

The Role of Electron Viscosity on Plasma-Wave Instability in HEMTs

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Abstract — Conditions for terahertz (THz) radiation due to the plasma-wave instability in the channel of HEMTs are re-examined by considering the electron viscosity in carrier hydrodynamic transport. Not only the DC output I-V characteristics are affected, but also the window for plasma-wave instability is altered by the term with viscosity in the transport equation. The solution procedure and numerical study are presented. The analysis has been applied to recent experimental work and it is shown that the device parameters required for plasma-wave instability are more stringent than those reported in the up-to-date THz emission experiments.

Keywords - terahertz; plasma-wave instability; viscosity

I. INTRODUCTION

By treating the carrier transport in the channel of HEMTs (high electron mobility FETs) hydrodynamically together with the gradual channel approximation (GCA) and ballistic assumption, Dyakonov and Shur (D-S) in [1] predicted the existence of the channel plasma-wave (or called carrier charge-density wave) instability, which may lead to THz emission. When an asymmetric biasing condition (i.e., the channel source-end *ac* short while drain-end *ac* open) is applied, the charge density wave traveling along the channel has a reflection coefficient greater than unity at the drain end, leading to a gain mechanism for the charge density wave. Since then, a number of theoretical [2-4] and experimental [5-7] works have been reported along this line of thought.

In reality, however, there are no truly ballistic FETs. Thus the sources of damping during carrier transport must be taken into consideration in order to accurately predict and examine the threshold for the occurring of plasma-wave instability. In [1] it is pointed out that the damping sources include various carrier scattering mechanisms and further the electron viscosity also plays a certain role. A preliminary estimation of the effect on threshold is provided in [1]. Dmitriev in [2] re-examined these damping effects on the plasma wave, but still based on the perfect ballistic transport condition, i.e., DC charge density and drift velocity are constant along the channel. An much improved analysis was conducted in [3-4] by Cheremisin, who considered the effect of carrier scattering on the DC transport and proposed a threshold value of electron mobility for instability to occur. However, Cheremisin's work did not incorporate the electron viscosity as the energy loss term in carrier transport, which may play an important role in the damping of the plasma wave.

In this study, we investigated the effects of electron viscosity in determining the window and strength of plasma-wave instability, as well as its effects on the DC I-V characteristics of the device. We conducted our analysis mainly through the numerical simulation, and showed that the device behavior is changed significantly because of the viscosity originated from the electron-electron collision in 2DEG (2D electron gas). In addition, we analyzed two major experimental works using the developed theoretical model and examined the threshold conditions for the terahertz emission.

II. METHOD

Considering the electron scattering and viscosity in carrier transport in the 1D channel, the complete set of PDEs (partial differential equations) are:

$$\frac{\partial U}{\partial t} + \frac{\partial(UV)}{\partial x} = 0 \quad (1)$$

$$\frac{\partial V}{\partial t} + V \frac{\partial V}{\partial x} = -\frac{e}{m^*} \frac{\partial U}{\partial x} - \frac{V}{\tau_p} + \frac{\kappa}{U} \frac{\partial^2 V}{\partial x^2} \quad (2)$$

where $U(x,t)$ and $V(x,t)$ are two solution variables, representing the voltage between the gate and channel (we assume zero threshold voltage), and the electron drift velocity, respectively. Eq. (1) has assumed the gradual channel approximation, i.e. $n = C_{ox} U / e$. In Eq. (2), the term with τ_p (momentum relaxation time) is for the carrier-scattering loss, and that with κ , the coefficient for viscosity, is for the viscosity loss. Because of the constant voltage applied on the device between gate and source terminals, and constant-current source applied between source and drain, we have two boundary conditions as:

$$U_s = U(0,t) = \text{const.} \quad (3)$$

$$U_d V_d = U(L,t) V(L,t) = \frac{I}{C_{ox} W} \quad (4)$$

where subscripts s, d represents the source ($x=0$) and drain ($x=L$), respectively, and the biasing current I is given. The inclusion of viscosity term on the right side of Eq. (2) makes the equation a 2nd-order PDE for V , hence requiring an additional boundary condition for V . We simply let

$$\left. \frac{\partial V(x,t)}{\partial x} \right|_{x=0} = 0 \quad (5)$$

Analytical solutions to this set of PDEs do not exist for either the steady state or the time-dependent perturbation. We thus use numerical simulation in obtaining solutions, including both the DC output I - V characteristics and the small perturbation. From these solutions, the conditions for the plasma-wave instability can be further inferred.

III. DC CHARACTERISTICS

As shown in [3], we introduce the following dimensionless variables

$$\eta = \frac{x}{L}, \chi = \frac{\kappa}{U_s S_s L}, \gamma = \frac{L}{S_s \tau_p}, v = \frac{V}{S_s}, u = \frac{U}{U_s}$$

the plasma-wave velocity is a function of position along the channel, $S(\eta) = \sqrt{\frac{eU_0(\eta)}{m^*}}$ [1], and we use subscript s to

indicate the source-end position, 0 (as in U_0 , similar use below) to denote the steady-state solution. χ and γ reflect the strength of viscosity and scattering, respectively (note that they also depend on the gate bias through S_s). A bigger χ corresponds to a stronger viscosity, and a bigger γ means stronger scattering since γ is inversely proportional to τ_p , which is proportional to the carrier mobility. Using the condition of constant DC current (Eq. (4)), we can collapse Eqs. (1-2) to a single equation for v_0 as follows,

$$\chi \frac{d^2 v_0}{d\eta^2} = v_{0s} \left(1 - \frac{v_{0s}}{v_0^3}\right) \frac{dv_0}{d\eta} + \gamma v_{0s} \quad (6)$$

The boundary conditions are translated as

$$u_{0s} = 1, u_0 v_0 = u_{0s} v_{0s} = v_{0s} \quad (7)$$

$$\left. \frac{dv_0}{d\eta} \right|_{\eta=0} = 0 \quad (8)$$

Solving Eq. (6) numerically, we are able to get steady-state drift velocity v_0 along the channel for different values of γ and χ . Note that the channel current I and drain-to-source voltage U_{ds} are given by

$$I = v_0(\eta) C_{ox} W \sqrt{\frac{eU_s^3}{m^*}} \quad (9)$$

$$U_{ds} = U_s \left(1 - \frac{v_{0s}}{v_{0d}}\right) \quad (10)$$

Let $I_c = C_{ox} W \sqrt{\frac{eU_s^3}{m^*}}$, we have a simpler expression for the channel current

$$I = v_0(\eta) I_c \quad (11)$$

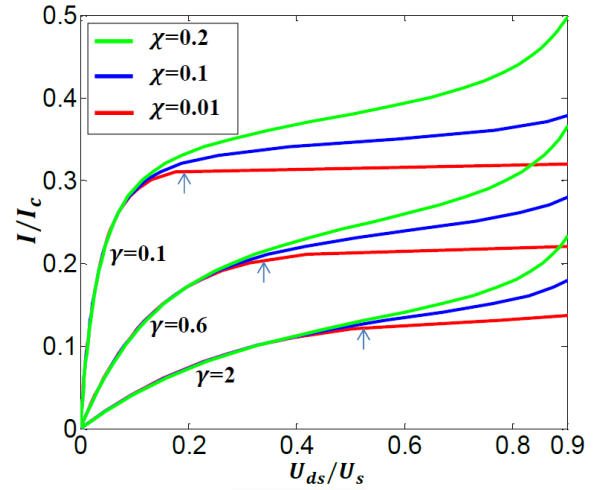


Fig. 1. Output I - V characteristics for different value of γ and χ . $U_s = U_{gs} - U_T$ and $I_c = C_{ox} W \sqrt{eU_s^3 / m^*}$ are constants for normalization. Hidden behind the red lines is the curve for $\chi = 0$. The arrows points to the ending points of $\chi = 0$ curves, beyond which no solutions exists.

Using such relations, we are able to find U_{ds} given gate voltage and drain current, which leads to I - V characteristics.

Before carrying out the calculation, it is necessary first to estimate the values of γ and χ . According to [4], the typical value of γ lies between 0.1~2. For the value of χ , recall that χ is related to κ , the coefficient for viscosity. According to [1], $\frac{\kappa}{U} = v_F \lambda_{ee}$, where v_F is Fermi velocity and $\lambda_{ee} \sim 1/\sqrt{n}$ is the mean free path between electron-electron collisions. Under the low temperature limit, $v_F = \frac{\hbar k_F}{m^*} \sim \frac{\hbar}{m^*} \sqrt{2\pi n}$, thus we have $\frac{\kappa}{U} \sim \frac{\hbar}{m^*} \sqrt{2\pi}$. When $S \sim 10^6$ m/s, $L \sim 0.1 \mu\text{m}$, χ can be estimated as close to 0.01. Taking the value of γ of 0.1, 0.6, 2 and χ of 0, 0.01, 0.1, 0.2, we plot the steady-state I - V characteristics in Fig. 1.

As expected, a larger γ leads to smaller drain current, but contrary to intuition, a bigger χ actually causes the drain current to increase. The reason is that the 2nd-order derivative term with χ on the left side of Eq. (6) slows down the increase of the drift velocity along the channel, resulting in a smaller U_{ds} for the same biasing current. Furthermore, χ has an even more fundamental role. Without the presence of χ , Eq. (6) degenerates to a 1st-order differential equation. When $v_{0d} = v_{0s}^{1/3}$, the drain current I becomes saturated, corresponding to Mach number at the drain, $M_d = V_{0d} / S_d = 1$ (marked by arrows in Fig. 1). Further increase of U_{ds} leads choking [8] to happen near the drain end of the channel and no stationary solution for drain current to exist. While

when $\chi \neq 0$, this choking situation will never happen and the solution of I always exists.

IV. INSTABILITY

We superimpose small perturbations on the steady-state solution $u_0(\eta)$ as the initial condition

$$u(\eta, 0) = u_0(\eta) + A_0 \sin(\pi\eta) \quad (12)$$

$$v(\eta, 0) = v_0(\eta) \quad (13)$$

The constraints on time-dependent u are

$$u_d(t)v_d(t) = u_d(0)v_d(0) \quad (14)$$

$$u_s(t) = u_s(0) \quad (15)$$

which are resulted from the *ac* open condition at the drain end and *ac* short at the source end.

Using the above specifications, we numerically solve Eqs. (1-2) by the method described in [2]. Based on the time evolution of $u(\eta, t)$, we are able to judge if the instability exists. In this way, different values of damping strength γ and χ are tried, together with different current bias to see their effects on the plasma-wave generation, and the threshold for the plasma-wave instability can be obtained. In practice, we first fix the value of χ , and check different values of γ and current I . As a result of carrier scattering on the plasma wave velocity, for each value of I there will be a maximum value of γ , beyond which no instability will exist. We denote this value as γ_{cr} . The plot of γ_{cr} with respect to channel current I gives the instability threshold under certain viscosity value, as shown in Fig. 2 with the simulation results.

It can be observed from the left part of Fig. 2 that the curve of γ_{cr} merges with the low scatter limit derived in [2]:

$$\gamma_{cr} = 2 \left(\frac{I}{I_c} - \frac{\pi^2}{8} \chi \right) \quad (16)$$

Also our simulation of $\chi = 0$ limit (not shown) reproduces the result reported in [3], which agrees with the theoretical prediction. Thus the validity of our simulation is justified. It can be seen that that nonzero χ decreases γ_{cr} at lower bias current while increases γ_{cr} at high current. It means that the role of viscosity creates an additional damping effect at lower bias current. However at high current region, the viscosity helps increase the range of γ , thus broadening the instability window. It can be attributed to the fact that the presence of viscosity eliminates the choking effect entirely, and thus the choking threshold [3] will not exist for $\chi \neq 0$.

V. THEORY VS. EXPERIMENT

The experimental work exploring the D-S effect came out rather late after the first theoretical prediction in 1993 because of the difficulty of fabricating short channel FETs with high carrier mobility required for the plasma instability. In 2004,

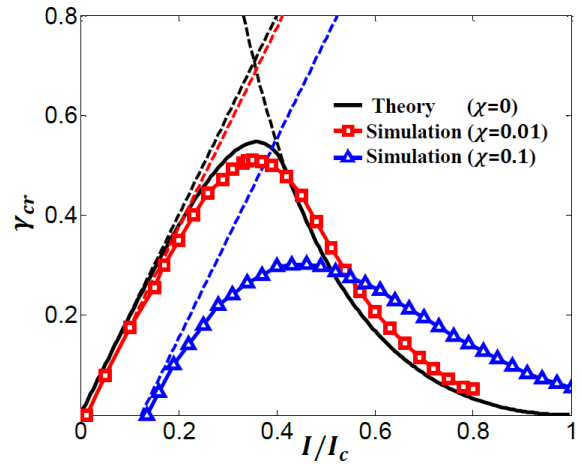


Fig. 2. The effect of viscosity on the instability threshold. The critical value of the scattering parameter γ_{cr} vs I for $\chi = 0$ (black line), 0.01 (red squares) and 0.1 (blue triangles). Dotted lines on the left are the asymptotes of low current limit as shown in Eq. (16); on the right corresponds to the choking threshold $M_d = 1$.

first observation of terahertz signal was reported [5], using a short-channel InGaAs HEMT. The output THz power is on the order of nW, but there is no frequency tunability at different gate bias. Highest output power of 1.8 μ W was reported in 2010 [6], using GaN-based HEMT with grating Omic source/drain contacts, which also plays a role of the antenna for emission. Also in 2010, first observation of voltage-tunability of frequency was reported [7].

We first give a detailed analysis of the experimental data from [5]. This InGaAs-based FET has a long ungated source-drain separation, leading to a non-negligible source/drain serial resistance R_s . Taking it into account, we have

$$U_s = U'_{gs} - U_T - \frac{IR_s}{2} \frac{L_{gs}}{L_{gs} + L_{gd}} \quad (17)$$

$$U_{ds} = U'_{ds} - IR_s \quad (18)$$

TABLE I. EXTRACTED PARAMETERS FROM DATA

Parameters	InGaAs/AlInAs ^a $m^* = 0.042m_e$	AlGaN/GaN ^b $m^* = 0.22m_e$
l/d (nm)	60 17	250 30
R_s (Ω)	13.5	83
U_T (V)	-0.152	-1.62
χ	0.1	0.01
γ_0 ($V^{-1/2}$)	1.15	1.56
$\gamma \gamma_{max}$	3.27 0.51	2.5 0.3
$\mu \mu_{min}$ ($cm^2/V \cdot s$)	1070 6860	1440 12000

^a 2004 experiment, see [5]

^b 2010 experiment, see [7]

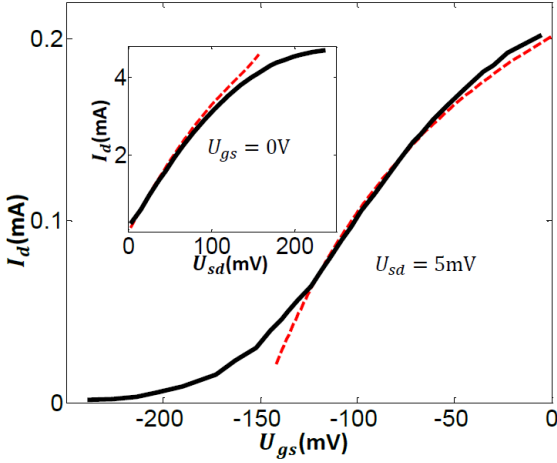


Fig. 3. Best fit (red line) to the transfer characteristics at $U'_{ds} = 5\text{mV}$ and I - V characteristics (inset) at $U'_{gs} = 0\text{V}$. The fitting does not include the sub-threshold region, where the gradual channel approximation is no longer valid.

where U'_{gs} , U'_{ds} are the measured terminal voltages which include the voltage drop on the serial resistance, and L_{gs} , L_{gd} are the gate-source/gate drain spacings. We extract all the parameters (R_s , U_T , χ , γ) from the transfer characteristics and I - V curve using the hydrodynamic model (Eq. (6)) instead of the drift-diffusion model as employed in [4]. Note that different I leads to different U_s in the presence of R_s .

According to the definition, $\gamma = \frac{L}{S_s \tau_p} = \frac{L}{\tau_p} \sqrt{\frac{m^*}{eU_s}}$, therefore

the value of γ is also affected by different I . For this reason, we extract the U_s -independent part of γ , defined as

$$\gamma_0 = \frac{\gamma}{\sqrt{U_s}} = \frac{L}{\tau_p} \sqrt{\frac{m^*}{e}} = \frac{L}{\mu} \sqrt{\frac{e}{m^*}} \quad (19)$$

Table I shows the extracted parameters. Our best fitting result for the transfer and I - V characteristics curves are shown in Fig. 3, which is in excellent agreement with the experimental data.

We then calculated γ at the starting point of THz radiation. We found that the extracted value of γ from experiment in 2004 [5] is too high compared with theoretical predicted value γ_{cr} . Since γ is inversely proportional to μ (see Eq. (19)), it means that the mobility is too low compared with theoretical requirements. The actual values γ , μ and theoretically required values γ_{cr} , μ_{min} are shown and can be compared in the last 2 rows of Table I.

The same treatment has been done for the data from experiment in [7], see the 3rd column of Table I. Similar to result in [5], the mobility is too low compared with required value. So it is not a sure thing to attribute the terahertz

emission to Dyakonov-Shur instability either. Furthermore, the prescribed boundary condition for D-S theory at the drain has never been satisfied in these experiments. In both experiments, the drain current bias is exerted on the device by voltage sources that drives the transistor into saturation region. Note that such methods cannot guarantee the constant current boundary condition at the drain end. Based on such evidences, we believe that the observed sub-THz emission in [5,7] may be caused by other mechanisms such as the hot carrier fluctuation effects.

CONCLUSIONS

We incorporated viscosity term in the hydrodynamic equations and examined its effect on the steady-state I - V characteristics as well as the plasma-wave instability. It was found that the viscosity term increases the saturation current and helps suppress the choking effect [8]. It narrows the instability window at lower current bias while broadening it at higher bias. However it does not significantly change the requirement on carrier mobility. Our results show that due to insufficient mobility compared with theoretical requirement, the origins of terahertz emission in two recent experiments may not be attributed to the plasma-wave instability. Further research is needed to explain the observed terahertz emission.

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