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Extraction of 3D interconnect impedances using edge elements without gauge condition

F.Charlet[a], J.F.Carpentier[b]
[a]CEA-DRT-LETI/DTS -CEA/GRE - 17, avenue des Martyrs. 38054 Grenoble CEDEX 9 – France
[b]STMicroelectronics, 850 rue Jean Monnet, 38291 Crolles, France

Abstract **-We present an efficient simulation method for the extraction of frequency dependant interconnect impedances. Our method is based on a symmetrical (A,T) system approximated by the use of edge finite elements. It doesn't need gauge condition and the matrix system has good convergence properties.**

Qualitative results on eddy current and return current effects are presented and a first numerical validation is made.

I. INTRODUCTION

Due to the increasing clock frequencies and the decreasing sizes, the evaluation of the electric parasitic effects becomes essential for the reliability of microelectronic circuits. After investigating the 3D calculation of capacitance matrices using a very efficient method [1], we now focus our work on the extraction of frequency dependant impedances of interconnects. At the present time, the most used methods are based on an integral equation approach where the unknowns are the current density "J" and the electric potential "V". The main advantage is to reduce the problem inside the lines, but these methods are known to lead to dense and non symmetrical matrices. Moreover they have to handle the current conservation constraint "div J=0" that is reputed to give numerical problems. Different methods have been developed to overcome these difficulties ([2][3][4]).

Another way to solve this problem is to use a volume finite element method coupling two electromagnetic unknowns on the whole domain. Different choices are available [5], but the (A,V) formulation is probably the most used(A is the magnetic vector). To obtain the unicity of (A,V) we need to add a gauge condition on "div A". When we use continuous nodal finite elements for A, gauging is an obligation to avoid numerical difficulties [6].

Recently, some authors proved that the use of edge "Nedelec-Whitney" elements allows to avoid gauging on vector magnetic potential ([9][6]). Even if the matrix is singular, the conjugate gradient method may converge [8] but in this case the convergence of the method is very sensitive to the approximation of the right hand side (RHS) of the system. To obtain a consistent system, the RHS need to be in the range of the "curl curl" matrix[8][5]. An other advantage is that the edge elements naturally take into account the normal discontinuity and the convenient boundary conditions on A [5].

II. (A,T) FORMULATION

From the previous considerations, we choose to express the problem using the (A,T) formulation given in [9], where A is the vector magnetic potential defined in the whole domain ($\nabla\times A=B$) and T is the vector electric potential such as ,inside the conductor regions, we have the relation "$\nabla\times T=J$",. The system is

$$(S)\quad \begin{cases} \nabla\times\nabla\times A-\mu\nabla\times T=0 \ \textit{in the whole space} \\ \nabla\times\sigma^{-1}\nabla\times T+j\omega\nabla\times A=0 \quad \textit{inside conductor} \end{cases}$$

where ω is the pulsation and σ the conductor's conductivity.

Approximation of (S) may give a symmetrical system. Using A and T as unknowns the terms of the system intrinsically have divergence equal to zero, then this formulation avoid the use of gauge constraints that may penalise the convergence.

II.1 Conditions on T

The vector electric potential is separated in two parts ($T=T_0+\tilde{T}$). T_0 is the source electric potential such as $J_0=\nabla\times T_0$ is the direct current (DC) inside the conductor. J_0 is obtained by solving the Laplace equation inside the conductor with scalar electric potential as unknown. J_0 is chosen to generate an imposed current I_0.

T_0 must be defined in the whole space. T0 may be chosen to be null in the Z direction ($T0=[T_{0x},T_{0y},0]$)[10] and $T_0=0$ on top of the structure. With these conditions T_0 is easy to obtain on a regular grid using the relations:

$$\begin{cases} T_{0x}(i,j,k)h_{ix}=T_{0x}(i,j,k+1)h_{ix}+J_{0y}(i,j,k)h_{ix}h_{kz} \\ T_{0y}(i,j,k)h_{jy}=T_{0y}(i,j,k+1)h_{jy}-J_{0x}(i,j,k)h_{jy}h_{kz} \end{cases}$$

where (h_{ix}, h_{iy}, h_{kz}) are the steps of the grid.

Most of the time, the structure to be considered is a truncated part of a conducting region, with given conditions on electrode parts. In this case, our method is equivalent to impose a return current path from the electrodes to the bottom of the grid. In figures 1.a, 1.b , we present an illustration on a single line example. J_0 is first calculated inside the line with given potential conditions on the electrodes E0, E1. Then T_0 is calculated in the whole space with the previous method.

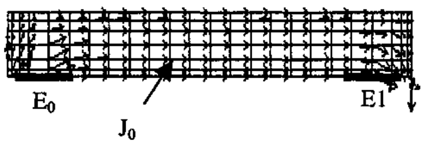

Fig.1.a : DC Density current J0 inside a single line

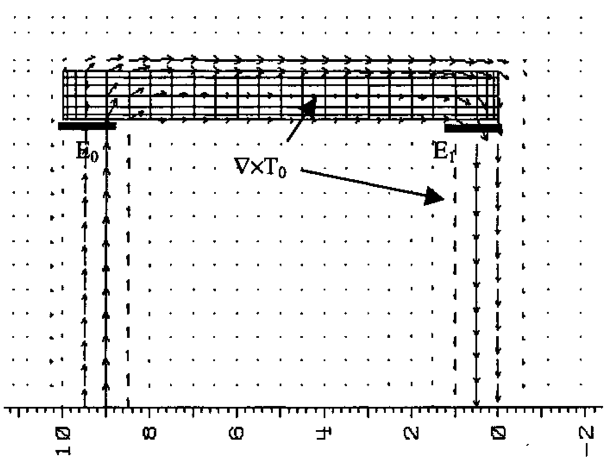

Fig 1.b: ∇×T0 derived from J0 of fig. 1.a

The second part of the vector electric potential $\tilde{T}$ is defined such as its total current is zero. The boundary conditions expressing that are "$\tilde{T}\times n=0$"on conductors frontiers, where n is the normal vector.

II.2 Boundary conditions on A

A is calculated on a parallelepiped box. On the boundaries of this box, we assume that magnetic field is the same than in the DC case. We use the "Biot-Savart" law on the DC to set the boundary conditions:

$$\vec{A}(P)=\frac{\mu_0}{4\pi}\int_{\Omega}\frac{\vec{\nabla}\times\vec{T}_0(Q)}{\|P-Q\|}dQ$$

where P belongs to the box boundaries

III.GROUND PLANE

When the conductor structure is in the free space, we choose the bottom of the grid domain far enough to have no influence on the results. When the structure set on a substrate or a conducting support, we may couple the previous problem in the dielectric domain with another in the conducting support. By this way, the return current in the support may be calculated. In figures 2.a, 2.b, 2.c we present the real part of the current density of the return current inside a metal plane underneath a single line. In high frequency domain, an attraction by the line may be observed(fig.2.c).

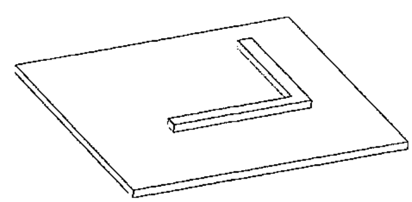

Fig.2.a: A single copper line (1μ width) set on a copper plane (plane-line distance is 2μ).

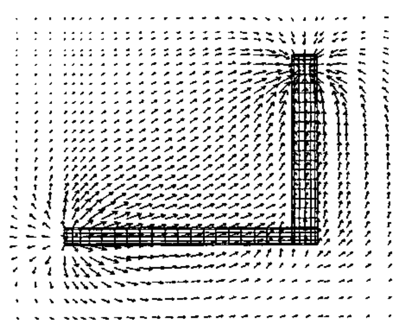

Fig.2.b: Geometry of fig.2, Frequency 1GHz:
Real part of the return density current inside the metal plane.

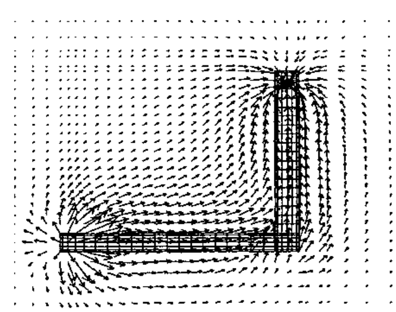

Fig.2.c: Geometry of fig.2, Frequency 10GHz:
Real part of the return density current inside the metal plane.

IV. MESHING

We use edge elements to approximate both A and T[9]. As the first equation of (S) is defined in the whole space, we use a regular 3D grid to approximate the magnetic potential A. Then assembling the rigidity matrix associated to "$\nabla\times\nabla\times A$" is not necessary. T is calculated on a unstructured mesh that models the inside of the conductors. A coarser grid for A than for T is used. The projection method from one grid to the other save the energy of the fields and the symmetry of the system. examples presenting the conductor mesh approximating the electric potential and the regular grid approximating the magnetic potential are in figures 3, 4.

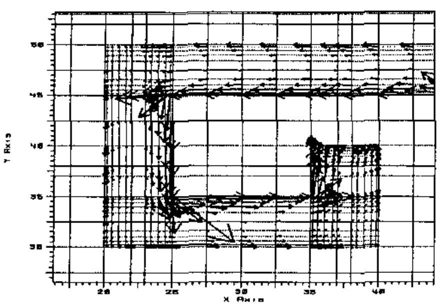

Fig.3 : Zoom on a plane view of the fig.5 structure : Real part of the current density at 20GHz. The conducting mesh and the regular grid are drawn . Eddy current effects are visible.

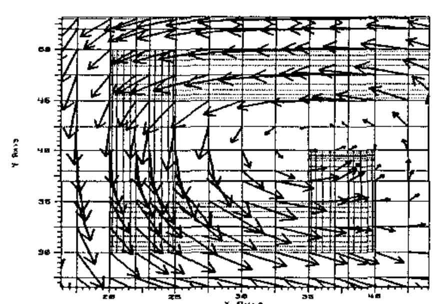

Fig.4 : Zoom on a plane view of the fig.5 structure: Real part of magnetic potential at 20GHz. The conducting mesh and the regular grid are drawn .

V. RESULTS

A first result on a induction loop set on a ground plane(fig.5) is presented. One end of the loop is connected to the ground plane and we calculate the impedance (R+jωL) at the other end. Geometry is not realistic, but this test case clearly shows the eddy current effects.

Fig.6 and Fig.7 present comparative results between our work and the program Fasthenry[3](freeware version 3.0). The inductance and the low frequency resistances are very close. High frequency resistances are higher in our program.

On this test case Fasthenry and our program have comparable computation times. Actually, our program is more performant when the geometry is more complex (Fasthenry becomes very expensive when the number of discretization filaments is important). Moreover, our program is more efficient to deal with low conductive and thick substrate or structures with "no filament" shapes.

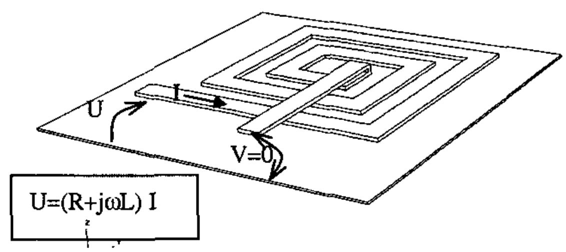

Fig.5 : Test case: Inductance loop on a ground plane. Lines and ground plane are Copper (σ=50 10^6 S/m) , lines are 5 µm width, 1 µm height. Lower level of the loop is 4 µm far from ground plane, Higher level is 6 µm far from the plane. Ground plane is 80 µm side and 1µm thick.

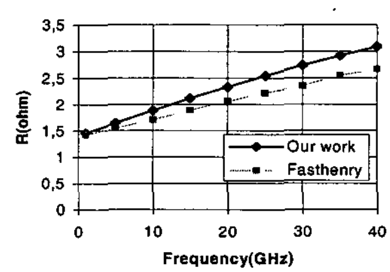

Fig .6 : Resistance of the test case of fig.5 Comparisons with Fasthenry

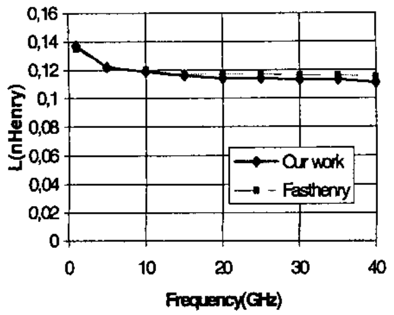

Fig .7 :Inductance of the test case of fig.5
Comparisons with Fasthenry

VI. SUMMARY AND CONCLUSION

A finite element method based on a A-T formulation and a edge approximation is presented. This formulation has good convergence properties due to the absence of gauge conditions. Two separated grids are used to approximate the fields which make the meshing easier and increases the software performances.

First numerical results have been presented that are going to be extended with measurement comparisons in multiturns spiral inductors.

ACKNOWLEGMENT

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