Modeling of High Frequency Noise in SiGe HBTs

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Abstract **---A compact model solution, consistent with the system theory for correlated base and collector shot noise sources, was derived and implemented in the bipolar transistor compact model HICUM using Verilog-AMS. Simulations were performed using ADS 2005A. Results were tested against measured noise parameters for high-speed conventional and LEC doped SiGe HBTs. Perfect agreement between simulated and measured data confirmed model validity.**

Index Terms **--- Noise, SiGe Heterojunction bipolar transistors, HICUM, correlation, shot noise, noise modeling, Verilog-AMS.**

I. Introduction

For bipolar transistors, noise behaviour at high frequency substantially differs from that at the low-frequency region due to the effect of correlation between the base and collector (i.e., the input and output) current shot noise sources [1][2]. For the bipolar transistors, correlation between base and collector shot noise plays a significant role at high frequencies [3][4][5] and is well described analytically in [2]. Unfortunately, implementation of correlated noise in a compact model is not straight-forward, and from a system theoretical approach a consistent implementation has neither been investigated yet, nor been done for bipolar transistors. Conventional noise computation, implemented in all simulators, can only handle uncorrelated noise sources. Even through the adjoined network concept [6], noise calculation does not handle correlation terms. From the point of view of SPICE implementation, a possible Verilog-AMS solution of correlated noise for MOS transistors has recently been given in [7]. However, a complete and systematic analysis of the bipolar transistor noise from the perspective of SPICE-like implementation is still missing. In this paper, we deal with a complete theoretical analysis of noise behaviour in bipolar transistors. Our solution handles the correlation terms in such a manner that existing SPICE-like simulators can additionally compute the correlated noise in a similar way they do for the uncorrelated noise sources. The problem was addressed almost two decades ago through the noise calculation of complex active filter circuits [8] and the corresponding solution provides a useful background for our investigation.

II. System Theory and Verilog-AMS implementation

Transfer of the noise signal in the linear network in frequency domain is described as follows[9]:

$$\mathbf{S}_{YY}(j\omega) = \mathbf{G}(j\omega)\cdot\mathbf{S}_{XX}(j\omega)\cdot\mathbf{G}^{+}(j\omega) \qquad (1)$$

where $\mathbf{G}(j\omega)$ is transfer function matrix of any noise source to a free of choice output in the circuit and $\mathbf{G}^{+}(j\omega)$ is the adjoint matrix of the transfer function. The $\mathbf{S}_{XX}(j\omega)$ is the power spectral density (PSD) matrix of noise sources and is defined as:

$$\mathbf{S}_{XX}(j\omega) = \begin{pmatrix} S_{i_{nb}} & S_{i_{nb}i_{nc}} \\ S_{i_{nc}i_{nb}} & S_{i_{nc}} \end{pmatrix} \qquad (2)$$

Off-diagonal elements of the matrix correspond to the cross-PSDs, which are not taken into account by conventional SPICE-like circuit simulators while simulating noise behaviour. Consequently, the correlation between noise sources in the noise computation is omitted. In order to force circuit simulators to compute correlation terms additionally, transformation of the input matrix into a diagonal matrix is performed. Here, the input matrix is expressed through a diagonal $\mathbf{D}_X(j\omega)$ and a transformation matrix $\mathbf{T}(j\omega)$:

$$\mathbf{D}_X(j\omega)=\begin{bmatrix} \mathbf{D}_1 & 0 \\ 0 & \mathbf{D}_2 \end{bmatrix}, \quad \mathbf{T}(j\omega)=\begin{bmatrix} 1 & 0 \\ t_{21} & 1 \end{bmatrix} \qquad (3)$$

$$\mathbf{S}_{XX}(j\omega) = \mathbf{T}(j\omega)\cdot\mathbf{D}_X(j\omega)\cdot\mathbf{T}^{+}(j\omega) \qquad (4)$$

where $\mathbf{T}^{+}(j\omega)$ is the corresponding adjoint matrix. Now the modified input matrix becomes:

$$\mathbf{S}_{XX}(j\omega)=\begin{bmatrix} \mathbf{D}_1 & \mathbf{D}_1 t_{21}^{*} \\ \mathbf{D}_1 t_{21} & \mathbf{D}_1|t_{21}|^2+\mathbf{D}_2 \end{bmatrix} \qquad (5)$$

Comparing matrix elements of eq. (5) and eq. (2), one obtains: $\mathbf{D}_1 = S_{i_{nb}}$, $\mathbf{D}_2 = S_{i_{nc}} - S_{i_{nb}}|t_{21}|^2$. The cross-PSD is:

$$S_{i_{nc}i_{nb}} = S_{i_{nb}} t_{21} \qquad (6)$$

The expression for the control factor t_{21} can be found later along with a controlled source in the equivalent circuit (EC).

Interpretation of the above matrix manipulation into a noise EC model can provide a better understanding for further implementation. After transforming $\mathbf{S}_{XX}(j\omega)$ into a diagonal matrix, the practically correlated noise sources i_{nb} and i_{nc} (see Fig. 1a) take the form of three uncorrelated ones $\overline{i_{nb}}$, $\overline{i_{nc}}$ and $t_{21}i$ (see Fig.1b). The transformation yields an additional noise source, which carries the correlation. This is understood as the inclusion of the additional controlled source during the transformation through the control factor t_{21}, which contains the correlation between the noise sources i_{nb} and i_{nc}. This additional controlled current source is tagged in parallel to the output noise source (see Fig.1b) keeping consistency with the system theory [9].

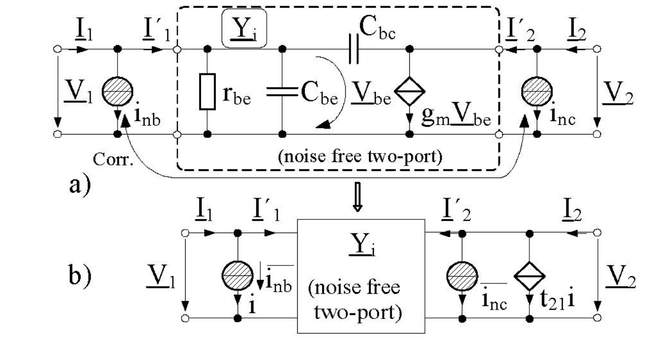

Fig. 1. a) Noise free two-port with small signal equivalent circuit (SSEC) of the internal BJT in the dashed box and correlated noise sources i_{nb} and i_{nc}, at the input and output respectively b) SSEC with three modified uncorrelated noise sources. Note that $i \equiv \overline{i_{nb}}$.

Now the input voltage noise source in Fig.2.b will be expressed as a function $v_{n\_ers} = f(\overline{i_{nb}}, \overline{i_{nc}}, t_{21}i)$ of uncorrelated noise sources in Fig.2.a. This further analysis is carried out in connection with the noise computational methods adopted in conventional SPICE-like circuit simulators. The two-port Y-parameter representation for circuits in Fig.2a/b is given by,

$$\begin{aligned} \underline{I}_1 &= \underline{Y}_{11}\underline{V}_1 + \underline{Y}_{12}\underline{V}_2 + \overline{i_{nb}} \quad , \underline{V}_1 = v_{nRs} - R_S\underline{I}_1 \\ \underline{I}_2 &= \underline{Y}_{21}\underline{V}_1 + \underline{Y}_{22}\underline{V}_2 + \overline{i_{nc}} + \underline{t}_{21}i \end{aligned} \qquad \text{(Fig. 2a)} \quad (7)$$

$$\begin{aligned} \underline{I}_1 &= \underline{Y}_{11}\underline{V}_1 + \underline{Y}_{12}\underline{V}_2 \ , \ \underline{V}_1 = v_{n\_ers} - R_S\underline{I}_1 \\ \underline{I}_2 &= \underline{Y}_{21}\underline{V}_1 + \underline{Y}_{22}\underline{V}_2 \end{aligned} \qquad \text{(Fig. 2b)} \quad (8)$$

Setting $\underline{V}_2 = 0$ (output shorted) in both eq. (7) and eq. (8) and inserting the result for $\underline{I}_2$, one obtains,

$$v_{n\_ers} = \underbrace{1}_{G1} v_{nRs} + \left(\underline{t}_{21} \underbrace{\frac{1 + R_S\underline{Y}_{11}}{\underline{Y}_{21}}}_{G3} + \underbrace{\left(-R_S\right)}_{G2} \right) \overline{i_{nb}} + \underbrace{\frac{1 + R_S\underline{Y}_{11}}{\underline{Y}_{21}}}_{G3} \cdot \overline{i_{nc}} \qquad (9)$$

Expressing eq. (9) in terms of power spectral densities, i.e. eq. (1), the final expression for PSD is:

$$S_{v_{n\_ers}} = |G_1|^2 S_{v_{nRs}} + |\underline{t}_{21}G_3 + G_2|^2 S_{\overline{i_{nb}}} + |G_3|^2 S_{\overline{i_{nc}}} \ , \qquad (10)$$

where $\quad S_{v_{nRs}} = 4kTR_S,\ S_{\overline{i_{nb}}} = 2qI_B \qquad (11)$

$$S_{\overline{i_{nc}}} = S_{i_{nc}} - |\underline{t}_{21}|^2 S_{i_{nb}} = S_{i_{nc}}\left(1 - B_f\left(\omega\frac{\tau_{Bf}}{3}\right)^2\right) \qquad (12)$$

with $\quad S_{i_{nb}} = 2qI_B,\ S_{i_{nc}} = 2qI_C \qquad (13)$

where B_f is the DC gain and τ_{Bf} is the base transit time. The control factor $\underline{t}_{21}$ in eq. (10), defined by eq. (6) follows as,

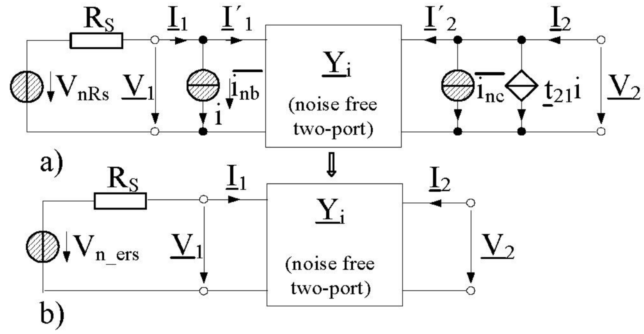

Fig. 2. a) modified SSEC (cf. dashed box in Fig. 1 a)) with uncorrelated noise sources $\overline{i_{nb}}$, $\overline{i_{nc}}$ and $\underline{t}_{21}i$, b) SSEC with equivalent noise voltage source v_{n_ers} at the input. R_S is the source resistance.

$$\underline{t}_{21} = \frac{\underline{S}_{i_{nc}i_{nb}}}{S_{i_{nb}}} = -j\omega\frac{\tau_{Bf}}{3}\frac{I_C}{I_B} \text{ with } \underline{S}_{i_{nc}i_{nb}} \approx -2qI_C\left(j\omega\frac{\tau_{Bf}}{3}\right) \qquad (14)$$

where I_B and I_C are the direct base (intrinsic base emitter diode) and direct collector (forward transfer) currents. The derivation of the cross-PSD $\underline{S}_{i_{nc}i_{nb}}$ (eq. (14)) may be found in [1][2].

Now one may try to implement the above mentioned method of correlated noise computation in a compact model and thereafter verify the implementation with a circuit simulator. Emergence of Verilog-AMS as a preferred language for writing compact models [10], enables model developers to implement and test any new concept fairly quickly. For the noise simulation in Verilog-AMS, only a limited number of functions are supported, among which “white_noise ( )” and “flicker_noise ( )” are useful. The language reference manual (LRM) has an example of correlated noise implementation with a real valued correlation coefficient. However, one can use the “ddt ( )” operator with a concept of capacitive coupling to implement any imaginary correlation [7]. Inside “white_noise ( )”, it is not obviously permitted to specify any argument that depends upon frequency; also the argument should not be negative in any case. As per our theoretical discussion, one is required to implement three controlled sources, one at the input and the other two at the output. The source at the input side is the same as the base current uncorrelated noise source. At the output side, one controlled source can be implemented by tagging a capacitor (with capacitance value of noise transit time) from the input noise source maintaining a proper sign of the control factor $\underline{t}_{21}$. However, the main problem, according to eq. (12), is to implement the remaining noise source in the output with the proper PSD, part of which depends upon the square of the frequency preceded by a negative sign! According to [7], we understand that to obtain a “negative sign” inside the noise PSD is not possible from simple addition and subtraction of controlled sources and one can not multiply controlled sources in any SPICE-like implementation of noise. Therefore, it is not possible to straightforwardly realize the new PSD $S_{\overline{i_{nc}}}$ in Verilog-AMS, since it is clear that any noise PSD corresponds to a squared quantity cancelling any negative sign. However for this case,

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we can use an approximation as in eq. (15), which can imitate the theoretical prediction:

$$S_{\overline{i_{nc}}} \approx S_{i_{nc}}\left(1-\frac{B_f}{2}\left(\omega\frac{\tau_{Bf}}{3}\right)^2\right)^2 = S_{i_{nc}} - |t_{21}|^2 S_{i_{nb}} + \underbrace{\frac{|t_{21}|^4}{4B_f} S_{i_{nb}}}_{\text{error term}} \qquad (15)$$

It is found that an error (last term in eq. (15)) does not exceed a 10% limit in the calculation of the minimum noise figure (NF_{min}) (for 150 GHz SiGe as well as for 80 GHz LEC HBTs). It is also worth to mention that this PSD is part of the total noise correlation, which is again part of the total noise of the transistor. Putting eq. (15) into eq. (10), and we obtain:

$$S_{v_{n\_ers}} \approx |G_1|^2 A + |G_2|^2 B + |G_3|^2 C + 2Re\left\{G_3 G_2^* t_{21}\right\} B + K \qquad (16)$$

$$A = S_{v_{nRs}}\ ,\quad B = S_{i_{nb}}\ ,\quad C = S_{i_{nc}}\ ,\quad K = |G_3|^2 \frac{|t_{21}|^4}{B_f} B$$

In eq. (12), factor 1/3 is relevant for pure diffusion transistors. In modern transistor models like HICUM [11], a bias dependent total transit time is formulated including the contributions from the emitter and the collector regions [12]. Therefore the factor 1/3 may be found a little smaller than the one actually required from device physics, if the total transit time is used in the implementation. To maintain consistency and generality for all other processes including Si-based ones, in our implementation we used the bias dependent total transit time and a parameter instead of the factor 1/3. The VCCSs, shown in Fig. 3, are dependent on the voltages V(b_n1) and V(b_n2):

$$I(bi, ei) = gV(b\_n1)\ ,\quad V(b\_n1) = \frac{1}{g}\sqrt{2 \cdot q \cdot i_{bei}}\ ,$$

$$I(ci, ei)_1 = \left(1 - \frac{B_f}{2}(alit \cdot Tf \cdot \omega)^2\right) \cdot g \cdot V(b\_n2)\ , \qquad (17)$$

$$V(b\_n2) = \frac{1}{g}\sqrt{2 \cdot q \cdot i_t}\ ,$$

$$I(ci, ei)_2 = -j\omega \cdot B_f \cdot alit \cdot Tf \cdot g \cdot V(b\_n1)$$

Fig. 3 shows a SPICE-like implementation of transistor noise (including correlation). where "g" corresponds to a uniform conductance of 1S, "*alit*" is a parameter dependent on the process technology, "*Tf*" is the bias dependent total transit time that takes into account the total delay for the carrier in emitter base, and collector regions, "*q*" is the elementary charge and "B_f" is equivalent to dc gain, "i_{bei}" is internal base and "i_t" is transfer current, as defined in HICUM. Now we can get the PSDs as:

$$S_{I(bi,\, ei)} = 2qi_{bei} = S_{\overline{i_{nb}}} = S_{i_{nb}} \qquad (18)$$

$$S_{I(ci,\, ei)_1} = 2qi_t \cdot \left(1 - \frac{B_f}{2}(alit \cdot Tf \cdot \omega)^2\right)^2 \approx S_{\overline{i_{nc}}}$$

$$S_{I(ci,\, ei)_2} = (2qi_{bei}(B_f \cdot alit \cdot Tf \cdot \omega)^2) = |t_{21}|^2 S_{\overline{i_{nb}}} = |t_{21}|^2 S_{i_{nb}}$$

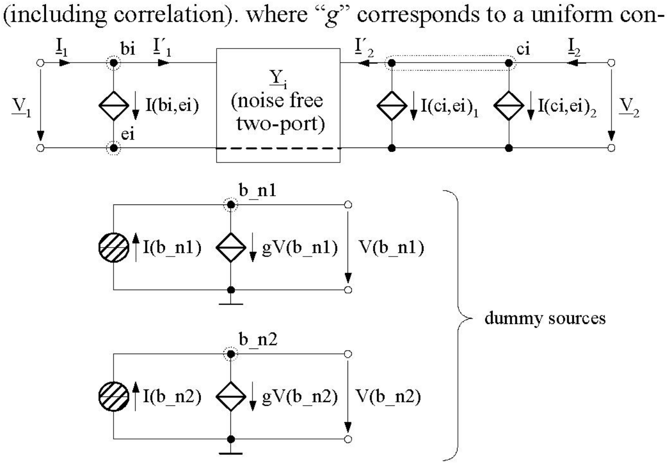

Fig. 3. Realization of correlation in CM from a system theory perspective.

III. Model verification

Noise parameters (NP) were measured for SiGe-BiCMOS HBTs [13] with a peak transit frequency of f_T=150 GHz CBEBC configuration and emitter window A_{E0}=0.2∗10.16 µm^2 and LEC (light emitter concentration, "true HBT") HBTs [14] with f_T=80 GHz, CEB, A_{E0}=0.3∗20.3 µm^2, both at V_{CE}=1.5V. Before any AC and noise analysis a standard DC model verification was performed. Both HICUM/L2v2.2 and GALENE yielded good agreement for output DC characteristics as well as for forward Gummel plot, for example LEC HBTs (Fig.4). Noise parameters were measured in the 2-26 GHz frequency band with ATN and at low frequency (high reflection conditions) with high matching Maury tuners. NPs were de-embedded to DUT level using correlation matrix technique. Simulations were performed with ADS 2005A using Verilog-AMS HICUM level 2.2 code. Since the Verilog-AMS based model can be easily modified and compiled again it was used to investigate the impact on noise parameters of correlation between the base and the collector current shot noise sources as well as to evaluate impact of selfheating and decompose NF_{min} into different noise sources. No impact of correlation is observed at 2 GHz for both HBT (Fig.5,6): curves with correlation and without coincide, while beyond ~10 GHz a significant deviation is present if correlation is omitted. Including the effect of correlation yields perfect agreement at least up to 26 GHz.

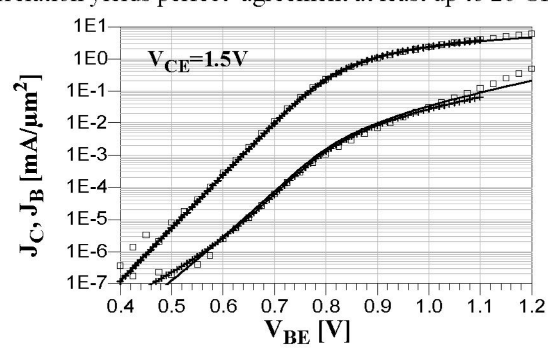

Fig. 4. Forward Gummel plot for 80 GHz LEC SiGe HBTs, symbols: meas., lines: HICUM, line with crosses is GALENE simulation.

LEC HBTs (Fig. 6) exhibit smaller impact of correlation due to the B/E junction peculiarities and, therefore lower base shot noise. This was confirmed by the decomposed noise sources. Deviation of the NF_{min} for GALENE shows that external HBT

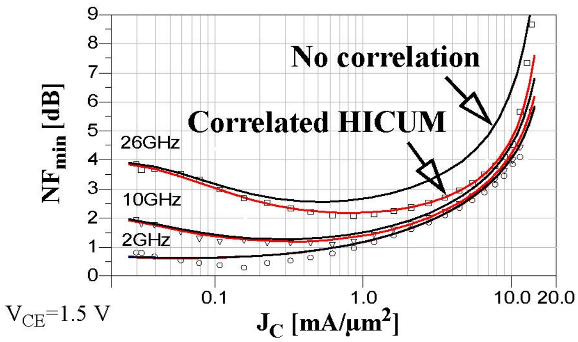

Fig. 5. NF_{min} for 150 GHz SiGe HBTs, symbols: meas., lines: sim.

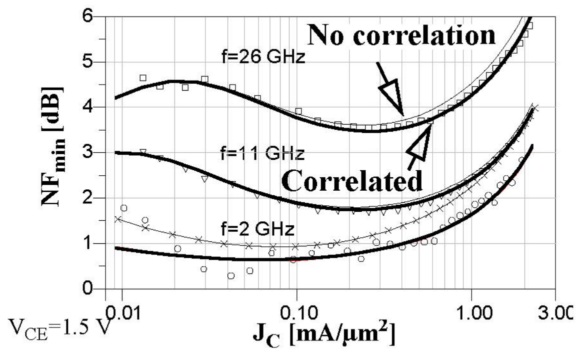

Fig. 6. NF_{min} for 80 GHz LEC SiGe HBTs, symbols: meas., lines: sim, line with crosses is microscopic simulation at 2 GHz.,

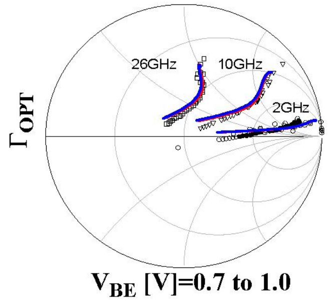

Fig. 7. Γ_{OPT} for 80 GHz LEC SiGe HBTs, symbols: meas., lines: sim.

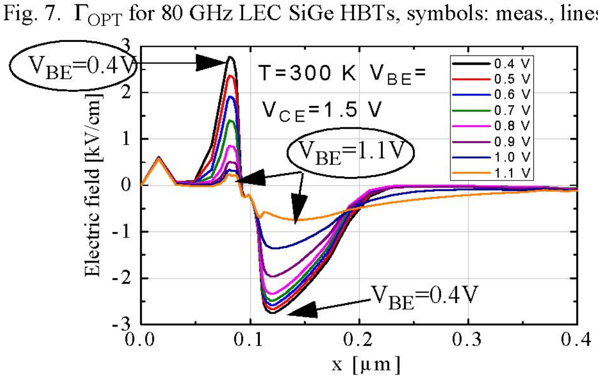

Fig. 8. Vertical Electric field for LEC SiGe HBT.

circuit elements, such as base/emitter, base/collector external capacitances, substrate resistance and selfheating are important, what can be seen from optimum source reflection coefficient (Γ_{opt}), (Fig.7). The derived noise model within HICUM ensured a perfect agreement for (R_n) and Γ_{opt} for both device types. Well calibrated microscopic model within the GALENE device simulator, yielded fair agreement for LEC HBT in terms of standard DC and AC characteristics. A possible impact of the B/C heterobarrier effect on noise due to a charge dipole, as for $A_{III}B_V$ [15], was not confirmed in microscopic level at V_{CE}=1.5 V (Fig. 8), but it was present at V_{CE}=0.5 V.

IV. Conclusions

Based on system theory, a bipolar transistor noise representation is formulated. The developed equations include noise correlation terms in such a way that the approach can be implemented into a compact model to be used with any conventional SPICE-like circuit simulator. The concept is realized in Verilog-AMS code for HICUM and verified against measured noise data of conventional and LEC doping SiGe HBTs.

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