Improved stochastic SPAD quenching model including build-up field effect

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***Abstract*— We present an improved stochastic model for simulating avalanche quenching in Single-Photon Avalanche Diodes (SPADs), incorporating the build-up field effect caused by space-charge accumulation during avalanche events. Built upon a one-dimensional Monte Carlo framework, our model introduces a correction term that accounts for the transient electric field induced by the charge dipole, derived from self-consistent Advection-Diffusion Monte Carlo (ADMC) simulations.**

This hybrid approach preserves the computational efficiency of fast stochastic methods while capturing key physical effects often neglected in simplified models. The inclusion of the build-up field is shown to impact significantly the simulated voltage and current transients, enabling more accurate prediction of quenching behavior, especially in cases of delayed or failed quenching.

Comparison with full ADMC simulations shows strong agreement, validating the accuracy and relevance of the correction scheme. The proposed model provides a practical and scalable solution for SPAD simulation and design, particularly in applications requiring precise timing and high-frequency operation.

***Index Terms*— Single-Photon Avalanche Diodes (SPADs), Monte Carlo simulations, avalanche quenching, space-charge effects, device simulation, semiconductor devices.**

I. Introduction

Single-Photon Avalanche Diodes (SPADs) are semiconductor devices designed to detect individual photons with exceptional temporal precision. Operating in Geiger mode, SPADs are essentially p-n diodes reverse-biased above their breakdown voltage, allowing a single photon-induced carrier to initiate a self-sustaining avalanche multiplication process detected by a readout circuit [1].

A crucial aspect of SPAD operation is quenching, which terminates the avalanche current after photon detection, allowing the device to reset for subsequent events. Accurate modeling of the quenching process is essential for optimizing SPAD performance. Conventional methods, such as circuit-level SPICE models, reproduce electrical behavior but lack detailed physical insights [2]. More advanced approaches like Boltzmann-based Monte Carlo simulations model carrier dynamics and electrostatics during avalanche multiplication but significantly increase computational complexity [3], [4].

However, many existing models neglect the space-charge field generated during an avalanche [5], leading to inaccuracies in predicting quenching behavior and timing characteristics [6].

In this work, we introduce an improved stochastic simulation model for SPAD quenching. Building upon a one-dimensional particle-based Monte Carlo framework, we incorporate a correction for the electric field induced by the avalanche-induced charge dipole. This enhancement yields a more accurate representation of transient electric field dynamics, improving predictions of SPAD quenching behavior and performance metrics.

II. Methodology

In this study, we employed a combined modeling approach utilizing two distinct Monte Carlo simulation frameworks to accurately simulate avalanche quenching in Single-Photon Avalanche Diodes (SPADs). The primary goal is to retain computational efficiency while incorporating crucial physical effects such as space-charge build-up within the multiplication region (MR). Fig. 1 illustrates the SPAD structure and the quenching mechanism, where electrons and holes are generated in the MR and drift towards the anode and cathode, respectively, at their respective saturation drift velocities.

The first model, based on the work by Inoue [7], provides a fast and efficient stochastic Monte Carlo simulation of quenching mechanisms in SPADs. This model simulates carrier dynamics by tracking random impact ionization events and distinguishes between two quenching mechanisms: successful quenching (SQ), characterized by rapid quenching following the initial avalanche multiplication pulse, and unsuccessful quenching (UQ), identified by prolonged avalanche pulses due to stochastic fluctuations around equilibrium carrier densities. Despite its computational efficiency, this approach neglects the space-charge effects induced by the carriers generated during avalanche multiplication.

To overcome this limitation, we integrated insights from a second, self-consistent Advection Diffusion Monte Carlo (ADMC) approach detailed in our previous work [8]–[10]. This advanced method explicitly calculates the electric field generated by space-charge accumulation during the avalanche process. Carrier positions are tracked in the MR, and their contributions to the local charge density are computed iteratively. The resulting charge distribution is then used to solve Poisson's equation self-consistently, providing an accurate, dynamic representation of the transient electric field. The core equation that describes charges transport in the multiplication region is given by the Euler scheme for the advection-diffusion equation:

$$X_{n+1} = X_n + \mathbf{v}(X_n)\Delta t + \sqrt{2D(X_n)\Delta t}\,\xi_n. \quad (1)$$

Where X_n is the position of the carrier at time step n, $\mathbf{v}(X_n)$ is the drift velocity at that position, $D(X_n)$ is the diffusion coefficient, and Δt is the time step. The term ξ_n represents a Gaussian random variable with zero mean and unit variance, which introduces stochasticity into the carrier motion. Then the charge density ρ is computed over the device mesh and the electric field is updated by solving Poisson's equation with a finite element method (FEM) approach, using the new charge density as an input.

In our combined methodology, we leverage the computational speed of Inoue's model while incorporating space-charge effects via a parametrization derived from the self-consistent ADMC simulations. Specifically, during the fast Monte Carlo simulations, we compute a global carrier density within the MR at each time step. This density serves as an input to dynamically modify the electric field according to a predetermined correction factor derived from the self-consistent Monte Carlo simulations. Thus, the modified electric field reflects the transient influence of space-charge accumulation without impacting computational performance.

At each time step, we compute an effective voltage V_{eff} that accounts for the space-charge effect as follows:

$$V_{eff}(t) = V(t) - \sigma_{SC} \cdot \rho(t) \quad (2)$$

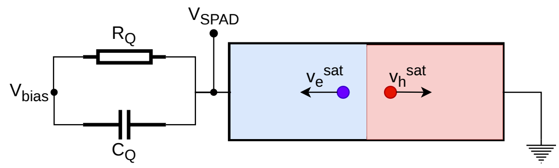

Fig. 1. SPAD structure and quenching mechanism. Electrons and holes are generated in the multiplication region (MR) and drift towards the anode and cathode, respectively, at their respective saturation drift velocities. The multiplication process is modeled by a stochastic process where the carriers can generate new electron-hole pairs according to the impact ionization coefficient.

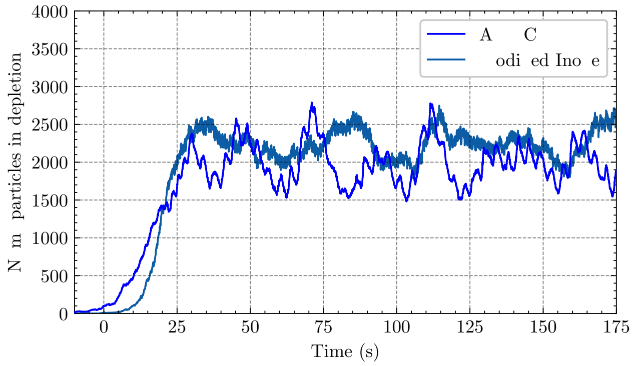

Fig. 2. Number of carriers in the multiplication region (MR) as a function of time during an avalanche without a quench circuit. The history effect enables the model to capture the oscillatory behavior of the space-charge effect [7].

where $V(t)$ is the voltage at time t, σ_{SC} is a correction factor derived from the self-consistent simulations, and $\rho(t)$ is the carrier density at time t. The correction factor is an empirical parameter that can be adjusted based on the specific device characteristics and the results of the self-consistent simulations.

As shown in Fig. 2, an oscillatory behavior of the space charge effect may be observed, due to the fact that generated carriers first induce a field drop where they are generated and are then drifted away, which leads to a reincrease in the field, and the process repeats. To capture this oscillatory behavior, we included the history of the drop as follows

$$V_{eff}(t+\Delta t) = \alpha \cdot V_{eff}(t) + (1-\alpha) \cdot (V(t) - \sigma_{SC} \cdot \rho(t)) \quad (3)$$

where α is a parameter that controls the history effect.

III. Results and Discussion

Fig. 3 illustrates typical voltage curves, $V(t)$, observed in SPAD operation. Three scenarios are depicted: no avalanche occurrence, a successfully quenched avalanche, and a failed quench. These results underscore the stochastic nature of the quenching process and the importance of accurately modeling

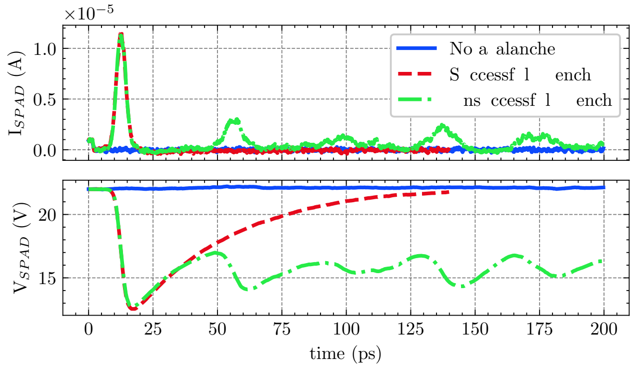

Fig. 3. Voltage and current evolution during three different types of simulation outcomes: no avalanche, single avalanche with successful quenching, and failed quenching.

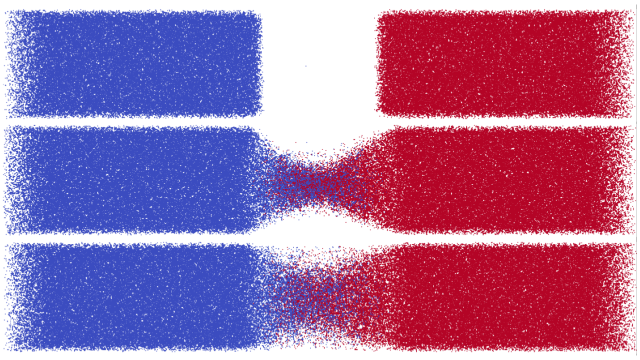

Fig. 4. Avalanche in a SPAD at three different times: when the particle is injected, after 20 ps, and after 100 ps. Electrons and holes are shown in blue and red, respectively. Simulated with the Self-Consistent ADMC simulation.

carrier dynamics.

Fig. 4 provides a visual representation of a three-dimensional Monte Carlo simulation of an avalanche event within the SPAD junction. The visualization clearly shows the significant accumulation of carriers, which alters the local electrostatics as it can be seen in Fig. 5.

The effect of incorporating the build-up field (space-charge) effect into the avalanche quenching simulations is explicitly demonstrated in Fig. 6. Here, we compare current ($I(t)$) and voltage ($V(t)$) transients generated using the self-consistent Advection-Diffusion Monte Carlo (ADMC) method. Including the build-up field effect notably affects both current and voltage waveforms, highlighting the importance of accounting for this phenomenon.

Further, Fig. 7 depicts voltage transients obtained from the Inoue stochastic Monte Carlo model with and without the proposed correction factor derived from the self-consistent approach. The corrected model more accurately reflects the build-up field effect, demonstrating an improved representation of the transient dynamics of the avalanche and subsequent quenching.

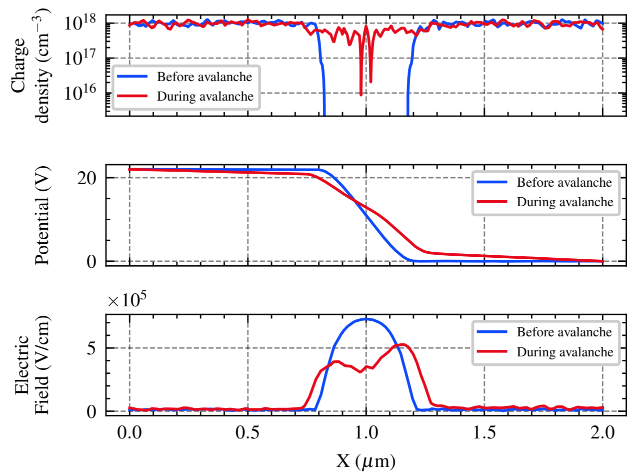

Fig. 5. Charge density (top), electrostatic potential (middle), and electric field (bottom) in the device at two different times : t=0 ps and t=30 ps (during the avalanche process) made with the Self-Consistent ADMC simulation. The electrostatics during the avalanche process is significantly affected by the charge density.

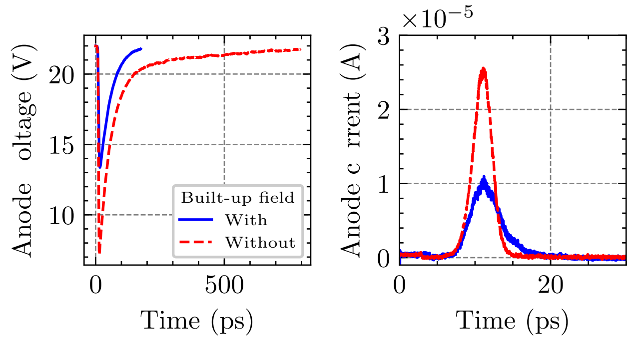

Fig. 6. $V(t)$ and $I(t)$ extracted from ADMC simulation with and without the build-up field effect. The build-up field effect can be removed from the ADMC simulations by giving a zero Poisson weight to particles generated during the avalanche.

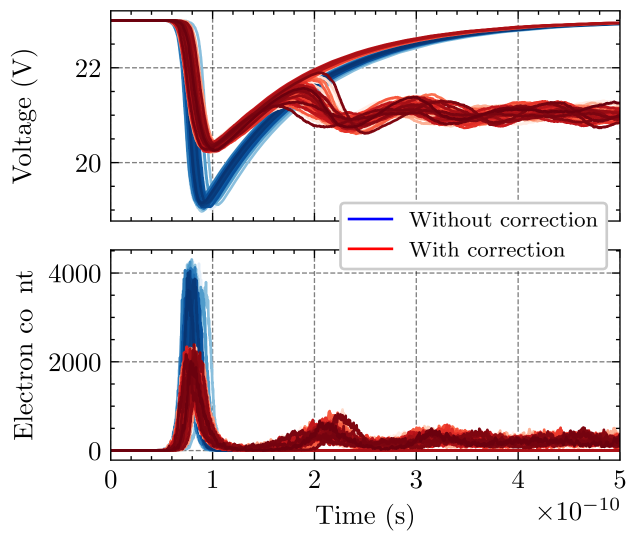

Fig. 7. Effect of the correction factor on the temporal response of the SPAD. As desired, the correction factor limits the current, hence the voltage drop. This has the effect of making quenching less likely to occur.

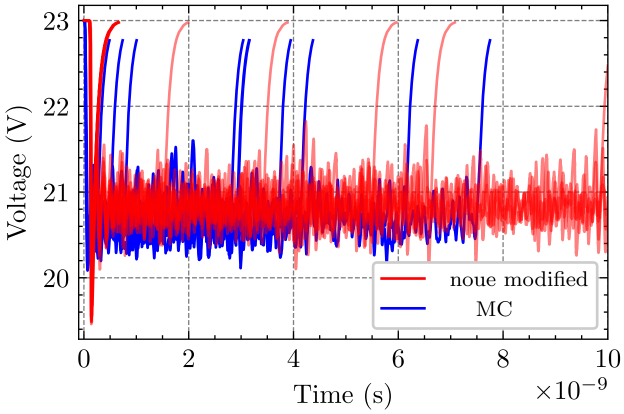

Fig. 8. Voltage transients obtained from the fast Inoue Monte Carlo model with the correction factor and the computationally intensive Self-Consistent ADMC simulations. The modified Inoue model is able to reproduce the voltage transient waveforms obtained from the ADMC simulations, in particular the late quenching events.

Finally, Fig. 8 presents a direct comparison between voltage waveforms obtained from the fast Inoue Monte Carlo model with the implemented correction factor and the computationally intensive ADMC simulations. The excellent agreement between the two methodologies validates the effectiveness of our hybrid approach. This demonstrates that our corrected Inoue-based model successfully captures the build-up field effect with significantly reduced computational overhead, offering a robust and efficient tool for SPAD quenching simulations.

IV. Conclusion

We have presented an improved stochastic quenching model for Single-Photon Avalanche Diodes (SPADs) that incorporates the build-up electric field effect caused by space-charge accumulation during avalanche events. By combining a fast, one-dimensional stochastic Monte Carlo framework with a correction factor derived from self-consistent ADMC simulations, our approach successfully captures the transient electrostatic effects that influence avalanche dynamics and quenching behavior.

The proposed model maintains computational efficiency while improving the physical realism of SPAD simulations. Results show strong agreement between the corrected stochastic model and full self-consistent simulations, validating the effectiveness of the correction methodology. This hybrid approach provides a practical and scalable solution for accurately modeling SPAD behavior in advanced device design and optimization, particularly in timing-critical and high-rate applications.

Future work may extend this framework to two- or three-dimensional geometries and explore temperature dependence or circuit-level integration to further enhance predictive capabilities.

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