

Revisiting Random Dopant Fluctuation at Cryogenic Temperature: the Effect of Incomplete Ionization

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Abstract—The classical Pelgrom law underestimates cryogenic V_t variability because it neglects the ionization ratio fluctuation (IRF) arising from incomplete ionization. Using calibrated 3D Statistical Impedance Field Method (SIFM) simulations, we verified this mechanism and developed an analytical σ_{V_t} model that incorporates ionization-ratio variance. The model predicts a stronger random dopant fluctuation (RDF) at low temperatures and matches the SIFM results from 300 K to 77 K, providing a physics-based, predictive Pelgrom-type law for cryogenic CMOS.

Index Terms—Cryogenic CMOS, Random Dopant Fluctuation (RDF), Pelgrom law, Incomplete Ionization, TCAD simulation

I. INTRODUCTION

Cryogenic CMOS is essential for quantum computing and deep-space electronics, offering improved mobility, near-ideal subthreshold swing, and reduced leakage at low temperatures [1]. However, as devices scale down, random dopant fluctuation (RDF) poses a critical variability challenge. Recent measurements show that the RDF-induced threshold voltage variation (σ_{V_t}) increases sharply upon cooling, directly contradicting the conventional Pelgrom prediction of a decrease [2]–[4]. This behavior severely limits cryogenic circuit design margins and calls for a physical re-examination of RDF at low temperatures.

The classic Pelgrom law for RDF, which assumes complete dopant ionization, forecasts a slight reduction in σ_{V_t} as temperature decreases. At cryogenic temperatures, however, dopants are only partially ionized (incomplete ionization), rendering this framework invalid. The absence of a predictive model that captures the experimentally observed rise in σ_{V_t} leaves a critical gap in cryogenic design methodologies and motivates a physically grounded re-assessment of RDF. In this work, we revisit RDF at cryogenic temperatures to uncover the physical origin of this variability increase. Rather than

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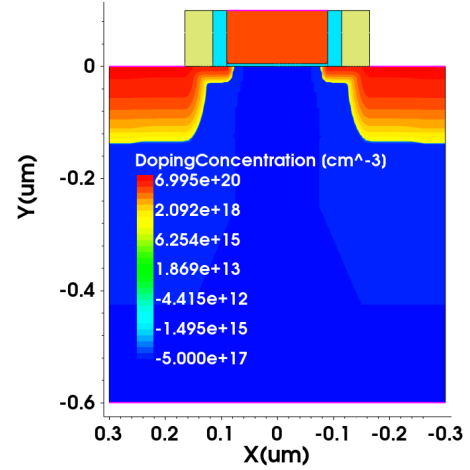


Fig. 1. Cross-section of the simulated 0.18 μm NMOS.

relying on exhaustive and time-consuming statistical cryogenic measurements, we employ calibrated 3D TCAD simulations integrated with the Statistical Impedance Field Method (SIFM). Since the SIFM incorporates random dopant positions without any tunable parameters, its variability predictions become intrinsically reliable once the baseline I–V simulation is rigorously calibrated [5]. Our analysis reveals that incomplete ionization at low temperature introduces a previously unrecognized fluctuation mechanism: ionization ratio fluctuation, which emerges as the primary driver of the degraded σ_{V_t} . Building on this insight, we derive a modified Pelgrom law for RDF-induced σ_{V_t} that explicitly captures the statistics of temperature-dependent incomplete ionization. The proposed model is validated against SIFM results across a wide range of temperatures and device geometries, demonstrating excellent agreement and establishing a predictive foundation for cryogenic variability assessment.

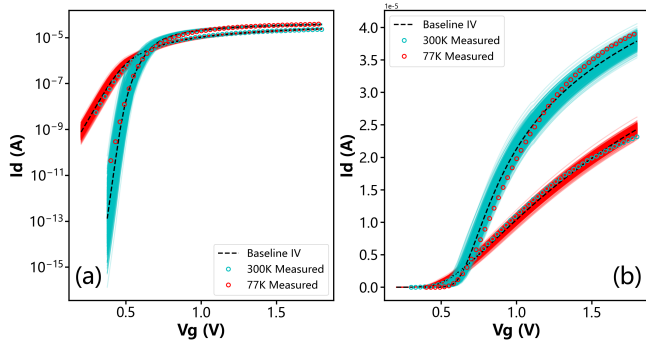


Fig. 2. Calibrated TCAD baseline models for NMOS I_d-V_g ($V_{ds} = 0.05V$), experimental data (circles), simulated baseline IV curves (dash line) and SIFM simulation data (band of curves): (a) Log scale; (b) Linear scale.

II. TCAD SIMULATION OF VARIABILITY

In this work, Sentaurus TCAD is used for device simulation. The transfer characteristics (I_d-V_g) were measured on a device with $W/L = 0.22 \mu\text{m}/0.18 \mu\text{m}$ at 300 K and 77 K, with a drain voltage of $V_D = 0.05 V$. Fig. 1 shows the simulated device structure. The gate oxide thickness is 5.2 nm.

To obtain accurate simulation results, we apply quasi-stationary simulation with extrapolation is used. For good convergence and numerical stability, an iterative linear solver and 80-bit extended precision is used with the minimum carrier density set to 10^{-200}cm^{-3} .

In the simulations, Fermi-Dirac statistics and the incomplete ionization model is turned on. For the carrier mobility, we employ the modified Klaassen model previously refined to capture low-temperature Coulomb scattering [6], combined with the electric-field-dependent Lombardi model. Because the transfer characteristics are simulated at a low drain voltage, velocity saturation is not activated.

To calibrate the measured $I-V$ curves, we first assign $1 \times 10^{13} \text{cm}^{-2} \text{eV}^{-1}$ donor-type interface traps with an exponential energy distribution centered at the conduction band edge and a characteristic energy of 30 mV. With this setting, the threshold voltage and subthreshold swing of the baseline $I-V$ curves are well matched. Subsequently, to fit the ON-state current, we calibrate acoustic phonon scattering and surface roughness scattering in the Lombardi model [7].

We compute the variation of 1000 samples by RDF using SIFM. Fig. 2 and Fig. 3 show SIFM-generated $I_d - V_g$ curves (log/linear) and V_t distributions at 300K and 77K. The σ_{V_t} is clearly degraded from 10.7mV to 13.1mV at 77K, underscoring that SIFM captures additional variability mechanisms at low temperature. For reference, σ_{V_t} predicted by the classical Pelgrom law decreases with temperature from 10.7mV to 9.84 mV, as also shown in Fig. 3. V_t is extracted by the Y-function ($I_d/\sqrt{g_m}$) method to preserve statistical randomness.

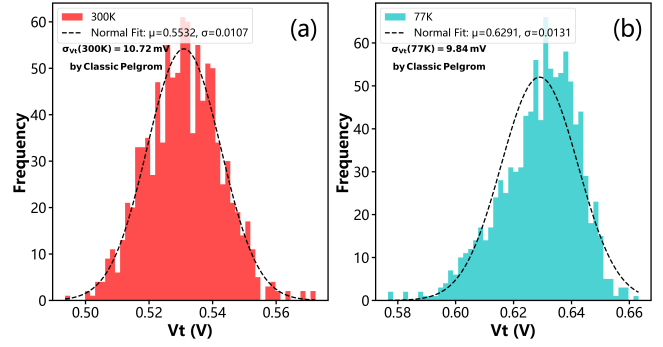


Fig. 3. V_t distribuion extracted from SIFM simulation: (a) 300K ; (b) 77K.

III. CRYOGENIC RDF MODELING

In p-doped silicon, the acceptor ionization ratio is

$$f_A = \frac{1}{1 + g_A \exp\left(\frac{E_A - E_F}{kT}\right)} \quad (1)$$

where E_A is the acceptor energy level. As temperature decreases, the Fermi level (E_F) moves toward the valence band. Near the surface, E_F lies above E_A , so $f_A \approx 1$ (full ionization); deeper into the bulk, however, E_F drops below E_A , leading to partial ionization and $f_A < 1$. As indicated by Eq. (1), with the transition from full to partial ionization f_A begins to drop appreciably [8].

To investigate the impact of RDF on the incomplete ionization of acceptors at cryogenic temperatures, we perform statistical ensemble simulations using the calibrated 3D TCAD framework. A set of 30 device samples with statistically independent random dopant distributions is generated by the cloud-in-cell (CIC) method, while keeping all other structural parameters identical to the nominal device [9]. The CIC method generates local doping concentrations by assigning to each grid cell a random number of dopant atoms drawn from a Poisson distribution. One example is presented in Fig. 4. For each sample, the transfer characteristics are simulated

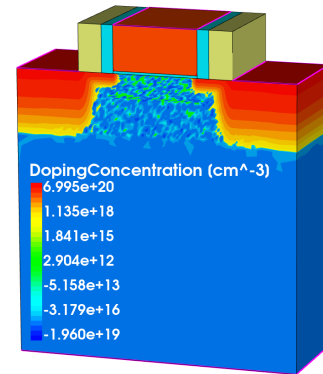


Fig. 4. A device sample with atomistic RDF generated via the cloud-in-cell method.

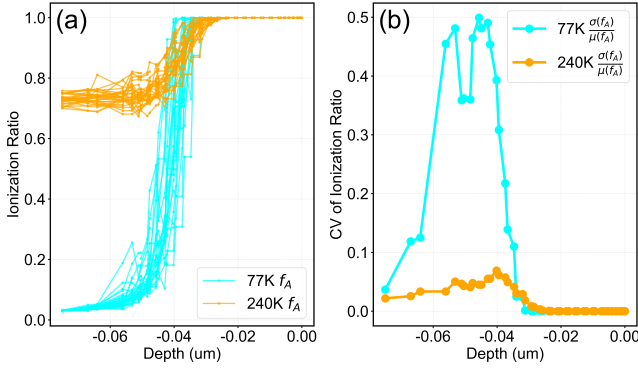


Fig. 5. Ionization ratio statistics from 30 samples: (a) Distribution along channel; (b) Coefficient of variation (CV).

at low temperatures under the same bias conditions as in the calibration, and the local acceptor ionization ratio f_A is extracted from the converged solution. This atomistic approach allows us to directly assess the variability of f_A , beyond the conventional fluctuations in the number of dopant atoms.

Statistical ensemble simulations of 30 RDF samples using the calibrated 3D TCAD reveal a clear trend in the acceptor ionization ratio, as shown in Fig. 5(a). Since the impurity activation energy depends on the local dopant concentration via $E_A = E_{A0} - \alpha N_A^{1/3}$, any fluctuation in N_A not only alters the number of dopants but also modulates the ionization ratio itself. This coupling gives rise to an additional source of fluctuation important in the cryogenic regime. The spatial distribution of the coefficient of variation of f_A (defined as σ_{f_A}/μ_{f_A} , i.e., the ratio of standard deviation to mean, which directly quantifies the relative fluctuation of the ionization ratio) presented in Fig. 5(b) confirms the presence of this ionization ratio fluctuation. Analytically, a fluctuation $\delta N_A(y)$ in doping concentration induces a change in ionized concentration $\delta N_A^-(y)$ given by

$$\delta N_A^-(y) = [f_A + N_A(y) \frac{\partial f_A}{\partial N_A}] \delta N_A(y) \equiv \gamma(y) \delta N_A(y) \quad (2)$$

Here, $\gamma(y)$ is the local ionization fluctuation response factor that represents the relation between a dopant fluctuation and an ionized charge fluctuation; owing to the concentration-dependent activation energy, $\gamma(y)$ exceeds the bare ionization ratio f_A at cryogenic temperatures.

$$\gamma(y) = f_A(y) + \frac{f_A(y)(1 - f_A(y))\alpha N_A^{1/3}}{3kT} \quad (3)$$

So, applying the weighting integral in Ref. [10], where y is the depth from the oxide/channel interface and $W_{d,ii}$ the depletion width under incomplete ionization—the resulting variance of V_t due to RDF under incomplete ionization is given by

$$\sigma_{V_t}^2 = \frac{q^2}{C_{ox}^2 WL} \int_0^{W_{d,ii}} (1 - \frac{y}{W_{d,ii}})^2 \gamma(y) N_A(y) dy \quad (4)$$

Assuming uniform doping and a constant ionization ratio which is defined as $\tilde{f}_A = (W_d/W_{d,ii})^2$. Substituting into Eq. (4) gives

$$\sigma_{V_t} = \sqrt{\frac{q^2 \gamma N_A W_{d,ii}}{3C_{ox}^2 WL}} = \frac{A'_{VT}}{\sqrt{WL}} \quad (5)$$

From Eq. (5), we can see the variation of V_t by RDF still follow the Pelgrom law at cryogenic temperature. And its coefficient should be modified as

$$A'_{VT} = \sqrt{\frac{\gamma/\tilde{f}_A}{W_d/W_{d,ii}}} \cdot \sqrt{\frac{q^2 \tilde{f}_A N_A W_d}{3C_{ox}^2}} = \sqrt{\frac{\gamma/\tilde{f}_A}{\tilde{f}_A^{1/2}}} \cdot A_{VT} \quad (6)$$

This expression separates the cryogenic V_t variability into the classical Pelgrom coefficient A_{VT} and a low temperature correction factor. The numerator captures the additional variability contributed by ionization ratio fluctuation (IRF), which is absent in the classical treatment.

$$\gamma/\tilde{f}_A = 1 + \frac{(1 - \tilde{f}_A)\alpha N_A^{1/3}}{3kT} \quad (7)$$

The denominator $\tilde{f}_A^{1/2}$ in Eq. (6) accounts for the well-known depletion-layer broadening at low temperatures, which brings

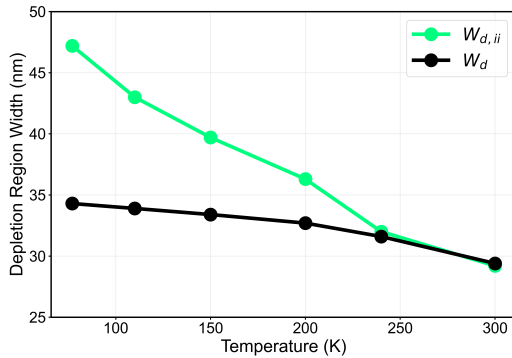


Fig. 6. Temperature dependence of the depletion region width.

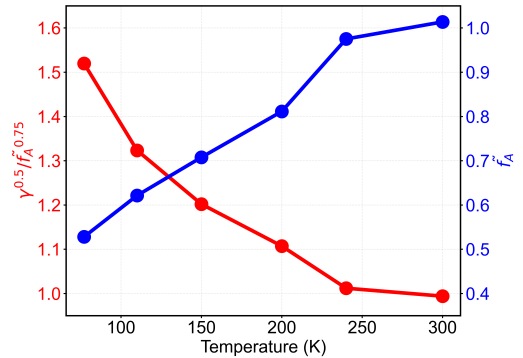


Fig. 7. Temperature dependence of A_{VT} correction factor and ionization ratio.

more ionized dopant fluctuations into the weighting integral for V_t and magnifies the impact of IRF. As illustrated in Fig. 6 and Fig. 7, both effects become more pronounced with decreasing temperature, accounting for the observed increase in A'_{VT} .

IV. MODEL VALIDATION

To validate the proposed model, we compare its predictions against our SIFM simulation results. As shown in Fig. 8, The SIFM data exhibit a pronounced increase in variability as temperature decreases, a trend that the classical Pelgrom law fails to reproduce. In contrast, the proposed model, which incorporates IRF, closely tracks the SIFM results. Minor discrepancies between the model and SIFM data arise primarily from the assumption of a constant ionization ratio f_A , which neglects its spatial variation within the depletion layer. Fig. 9 presents the geometry dependence at 300K and 77K. The SIFM results exhibit a larger coefficient A'_{VT} , which implies that the Pelgrom scaling relation steepens with decreasing temperature. Our model accurately predicts this slope, demonstrating that the correction factor derived in Eq. (6) properly accounts for the incomplete-ionization effects across different device dimensions.

V. CONCLUSIONS

We have developed a physics-based analytical model for cryogenic RDF σ_{V_t} that corrects the classical Pelgrom law's underestimation. Using an ensemble of 30 RDF samples generated via the cloud-in-cell method, we extract the local acceptor ionization ratio and reveal ionization ratio fluctuation (IRF) as a new source of variability. Combined with the broadened depletion layer inherent to incomplete ionization, this mechanism explains the increased RDF σ_{V_t} . The model takes the form of Pelgrom Law with more complex coefficient that depends on ionization ratio. The model is validated against 3D SIFM simulations from 300K to 77K and provides a predictive tool for variability-aware cryogenic CMOS design.

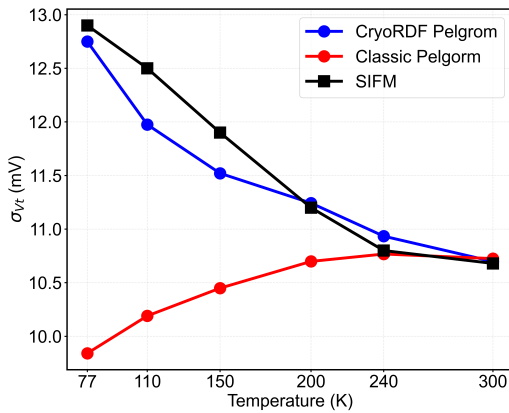


Fig. 8. Comparison of σ_{V_t} : temperature dependence of NMOS baseline models.

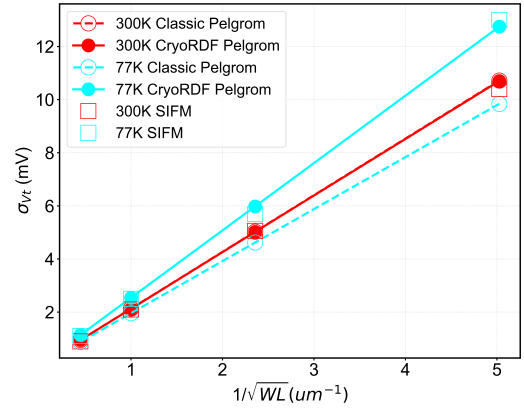


Fig. 9. Comparison of σ_{V_t} : Pelgrom geometry dependence.

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