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On the noise-sensitivity of entangling quantum logic operations implemented with a semiconductor quantum dot platform☆

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ABSTRACT

A full-stack modeling study is conducted against three types of 2-qubit (quantum bit) entangling logic operations that are implemented with a realistically sized electrode-driven Silicon (Si) quantum dot system. Using device simulations based on the bulk physics augmented with electronic structure calculations based on the effective mass theory, we computationally explore the design of a fast SWAP$^{1/2}$ (S$^{1/2}$), Controlled-Z (CZ), and Controlled-X (CNOT) gate, and particularly study their operational sensitivity to the charge noise that is fundamentally not easy to be removed from semiconductors. Our results indicate that a recently reported one-step CNOT logic that is implemented with a single microwave pulse, is much more sensitive to the noise than S$^{1/2}$ & CZ gate are, recommending basic entangling blocks that would be desirable for designs of quantum circuits based on a Si quantum dot platform.

1. Introduction

Electron spins in isotopically purified Silicon (Si) are promising for designs of gate-based quantum circuits due to their long coherence time [1]. Electrode-driven Si quantum dot (QD) systems has obtained huge attention for realization of electron spin quantum bits (qubits) as they can be fabricated with classical control hardware using industry-standard processes. Much effort has been particularly put to implement entangling logic gates, and, recently, a fast single-step controlled-X (CNOT) logic, whose gating is achieved with a single microwave pulse in ∼150 nanoseconds (ns) has been realized in a Si double QD (DQD) system [2] based on a theoretical background [3]. However, in general, operations of spin qubits must be sensitive to charge noise, which is defined as fluctuation of electrostatic charge and commonly exists in semiconductor devices [4].

Here we computationally explore how charge noise affects entangling logics implemented with a Si DQD system including the reported CNOT one [2]. With a focus on three 2-qubit logics — SWAP$^{1/2}$ (S$^{1/2}$), controlled-Z (CZ) and CNOT, we quantify the sensitivity of fidelity to the noise to present a practical guideline for choice of basic entangling units for designs of quantum circuits using Si QD systems.

2. Methods

We aim to model the physically realized DQD system to investigate the noise-sensitivity of entangling logics, which can be represented in a 2D manner with a periodic boundary condition as illustrated in Fig. 1(a) since the reported structure is longer than 100 nanometer along the [001] (Z) direction [2]. The bias-dependent electrostatic charge and potential are simulated with bulk physics augmented with electronic structure calculations based on the effective mass model [5,6]. A static (DC) magnetic field (B_Z), being generated along the Z-direction with a horseshoe-shaped micro-magnet in the real experiment, is incorporated into simulations with a spatial distribution driven by Neumann *et al.* [7], and the time responses of spin qubits are calculated with the Heisenberg model of a 2-spin Hamiltonian [3] whose matrix elements are determined with device simulations and time-varying control pulses. The noise is incorporated into modeling by disturbing the DQD potential profile with random values generated under a zero-mean gaussian distribution of standard deviation σ. 1,000 simulations are conducted per each value of σ, and a temperature of 1.5 K is used.

It is worth noting that the modeling method based on bulk physics augmented with electronic structure simulations) can be cost-efficient since it is not necessary to solve electronic structure simulations for the whole domain. In the DQD system, electronic structure simulations are needed to predict to electron density profiles in the vicinity of the middle Si layer where most of electron spins will stay, while it is fair to use bulk physics for description of the other region where there will be almost no electrons in the conduction band.

3. Results and discussion

The left subfigure of Fig. 1(b) shows the charge stability diagram that is simulated at a barrier gate bias (V_B) of 200 mV and a middle

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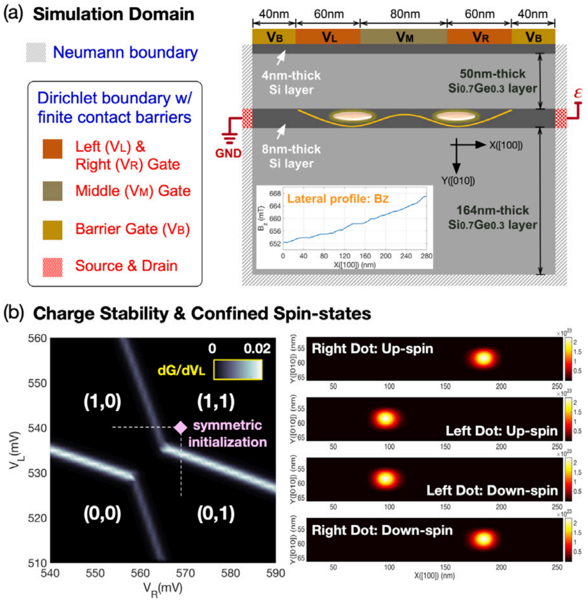

Fig. 1. Simulation domain and DQD initialization. (a) A 2D domain representing a Si DQD system that is assumed be infinitely long along the [001] direction. Quantum confinement along the [100] direction is formed with biases imposed on top electrodes. (b) A charge stability calculated as a function of V_L and V_R (V_M = 400 mV, V_B = 200 mV), and electron density profile at lowest $|\uparrow\rangle$ and $|\downarrow\rangle$ states of two QDs. At (V_L, V_R) = (540 mV, 570 mV), QDs are symmetrically initialized, and their Zeeman-splitting energies become 18.45 GHz (right) and 18.31 GHz (left).

gate bias (V_M) of 400 mV, where two numbers in each parenthesis represent electron populations of two QDs. The starting step of qubit operations is to initialize the DQD system such that the lowest down-spin ($|\downarrow\rangle$) state of each QD is filled with an electron. Here we initialize the system to a 2-qubit $|\downarrow\downarrow\rangle$ state (= $|\downarrow\rangle \otimes |\downarrow\rangle$) by setting a left ($V_L$) and a right gate ($V_R$) bias to 540 mV and 570 mV, respectively since it is known that *symmetric − biasing* (*i.e.*, potential shapes of two QDs are same) is beneficial for reducing the sensitivity of qubit interactions to noise [4]. The right subfigure of Fig. 1(b) shows the spatial distribution of electrons at $|\uparrow\rangle$ (lowest up-spin) and $|\downarrow\rangle$ state of two QDs. The Zeeman-splitting energy in the right (E_{ZR}) and left QD (E_{ZL}) becomes 18.45 GHz and 18.31 GHz due to the inhomogeneous B_Z profile along the [100] (X) direction. Note that the Zeeman-splitting energies and the charge stability diagram (Fig. 1(b)) are well connected to the results reported for the physical DQD device [2].

Fig. 2(a) shows how E_{ZL}, E_{ZR}, and exchange interaction between the left $|\downarrow\rangle$ and right $|\downarrow\rangle$ state (J) changes with increasing V_M (at V_L = 540 mV, V_R = 570 mV). Varying V_M by several mVs (affecting the barrier height between QDs) does not drive remarkable changes of E_{ZL} and E_{ZR}. The sensitivity of J to V_M, however, is large so J at V_M = 400 mV and 408.1 mV is calculated as 75.6 kHz and 19.3MHz, respectively. At V_M = 400 mV where J is in the order of kHz, we already have shown that both QDs can be individually addressed [5]. When J reaches 19.3 MHz at V_M = 408.1 mV, the interaction is not negligible and we have possibility for implementation of entangling logics. Fig. 2(b) shows the 2-qubit time response calculated against the 4 input states ($|\downarrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\uparrow\uparrow\rangle$). Due to the non-negligible J, the resonance frequency of the right QD depends on the occupied spin state of the left QD, and the CNOT logic can be secured at 100.4 ns with a single [010]-oriented AC magnetic pulse ($B_Y(t)$) whose amplitude and driving frequency are 4.98 MHz and 1.83 GHz, respectively. Again, we note that $B_Y(t)$ is incorporated into the Heisenberg Hamiltonian, and that the time response shown in Fig. 2(b) establishes a sound connection to the reported experimental work [2].

Another universal entangling logic useful for designs of gate-based quantum circuits, the CZ operation, can be also implemented with the same DQD structure. Unlike the 1-step CNOT operation, DQD implementation of the CZ logic does not require time-varying magnetic pulses so the control becomes simpler. Fig. 2(c) shows the 2-qubit time response that is simulated only with the above-mentioned DC control signals (DC biases imposed on top electrodes and B_Z profile), and our results indicate that the CZ logic is secured at ~32 ns that is about three times faster than the CNOT case. Although the interaction strength between two QDs remains the same, the CZ logic becomes faster since its time response is solely determined with the DC magnetic field B_Z that is much larger in magnitude than the time-varying pulse $B_Y(t)$ that determines the synchronized Rabi frequency of 2-qubit time responses for the 1-step CNOT case [3].

The $S^{1/2}$ gate performs half-way of a 2-qubit SWAP logic. As the cases of CNOT and CZ are, the $S^{1/2}$ logic is also universal since it is well known that any quantum circuits in principle can be constructed with $S^{1/2}$ and single qubit gates [8]. But, in a strict sense, the SWAP$^{\alpha}$ ($0 < \alpha \leq 1$) gating can be precisely implemented using a DQD platform *if and if only* both QDs have the same Zeeman-splitting energy (*i.e.*, B_Z has no gradient along the X direction) [9], which is not our case because we employed a laterally inhomogeneous external magnetic field as shown in Fig. 1(a). On the other hand, if we use a laterally homogeneous external field to implement the SWAP$^{\alpha}$ gate, more fundamental problem happens — all the spin qubits in QDs become indistinguishable and individual qubit addressing may not be feasible [3,5]. Not to hurt the programmability of our Si DQD system, here we increase the interaction strength J in order that the nonlinearity between the Zeeman term and total spin of the system Hamiltonian can be negligible (*i.e.*, $J \gg |E_{ZR}$ - $E_{ZL}|$). For this purpose, we only changed V_M to 412 mV from the control set used to implement the CZ logic, where J and $|E_{ZR}$ - $E_{ZL}|$ become ~266 MHz and ~132 MHz, respectively (see Fig. 2(a)). Corresponding 2-qubit time response is calculated and is shown in Fig. 2(d). At V_M = 412 mV, we observe that the fastest (first) $S^{1/2}$ operation is achieved within 1 ns, which becomes much faster than the CZ case mainly due to the increased strength of spin interaction.

As addressed in the Introduction section, charge noise refers to the fluctuation of charge densities and therefore potential energies that happens in a certain system. The fluctuation of electric field causes unintentional variation on E_{ZR}, E_{ZL} and J of spin qubits since QDs are created by the electrode-driven potential well in our case. To computationally quantify how charge noise affects preciseness of the three entangling operations, we disturbed the self-consistently determined potential profile with randomly generated noisy values as described in the Methods section. Being represented by the standard deviation σ of random noisy potential energies, today's charge noise in Si/SiGe devices is in the range of 4–11 μeV [10]. In a noise-free condition, we observe the fidelity of 1-step CNOT and CZ logic becomes 98.34% and 99.94%, respectively. In the case of $S^{1/2}$ logic, the noise-free fidelity turns out to be 96.86%, being a little bit lower than the other two cases due to the inhomogeneous distribution of external magnetic field B_Z. Once we incorporate charge noise into simulations with σ ranging from 10^{-2} to 5 μeV, the fidelity starts to drop for all the three entangling logics Fig. 3 shows.

Though the experimentally reported CNOT gate [2] is obviously advantageous since it can be implemented with a single-step control as also shown in Fig. 2(b), it turns out to be vulnerable to charge noise so the gate fidelity drops to 32.84 ± 0.54% when σ is 5 μeV. Compared to the case of CNOT, the noise-robustness significantly improves in the case of CZ gate (the fidelity = 67.13 ± 0.01% at σ = 5 μeV). The $S^{1/2}$ logic shows the best performance among the three cases, and its fidelity drops to 77.18 ± 0.006% when σ is 5 μeV. Note that, as shown in the inset of Fig. 3, the fidelity of CNOT, CZ, and $S^{1/2}$ logic at σ = 5 μeV is 33.34%, 67.17%, 79.68% of the noise-free value.

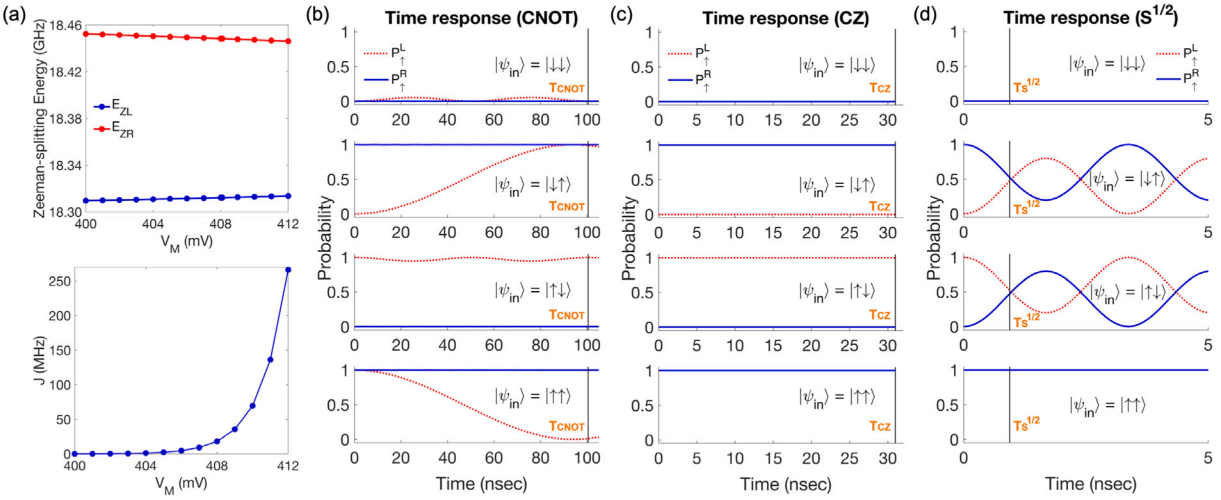

Fig. 2. Dependency of major control parameters on V_M and 2-qubit time responses of entangling logics. (a) Interaction between inter-QD spin states is quite sensitive to V_M, so ΔV_M of a few mV changes J by an order of magnitude. The Zeeman-splitting energy of electron spin in each QD, however, rarely depends on V_M, so J can be controlled almost independently of E_{ZL} and E_{ZR}. (b) 2-qubit time response obtained at J = 19.3 MHz (V_M =~408 mV) with a [010]-oriented AC magnetic pulse (CNOT) and (c) with no AC magnetic pulses (CZ) (d) 2-qubit time response at J = 266.1 MHz (V_M = 412 mV) with no AC magnetic pulses ($S^{1/2}$). The CNOT, CZ, and $S^{1/2}$ entangling logic are secured at 100.4 ns (T_{CNOT}), 32 ns (T_{CZ}), and 0.94 ns ($T_{S^{1/2}}$), respectively.

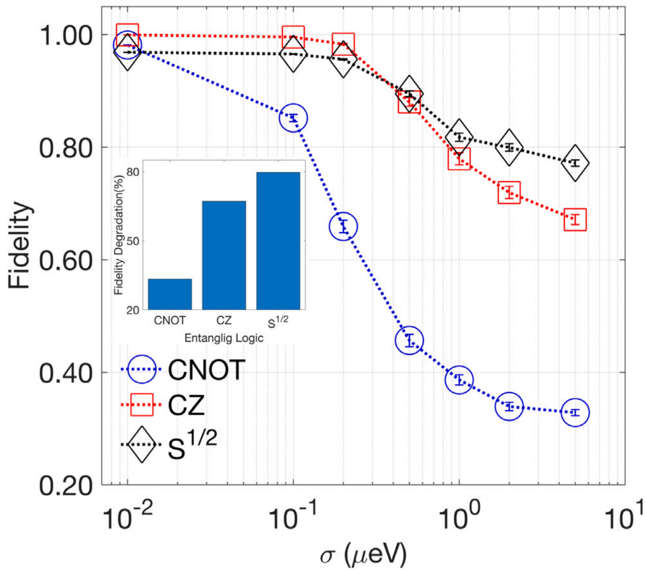

Fig. 3. Noise-sensitivity of the tree entangling logic operations. When the DQD system is free from charge noise, the fidelity of 1-step CNOT, CZ, and $S^{1/2}$ logic marks 98.34%, 99.94%, and 96.86% respectively. However, All the fidelities start to reduce once we incorporate charge noise into simulations, which is done by adding randomly generated noisy potential profiles to noise-free solutions. Simulation results clearly show that DC gates (CZ and $S^{1/2}$) have much stronger noise-robustness than the 1-step CNOT gate. $S^{1/2}$ is again remarkably better than CZ, so the fidelity at σ = 5μ eV is 79.68% of its noise-free value while the CZ shows 67.17% (see the inset).

4. Conclusion

In the current situation where diverse physical platforms such as superconducting circuits, trapped ions, and semiconductor quantum dots (QDs) are being actively utilized for realization of scalable gate-based quantum circuits, solid answers to the following question – "what would be the best block gate (or the best combination of block gates) for realization of a certain quantum logic operation in a certain physical platform?" – must be rigorously pursued. To enrich answers for the case of semiconductor QD systems using our in-house device simulations, here we present a preliminary modeling study on the noise-sensitivity of the representative 2-qubit (quantum bits) three universal entangling logics (controlled-X, controlled-Z, and SWAP$^{1/2}$ gate) that are implemented in a silicon double QD platform.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available from the corresponding author upon reasonable requests.

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