Modeling SOT-Driven Domain Wall Motion in MTJ Switching

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***Abstract*—Spin orbit torque (SOT)-based magnetic tunneling junction (MTJ) switching is central to magnetic random-access memories (MRAMs) where additional physical effects such as domain wall (DW) motion often coexist. This work provides an analytical framework for Dzyaloshinski-Moriya-Interaction (DMI)-induced DW propagation in confined MTJ structures using the Euler-Lagrange variational approach. We highlight the key implication of varying DW length in confined MTJs and propose a pseudo-steady-state solution for estimating the DW velocity. Our model is verified by micromagnetic simulations across a wide range of parameter space and thus sets up a solid foundation towards circuit-compatible SOT-MRAM compact models.**

***Index Terms*—SOT, MRAM, MTJ, magnetic domain wall, DMI**

I. Introduction

SOT MRAMs have raised great technological interest thanks to its fast switching and improved read endurance [1]. In general SOT switching involves a variety of physical effects from broken inversion symmetry : Spin Hall effect, Rashba effect,antisymmetric exchange Dzyaloshinski-Moriya Interaction (DMI) etc at ferromagnet (FM)/Heavy metal (HM) interface. [2]. For practical SOT-MTJs with a realistic diameter, the role of DMI has been brought to increasing attention due to the commonly occurred incoherent switching via chiral Néel DW [3] motion. Unfortunately, most existing SOT-MRAM switching models so far have been macro-spin-based that solely focus on Spin-Hall effect (SHE) in coherent switching, whereas the incoherent DW motion typically only comes in computationally cumbersome micromagnetic simulations.Though the DW aware model for a fictious infinite stripe already exists but model for DW motion in confined MTJ structure needed for guiding SOT technology development is lacking.

In this paper, we present an analytical modeling framework for SOT-driven DW motion inside a circular nanomagnet in presence of DMI. We extend the collected coordinate approach to a confined MTJ highlighting the key differences compare to fictitious infinite stripe [4] and crucially introduce a “pseudo steady state” as describe in Section II and III to calculate DW velocity and corresponding switching time. We further justify the validity and theoretical value of the model by successfully capturing the switching time predicted in micromagnetic simulations as discussed in Section IV .

II. Model Description

DW motion-based incoherent switching generally consists of nucleation and propagation (Fig. 1). DW nucleation has already been studied using 1D energy landscape model [5].Here we are solely concerned with the propagation of DW present at the edge of a MTJ in the confined disc like structure.

A. Mathematical Formulation and Constraints

We model the DW propagation after the fictitious infinite track in [4], using the Lagrange variational approach and the generalized coordinates $\{q,\phi\}$ (Fig. 2; [5]), where q is the domain wall position and ϕ is the angle of in-plane magnetic moment with respect to DW normal. In general DW is tilted and as a result the DW motion is asymmetric with respect to the direction of current due to the combined effect of DMI and SOT [6]. To simplify our model here we assume that DW is straight, rigid and the profile is conserved while it propagates with respect to time, pursuant to a low DMI limit which can be tuned in SOT HM/FM systems. The domain wall profile is induced by magnetization orientation with respect to polar angle and domain wall width δ.

$$\theta(x,t) = 2\tan^{-1}[\exp\frac{x-q(t)}{\delta}] \tag{1}$$

DW dynamics is captured by deriving Equation of Motions (E.O.Ms) using lagrange formalism:

$$\frac{d}{dt}(\frac{\partial L}{\partial \dot{X}}) - \frac{\partial L}{\partial X} = -\frac{\partial F}{\partial \dot{X}} \tag{2}$$

Here X represents generalized coordinates (q,ϕ) and F is dissipative force term related to damping and SOT torque.

$$F = \frac{\alpha M_s}{2\gamma}(\frac{d}{dt}\vec{m} + \frac{\gamma H_{sh}}{\alpha}(\vec{m}\times\vec{\sigma}))^2 \tag{3}$$

Here H_{sh} is the effective SOT field and σ is spin polarization direction.

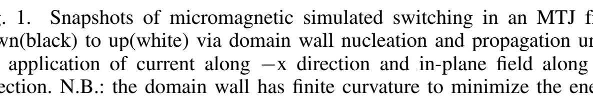

Fig. 1. Snapshots of micromagnetic simulated switching in an MTJ from down(black) to up(white) via domain wall nucleation and propagation under the application of current along −x direction and in-plane field along +x direction. N.B.: the domain wall has finite curvature to minimize the energy of the system

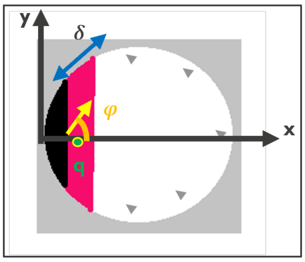

Fig. 2. Simplified representation of a domain wall in an MTJ. q is the position of DW from edge, ϕ is the angle of in-plane magnetic moment with respect to DW normal. δ is the DW width.

Lagrangian of a system is the difference between kinetic energy (T) and potential energy(U).

$$T = \frac{M_s}{\gamma}\int_{disc}\dot{\phi}(1-\cos\theta)d^3r \quad (4)$$

$$U = \int_{disc}(u_{anis}+u_{exchange}+u_{DMI}+u_{ex}+u_{DW})d^3r \quad (5)$$

Here total potential energy contribution comes from the effective anisotropy energy density u_{anis}=$K_{eff}\sin^2\theta$, the exchange energy density u_{ex}=$A_{ex}(\frac{\partial\theta}{\partial x})^2$, the effective DMI energy density u_{DMI}=$D\frac{\partial\theta}{\partial x}$, the effective energy due to applied in-plane external field u_{ex}=$\mu_0 M_s H_x \sin\theta\cos\phi$ and the in-plane effective DW anisotropy energy density u_{DW}=$\frac{\mu_0 M_s}{2}H_{DW}\sin^2\phi$, where H_{DW}=$\frac{M_s}{2}(N_y-N_x)$. N_y and N_x are demagnetizing factors defined as [7]:

$$N_y = \frac{t_{FM}}{t_{FM}+2R} \quad and \quad N_x = \frac{t_{FM}}{t_{FM}+\pi\delta} \quad (6)$$

Using the volume integration over disc $\int d^3r$=2 $t_{FM}\int_{-R}^{R}\sqrt{R^2-x^2}$ and using wall profile (1), E.O.Ms are derived which are two first order coupled differential equations of q and ϕ [8].

$$-\dot{\phi}+\alpha\frac{\dot{q}}{\delta} = -\frac{\gamma\delta}{M_s\int m_{xy}^2}\frac{\partial U}{\partial q}+\frac{\gamma H_{sh}\cos\phi\int m_{xy}}{\int m_{xy}^2} \quad (7)$$

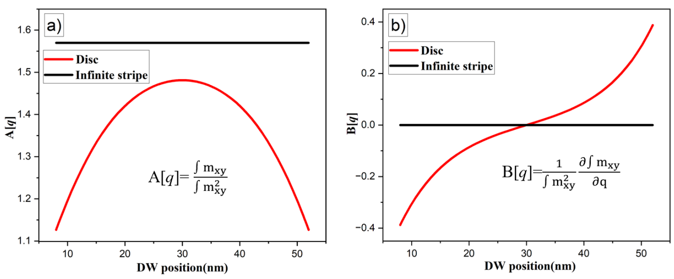

Fig. 3. Position factors a) A[q] and b) B(q] in the EOMs, with A[q] being the DW-area averaged in-plane magnetic moment and B[q] being its derivative to q. Note that in an infinite stripe both of them reduce to constants.

$$\alpha\dot{\phi}+\frac{\dot{q}}{\delta} = -\frac{\gamma}{M_s\int m_{xy}^2}(\frac{\partial U}{\partial\phi})+\frac{1}{\int m_{xy}^2}\frac{\partial\int m_{xy}(\gamma H_{sh}\sin\phi)}{\partial q} \quad (8)$$

TABLE I
DEFAULT SOT/MTJ PARAMETERS

Parameter	Value
Damping constant (α)	0.0086
Saturated Magnetization (M_s)	1.06×10^6 A/m
Uniaxial anisotropy constant (K_u)	844 kJ/m^3
Diameter of disc (2R)	60 nm
Spin hall angle (θ_{SH})	0.3
Thickness of free layer (t_{FM})	0.9 nm
In-plane field ($\mu_0 H_x$)	0.03 T

Here m_{xy} is the in-plane magnetic moment inside DW and integration of m_{xy}^2 is interpreted as DW effective area mutiplied by $2t_{FM}$.

B. Key difference from Infinite Stripe Model

Unlike existing treatment of the propagation of DW in an infinite stripe in [4], [5], the length of DW in a confined MTJ varies with its position corresponding to effective surface area A_s=$2\delta\sqrt{R^2-(R-q)^2}$.This leads to both q-dependent DW energies and q-dependent effective torque on the DW vector.

The energetic (exchange, Zeeman and anisotropy) effect is embodied in the modulating factors $\frac{\partial}{\partial q}$ in the (7) and is similar to the “stretch field” in Spin-Transfer Torque (STT)-MTJs [8], while the effective torques (DMI and SHE) scale with the effective in-plane magnetic moment $\int m_{xy}$ normalized to DW area , both of which are absent in infinite stripes that have a constant DW length.

After rearranging terms and redefining disc specific parameters (7),(8) is defined as follows:

$$-\dot{\phi}+\alpha\frac{\dot{q}}{\delta} = -\gamma(H_{stretch}-(\frac{D}{\delta}+\mu_0 M_s H_x\cos\phi)B[q]) + \gamma H_{sh}\cos\phi A[q] \quad (9)$$

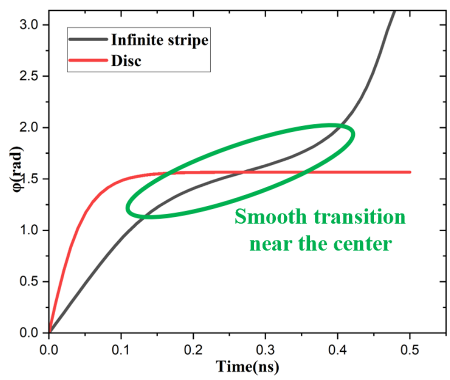

Fig. 4. Comparison between infinite stripe and confined MTJ of ϕ evolution with respect to time between wire and circular geometry. ϕ variation significantly slows down near the center of the MTJ (highlighted as green) due to DW energy minimization.

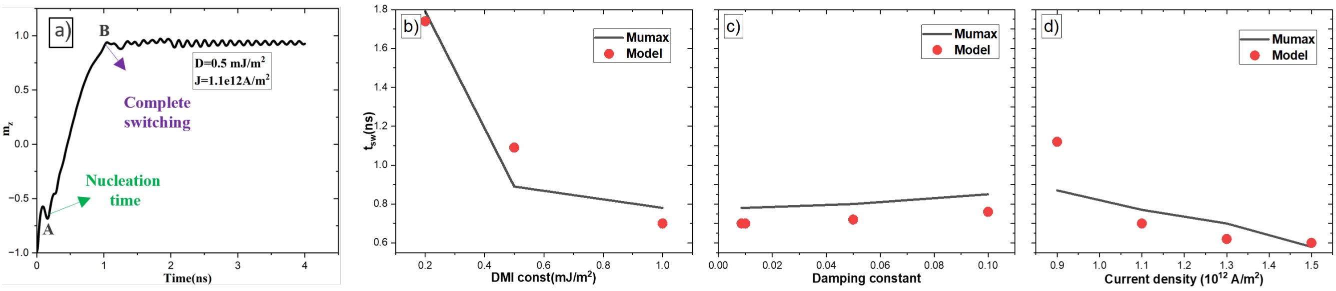

Fig. 5. a)Micromagnetic simulated dynamics of out of plane magnetization under the application of square pulse of 2 ns along with applied in-plane field $\mu_0 H_x$ = 0.03 T). The switching time (t_{sw}) is extracted by subtracting nucleation time(A) from complete switching time (B) and further used to validate with analytical model. b) Represents Comparison between micromagnetic simulation and analytical model of switching time by varying intrinsic parameters DMI constant, c) damping constant and extrinsic parameter d) applied current density.

$$\alpha\dot{\phi} + \frac{\dot{q}}{\delta} = -\gamma(-\frac{H_{DW}}{2}\sin 2\phi + (\frac{D}{\delta} + \mu_0 M_s H_x \sin\phi)A[q]) + \gamma H_{sh}\sin\phi B[q] \tag{10}$$

Here, A[q] and B[q] are DW area averaged m_{xy} and it's derivative respectively as function of q unlike the infinite stripe as shown in Fig. 3

C. Pseudo-steady state solution

Directly solving E.O.Ms can be computationally prohibitive due to it's complexity. Here we model after the "stationary approach" in [4] by finding the "pseudo-steady state" of ϕ; this is justified by the observation in the coupled solution of (7),(8) that ϕ(t) markedly slows down around the point when the DW crosses the center, i.e., is about to switch (Fig. 4). This leads to ϕ being a parameter for determining the DW velocity namely $\dot{q}$ and further for determining the switching time (t_{sw}).

III. MODEL IMPLEMENTATION

The "pseudo-steady state" is determined by solving the (9),(10) at $\dot{\phi}$ = 0 , which yields a " pseudo-steady angle" ϕ_{ps} as function of q defined as follows:

$$\cos\phi_{ps} = \frac{2\Lambda\Xi \pm \sqrt{4\Lambda^2\Xi^2 - 4(\Lambda^2 - 1)(\Xi^2 - 1)}}{2(\Xi^2 + 1)} \tag{11}$$

Here, Λ and Ξ redefine the ratio of the parameters of the effective fields as a function of q.

$$\Xi = \frac{1}{\alpha}\frac{H_{sh}A(q) + \frac{\gamma}{2}(H_x + \frac{H_D}{2})B[q]}{(H_x + \frac{H_D}{2})A[q] + \gamma\frac{H_{sh}}{2}B[q]} \tag{12}$$

and

$$\Lambda = \frac{1}{\alpha}\frac{(H_k + \frac{1}{4}H_{DW})\frac{\delta(R-q)}{q(2R-q)}}{(H_x + \frac{H_D}{2})A[q] + \gamma\frac{H_{sh}}{2}B[q]} \tag{13}$$

The corresponding position-dependent DW velocity with parameter ϕ_{ps}, i.e., $\dot{q}$(q; ϕ_{ps}(q)) is written as:

$$\dot{q} = \frac{\gamma}{\alpha}(H_{sh}A(q)\delta\cos\phi_{ps} - (H_k + \frac{1}{4}H_{DW})\frac{\delta(R-q)}{q(2R-q)} + (H_x + \frac{H_D}{2})B[q]\cos\phi_{ps}) \tag{14}$$

Here we simplify the solution by settling the q-dependency at the initial DW position q_0 in (14) reffered to fig.[2].As a result, the resultant DW velocity $\dot{q}$ is expressed as a function of effective SHE/DMI-field, applied in-plane field $\mu_0 H_x$ and damping constant α (14). This approximated DW velocity provides a first-order estimation of the switching time t_{sw} as the ratio of the diameter of the MTJ to DW velocity as we confine the DW movement along x axis.

IV. MODEL VALIDATION

We calculate the DW velocity and ϕ_{ps} (14) using the parameters mentioned in TABLE I and corresponding t_{sw} as mentioned above and validate with microspin simulation result. Switching time solely related to DW propgation is extracted by subtracting the nucleation time as shown in Fig. 5a) in simulation .The model turns out capable of matching simulation results across the typical range of DMI constants and α in experiments (Fig. 5), with t_{sw} decreasing with DMI and increasing (albeit subtly) with α. The modeled t_{sw} is additionally verified at different SOT current.

V. CONCLUSION

We studied here the incoherent switching mechanism of SOT MRAM via DW nucleation and propagation, where we solely focused on DW propagation in confined MTJ structure. Here we revisited the collective coordinates approach q,ϕ used to describe the DW profile and lagrange variational approach to find E.O.Ms to capture the DW dynamics and identified the unique effect of varying DW length. In particular, we have creatively proposed a computationally friendly "pseudo-steady-state" method to calculate the DW velocity and the corresponding domain switching time. The model is verified with micromagnetic simulation over a wide range of varying parameters (DMI and α) with additional variation of applied current. We observed that the model agrees well with the simulated result. This agreement across geometrical and material parameters proves the usefulness of the proposal, helps to fill the gap between micromagnetics and compact models for SOT optimization.

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