# Modeling of Discrete Dopant Effects on Threshold Voltage Shift by Random Telegraph Signal

Ken'ichiro Sonoda, Kiyoshi Ishikawa, Takahisa Eimori, and Osamu Tsuchiya

Production and Technology Unit, Renesas Technology Corp., Mizuhara 4-1, Itami, Hyogo 664-8641, Japan

Email: sonoda.kenichiro@renesas.com, Telephone: +81-72-784-7325, Fax: +81-72-780-2693

Abstract— This paper discusses the discrete channel dopant effects on the threshold voltage shift by random telegraph signal (RTS) in MOSFETs. Appropriate grid spacing to incorporate discrete dopant effects in three dimensional device simulation is addressed to obtain consistent results with continuum limit. Considering discrete dopant effects, the threshold voltage shift of MOSFETs by RTS follows the log-normal distribution, while the threshold voltage itself follows the normal distribution. An analytical model for the distribution of the threshold voltage shift is also presented. The threshold voltage shift by RTS will become a serious concern in 50nm flash memories and beyond.

## I. INTRODUCTION

The random telegraph signal (RTS)[1] caused by trapping of a single carrier at the Si/SiO<sub>2</sub> interface will becoming a serious issue for MOSFETs with sub 100nm dimensions as a source of, not only low-frequency noise in analog circuits, but also functional error in digital logic circuits and memories. Flash memories, for example, are susceptible to RTS[2] owing to thick gate dielectrics.

The threshold voltage shift  $\Delta V_{\rm th}$  which is related to the current change  $\Delta I_{\rm d}$  by RTS as  $-\Delta I_{\rm d}/g_{\rm m}$  is sometimes anomalously larger than the value estimated by the conventional formula  $q/C_{\rm ox}W_{\rm eff}L_{\rm eff}$ [3], where  $g_{\rm m}$  is the transconductance, q is the elementary charge,  $C_{\rm ox}$  is the capacitance of the gate dielectric per unit area, and  $W_{\rm eff}$  and  $L_{\rm eff}$  are the effective channel width and the effective channel length, respectively. A probable explanation is that the strategically located traps in the inhomogeneous channel influence the magnitude and the spreading of the RTS amplitudes[1][3][4].

In this paper, the threshold voltage shift by RTS is analyzed using three dimensional device simulation considering inhomogeneous carrier concentration in the channel caused by random discrete dopant[5]. An analytical model for the distribution of the threshold voltage shift is also presented.

## II. MODELING

Before simulation of the threshold voltage of MOSFETs is done, a grid spacing dependence of conduction current is investigated to assess consistency between continuous and discrete cases. The conduction current in *p*-type silicon with the average acceptor concentration  $N_A$  is simulated. Uniform rectangular grid with spacing *d* and constant mobility are used throughout this work. In the discrete case, discrete random variables following the Poisson distribution with the average  $N_A d^3$  are used for the number of dopant atoms associated with each nodes.

Majority carrier current in the discrete case decreases as the grid spacing decreases as shown in Fig. 1, because the depth of the potential wells which originate in ionized dopant atoms becomes deeper and more majority carriers are trapped in the potential wells[7]. Minority carrier current shows opposite dependence on the grid spacing because of the mass action law. If the rectangular region with constant charge density is represented by a sphere of the radius  $r_{\rm imp}$  with the same volume of a grid as shown in Fig. 2, the depth of the potential wells is given by  $E_{\rm B} = (3/2)(q/4\pi \epsilon r_{\rm imp})$ . The radius  $r_{\rm imp}$  is obtained by solving  $(4/3)\pi r_{\rm imp}^3 = d^3$ .

The depth  $E_{\rm B}$  of the potential wells is regarded as the ionization energy of dopant atoms. The simulation result in the continuous case using the energy  $E_{\rm B}$  given by the above formula for the ionization energy  $E_{\rm B0}$  agrees with the average value of the simulation results in the discrete case as shown in Fig. 1. In order to obtain consistent simulation results between continuous and discrete cases, the width d of the rectangular region in the discrete case should be determined so that the energy  $E_{\rm B}$  is equal to the ionization energy of dopant atoms in the continuous case. When B is used as acceptors in silicon, the width d = 6.6nm should be used to match the ionization energy with the measured value 45meV[8]. If finer grid spacing is necessary to express spacial variation, the dopant atom is expanded so that the energy  $E_{\rm B}$  is equal to the ionization energy.

## III. RESULTS AND DISCUSSION

Discrete dopant effects on the threshold voltage of nMOSFETs and its shift by RTS are analyzed using three dimensional device simulation. The threshold voltage shift is defined as the difference between threshold voltages with and without an electron captured in an interface trap located at the center of the channel as shown in Fig. 3.

The threshold voltage  $V_{\rm th}$  in the discrete case fluctuates around the value in the continuous case as shown in Fig. 4, and follows the normal distribution as shown in Fig. 5 (a). The standard deviation is well described by the analytical model[9]. The threshold voltage shift  $\Delta V_{\rm th}$  in the discrete case also fluctuates around the value in the continuous case and follows the log-normal distribution as shown in Fig. 5 (b).

An analytical model for the fluctuation of  $\Delta V_{\rm th}$  is derived to explain the origin of the log-normal distribution. Moderate inversion is assumed because threshold voltage is discussed in the following. Using the total number of carriers in the channel N, the drain current of MOSFETs is expressed as  $I_{\rm d} = q\mu N V_{\rm ds}/L_{\rm eff}^2$  [10], where  $\mu$  is the carrier mobility and  $V_{\rm ds}$  is the drain-source voltage. Suppose the channel is divided into small cells[6] with the area  $A_{\rm t} = \pi r_{\rm c}^2$ , in which the conductivity is affected by trapping of a single carrier as shown in Fig. 6. The local threshold voltage  $V_{\text{th}i}$  of the cell *i* is defined so that the number of carriers in the cell is expressed as  $N_i = N_0 \exp(q(V_{\rm gs} - V_{\rm th})/nkT)$ , where  $V_{\rm gs}$  is the gate-source voltage,  $n(=1+C_{\rm dep}/C_{\rm ox})$  is the subthreshold factor,  $C_{dep}$  is the depletion layer capacitance per unit area under the channel, k is the Boltzmann constant, T is the temperature, and  $N_0$  is a constant. By trapping of a single carrier at the trapping site in the cell j, the local threshold voltage of the cell increases by  $\Delta V_{\text{th}i} = q/C_{\text{ox}}A_{\text{t}}$ , and the current change is  $\Delta I_{
m d} = (q \mu V_{
m ds} N_0 / L_{
m eff}^2) \exp(q (V_{
m gs} V_{{\rm th}j})/nkT)(\exp(-q\Delta V_{{\rm th}j}/nkT) - 1)$ . The corresponding threshold voltage shift is  $\Delta V_{
m th} = -\Delta I_{
m d}/g_{
m m}$  $(q/C_{\rm ox}W_{\rm eff}L_{\rm eff})N_{\rm total}/\sum_i \exp(q(V_{\rm th}j-V_{\rm th}i)/nkT)$ , where  $N_{\rm total} \equiv W_{\rm eff} L_{\rm eff} / A_{\rm t}$  is the total number of the cells. The logarithm of the  $\Delta V_{\rm th}$  is, therefore,

$$\log(\Delta V_{\rm th}) \sim \log\left(\frac{q}{C_{\rm ox}W_{\rm eff}L_{\rm eff}}\right) - \frac{q}{nkT}(V_{\rm th}j - V_{\rm thave}), (1)$$

where  $V_{\text{thave}}$  is the average of the local threshold voltage.

The derived model Eq. (1) suggests that the threshold voltage shift  $\Delta V_{\rm th}$  becomes larger than the conventional formula  $q/C_{\rm ox}W_{\rm eff}L_{\rm eff}$  if the local threshold voltage  $V_{{\rm th}j}$  of the cell that has a trapping site is lower than the average local threshold voltage, and vise versa. The strategically located trap at the site that has lower local threshold voltage and higher local carrier concentration enlarges the threshold voltage shift by RTS. The model Eq. (1) also suggests that  $\Delta V_{\rm th}$  obeys not normal but log-normal distribution if the local threshold voltage  $V_{{\rm th}j}$ fluctuates in normal distribution. The standard deviation of the local threshold voltage fluctuation is related to the surface potential fluctuation as  $\sigma_{V_{{\rm th}j}} = n\sigma_{\phi_{\rm s}}$ . The standard deviation of  $\log(\Delta V_{\rm th})$  is, therefore, expressed as

$$\sigma_{\log(\Delta V_{\rm th})} = \frac{q}{kT} \sigma_{\phi_{\rm s}}.$$
 (2)

The power spectrum of the surface potential fluctuation by random discrete dopant is calculated using the approximate Green's function for the electrostatic potential in the depletion layer[11]. The standard deviation of the surface potential is obtained by integrating the power spectrum as

$$\sigma_{\phi_{\rm s}} = \sqrt{\frac{N_{\rm A}}{4\pi}} \left(\frac{q}{\epsilon_{\rm ox} + \epsilon_{\rm Si}}\right) \sqrt{\int_{q_{\rm min}}^{q_{\rm max}} \frac{1 - e^{-qw}}{Q_{\rm s}^2 + q^2} dq} \quad (3)$$
$$\sim c_{\phi_{\rm s}} \sqrt{\frac{N_{\rm A}}{4\pi Q_{\rm s}}} \left(\frac{q}{\epsilon_{\rm ox} + \epsilon_{\rm Si}}\right)$$
$$\times \sqrt{\arctan\left(\frac{q_{\rm max}}{Q_{\rm s}}\right) - \arctan\left(\frac{q_{\rm min}}{Q_{\rm s}}\right)}, \quad (4)$$

where  $N_{\rm A}$  is the dopant concentration, w is the depletion layer width, and  $Q_{\rm s} = (C_{\rm ox} + C_{\rm dep})/(\epsilon_{\rm ox} + \epsilon_{\rm Si})$ . The minimum wavenumber,  $q_{\rm min}$ , is determined by the longer of the channel width and the channel length as shown in Fig. 7. The maximum wavenumber,  $q_{\text{max}}$ , is determined by the diameter of the area in which the conductivity is affected by trapping of a single carrier as shown in Fig. 6. The term  $1 - e^{-qw}$  was replaced with a constant  $c_{\phi_s}$  in order to integrate the power spectrum analytically. The constant  $c_{\phi_s}$  is treated as a fitting parameter and set to 0.2 in this work.

The proposed analytical model defined by Eqs. (2), (4) well describes not only the numerical simulation results as shown in Fig. 5 (b), but also the measured results as shown in Fig. 8. Substrate doping concentration dependence of  $\sigma_{\log(\Delta V_{\rm th})}$  is also expressed by the proposed model. Note that  $\sigma_{\log(\Delta V_{\rm th})}$  is proportional to  $N_{\rm A}^{1/2}$  as shown in Fig. 9 (b), while  $\sigma_{V_{\rm th}}$  is proportional to  $N_{\rm A}^{1/4}$  [9] as shown in Fig. 9 (a).

Finally, an impact of the threshold voltage shift on flash memories and SRAMs is estimated by the proposed model as shown in Fig. 10. The coupling coefficient of flash memories is assumed to be  $\alpha_{\rm cg} = 0.6$ . As the dimension shrinks, both 50 percentile and 99.9 percentile increase in proportional to  $1/W_{\rm eff}L_{\rm eff}$ . For 50nm-flash, 99.9 percentile exceeds 100mV, which suggests that the threshold voltage shift by RTS should be considered in flash memory design because the margin for the threshold voltage is only several hundreds mV in multi-level-cell flash[2][12]. In contrast, 99.9 percentile for 50nm-SRAM is far below 10mV and little attention would be necessary for SRAMs.

### IV. CONCLUSION

The discrete dopant effects on the threshold voltage shift of MOSFETs by RTS have been discussed. Considering discrete dopant effects, the threshold voltage shift by RTS follows the log-normal distribution. An analytical model for the distribution of the threshold voltage shift has been presented and confirmed by using numerical simulation results and measured results. The proposed model predicts that the threshold voltage shift by RTS should be considered in the design of flash memories of 50nm and beyond.

#### ACKNOWLEDGMENT

The authors would like to thank Shun'ichi Narumi, Yoshihiro Ikeda, Makoto Ogasawara, Shiro Kamohara, Yutaka Okuyama, Hideaki Kurata, Yoshitaka Sasago, and Hitoshi Kume for their technical supports and discussions.

#### REFERENCES

- [1] K. S. Ralls et al., Phys. Rev. Lett., vol.52, p. 228, 1984.
- [2] H. Kurata et al., Symposium on VLSI Circuits, 13.3, 2006.
- [3] M. J. Uren et al., Appl. Phys. Lett., vol.47, p. 1195, 1985.
- [4] L. K. J. Vandamme et al., Solid-State Electron., vol. 42, p. 901, 1998.
- [5] A. Asenov et al., IEEE Tarns. Electron Devices, vol. 50, p. 839, 2003.
- [6] R. W. Keyes, IEEE J. Solid-State Circuits, vol. SSC-10, p. 245, 1975.
- [7] N. Sano et al., IEDM Tech. Dig., p. 275, 2000.
- [8] S. M. Sze, Physics of Semiconductor Devices, second edition, John Wiley & Sons, 1981.
- [9] P. A. Stolk et al., IEEE Trans. Electron Devices, vol. 45, p. 1960, 1998.
- [10] M-. H. Tsai et al., IEEE Electron Device Lett., vol. 15, p. 504, 1994.
- [11] G. Slavcheva et al., J Appl. Phys., vol. 91, p. 4326, 2002.
- [12] S. Lee et al., ISSCC Dig. Tech. Papers, p. 52, 2004.



Fig. 1. Majority (upper) and minority (lower) carrier current flowing through a *p*-type silicon cube as a function of grid spacing *d* using three dimensional device simulation. Two electrodes are placed on the opposite sides of the cube.  $E_{\rm B0}$  is the ionization energy used in the calculation of incomplete ionization of dopants.  $r_{\rm imp}$  is a function of *d*. The band degeneracy factor is set to 1. The averages and the  $\pm 3\sigma$ -deviations of 30 distinct samples are shown for the discrete dopant case.







Fig. 3. Definition of the threshold voltage shift  $\Delta V_{\rm th}$  by RTS.



Fig. 4. Simulated  $V_{\rm g}$ - $I_{\rm d}$  characteristics of MOSFETs. 30 distinct samples are used for the discrete dopant case.



Fig. 5. The Cumulative distribution function of the simulated (a) threshold voltage  $V_{\rm th}$ , and (b) threshold voltage shift  $\Delta V_{\rm th}$  by RTS for 30 distinct samples. A trap is located at the center of the channel. The parameter  $c_{\phi_{\rm S}} = 0.2$  is used in the analytical model for  $\Delta V_{\rm th}$ .



Fig. 6. Schematic diagram of the mobile carrier concentration around the trapped electron.



Fig. 7. Schematic diagram of the minimum and the maximum wavenumbers of the surface potential which affects the local threshold voltage fluctuation.

Weff/Leff=24nm/50nm, tox=9nm, Na=5e17cm-3

-OMEAS. (CONFI. LEVEL=99%)

SIM. (Nt=5e10cm<sup>-2</sup>eV

 $10^{-2}$ 



(b) Fig. 9. The simulated standard deviation of (a) the threshold voltage  $V_{\rm th}$ , and (b) the logarithm of the threshold voltage shift  $\Delta V_{\rm th}$  by RTS as a function of substrate doping concentration. Error bars indicate 99% confidence intervals.



Fig. 8. Measured and simulated threshold voltage shift. Poisson distribution is assumed for the number of traps at the interface with the average  $N_{\rm t}(kT/q)W_{\rm eff}L_{\rm eff}$ , where  $N_{\rm t}$  is the trap density per unit area and unit energy.

ΔVth (V)

 $10^{-1}$ 

Fig. 10. The 50 and 99.9 percentiles of the simulated threshold voltage shift caused by single electron RTS.

99.99

99.9

99

95

90

80

70

<sub>50</sub> l

G

CDF (%)

3.0

2.0

1.0

سا<sub>0</sub>0.0 10