initial state.

Third, the switching probability is analyzed as a function of current to validate the predicted transition from low to high regime across the entire operating range (Fig. 5) for two different heavy metal layer W ( $\beta = 0.3$ ) and Ta ( $\beta = 1.05$ ). Switching probability is reliable for W after reaching  $P_{sw} = 1$ . But for Ta one, probability is degrading slightly over the range of applied current and not able to achieve maximum value. This behavior is expected because of having high field-like parameter as reported earlier [13]. Here the probability is calculated following a 2 ns relaxation period.

The model shows good quantitative agreement with the s-LLG solver for two different heavy metal W and Ta, highlighting the contribution of field like parameter on switching reliability. In addition, threshold current as function of pulse width is extracted from FP solver and s-LLG solver at  $P_{sw} = 0.5$  criterion. FP solver exhibits good qualitative agreement with s-LLG (Fig. 6), producing the same trend as depicted in (Fig. 3)

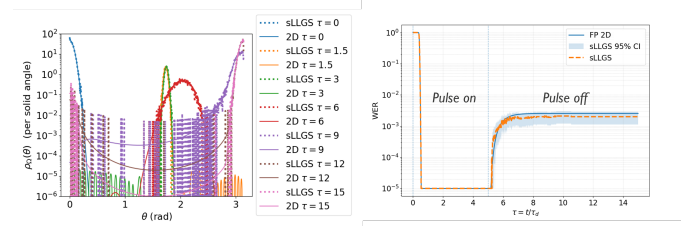


Fig. 4. Validation of Fokker-Planck model against s-LLG solver at a current of  $442 \mu A$  and  $42 \text{ mT}$  of in-plane field. a) Evolution of probability density distribution  $\rho(\theta)$  shows excellent agreement whereas peaks are getting shifted from  $\theta = 0$  to  $\pi$  over time which proves successful switching. b) Write Error Rate is in good agreement for both pulse and relaxed regime. Time is shown in normalized units ( $\tau = t/\tau_d$ ) where  $\tau_d = (1 + \alpha^2)/\alpha\gamma H_{k,eff}$ .

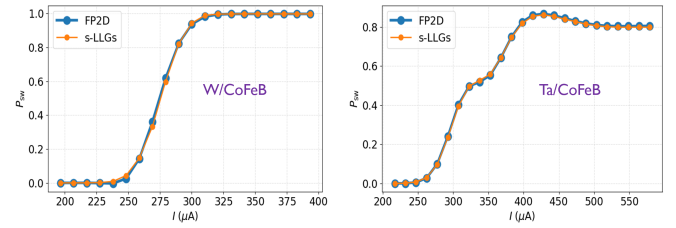


Fig. 5. Comparison of switching probability as function of applied current for W and Ta under  $40 \text{ mT}$  in-plane field with a  $1 \text{ ns}$  pulse followed by  $2 \text{ ns}$  relaxation period, showing good agreement across low and high current regimes. Switching probability is more reliable for W than Ta due to its smaller field like parameter.

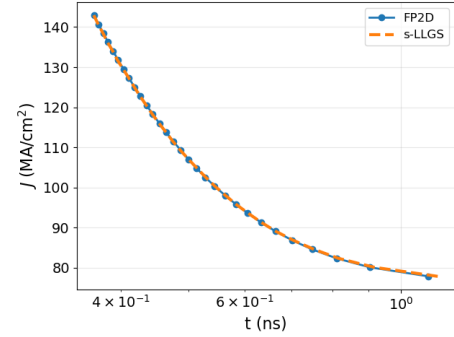


Fig. 6. Threshold current is evaluated as function of pulse width from  $P_{0.5}$  level for W/CoFeB stack, exhibits good agreement with s-LLG solver.

### IV. CONCLUSION

We developed analytical models for both deterministic and stochastic SOT switching that explicitly incorporate the field-like torque in addition to the damping-like torque. The deterministic framework incorporates the FLT-dependent azimuthal condition to accurately predict the threshold current as a function of pulse width showing good agreement with experimental data, while the stochastic framework introduces a 2D Fokker-Planck model having a unique advantage to track down the density evolution compared to  $10^5$  trajectories to estimate switching-error metrics using finite volume method

for better numerical stability. The model demonstrates good consistency with the stochastic LLG solver. Both models are validated for two different heavy metal W and Ta to assess the impact of field like parameter on the switching dynamics. Moreover, threshold current corresponding to  $P_{0.5}$  is extracted as function of pulse width shows the expected trend ( $\sim 1/t$ ) and consistent with s-LLG solver. Collectively, these models provide a robust foundation for an accurate and circuit-friendly compact model of SOT-MRAM.

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