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Automatic Order Reduction of Thermo-Electric Model for Micro-Ignition Unit

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***Abstract*-In this paper we present an automatic, Krylov-subspace-based order reduction of a thermo-electric model, describing a novel type of micropropulsion device. Model order reduction is essential for achieving easily to evaluate, yet accurate macromodel of the device, and is needed for simulating both the microthruster array and its driving circuitry. We present numerical simulation results of the full finite element model and the reduced order model that describes the transient thermo-electric behavior. A comparison between Krylov-subspace-based order reduction and order reduction using control theoretical approaches, such as Balanced Truncation Approximation (BTA), has been performed. For the first time a Single-Input-Single-Output (SISO) setup for the reduction algorithm was sufficient to approximate the complete time-dependent temperature distribution of the device.**

I. INTRODUCTION

A new class of high energy MEMS actuator integrates solid fuel with three silicon micromachined wafers [1]. It delivers either an impulse-bit thrust or pressure waves within a sub millimeter volume of silicon, by producing a high amount of energy from an ignitable substance contained within the microsystem. The microthruster fuel is ignited by passing an electric current through a polysilicon resistor embedded in a dielectric membrane, as shown in Fig. 1. After the ignition

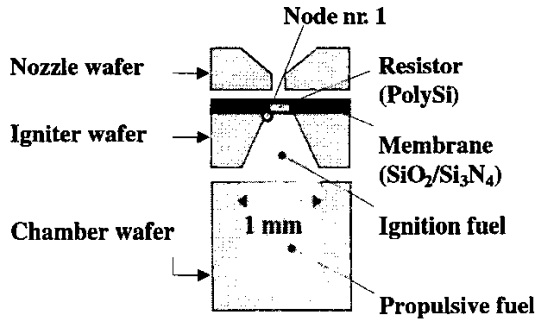


Fig. 1 Microthruster Structure.

phase, sustained combustion takes place and forms a high-pressure, high-temperature gas mixture. Under the pressure of the gas the membrane ruptures, and an impulse is imparted to the carrier frame as the gas escapes from the tank.

The present work considers the initial heating phase of the fuel, right up to the onset of ignition, described through the following equations:

$$\nabla\bullet(\kappa\nabla T) + Q - \rho C_p \frac{\partial T}{\partial t} = 0,\ Q = \frac{j^2}{\sigma} \qquad (1)$$

where κ is the thermal conductivity, C_p is the specific heat capacity, ρ is the mass density, T is the temperature distribution, Q is the heat generation, j is the electric current density vector and σ is the specific electric conductivity.

We use a two dimensional axi-symmetric model, which after the finite element (FE) based spatial discretization of the governing equations (1) results in a linear system of about 1000 ordinary differential equations (ODEs) of the form:

$$\begin{aligned} [C]\dot{T} + [K]T &= FI(t)^2R \\ y &= E^T \cdot T \end{aligned} \qquad (2)$$

where $[K],[C] \in R^{nxn}$ are the global heat conductivity and heat capacity matrix, $T(t),F,E \in R^n$ are the temperature (state), the load and the output vector respectively and n is the dimension of the system. The electric current $I(t)$ through the heater with electric resistivity R is the input to the system.

A numerical simulation result of the full finite element model is shown in Fig. 2.

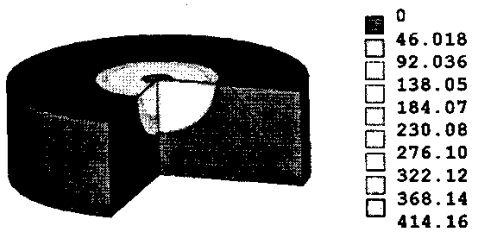


Fig. 2 Temperature distribution within the igniting wafer after 0.3s of heating with 80 mW power; $T_{ref} = 273K$.

As the above number of equations n is too large for an efficient system simulation, e.g., using behavioral simulators such as SABER or ELDO or a circuit simulator such as SPICE, a reduced order model:

$$[C]_r\dot{T}_r + [K]_r T_r = F_r I(t)^2 R$$
$$y_r = E_r^T \cdot T_r \tag{3}$$

with the dimension $r \ll n$ was generated.
The equations (2) and (3) as written above, represent a Single-Input-Single-Output system. The present work considers a special case when E is an identity matrix, that is, $y = T$, which we call a Single-Input-*Complete* Output (SICO) system.

II. MODEL ORDER REDUCTION

Conventionally, the reduction of thermo-electric models for MEMS devices is performed through a lumped-element decomposition of the model followed by parameter optimization [2]. Such a non-automatic approach requires the designer to choose the right reduced model structure without strict guidelines, and to perform a time-consuming parametrization including indispensable simulation of the full-scale model. We propose a different, automatic order reduction approach, suitable for the linear thermo-electric model of the microthruster device, and based on an Arnoldi algorithm [3].

Small Linear Systems

Control theory already has a number of well established tools for the automatic model reduction of stable linear systems [5]. Each linear dynamic system (2) of order n has n so-called Hankel singular values σ_i which can be computed by solving the Lyapunov equations:

$$AP + PA^T = -bb^T$$
$$A^T Q + QA = -EE^T \tag{4}$$

with $A = -[K]^{-1}[C]$, *and* $b = -[K]^{-1}F$ for the controllability grammian P and the observability grammian Q. Hereby P is to be seen as the connection between the input function and the state vector (in our case temperature), and Q as the connection between the state vector and the system output. They indicate which states can be controlled and which ones can be observed. The Hankel singular values of the original dynamic system (2) are equal to the square root of the eigenvalues of the product of P and Q.

$$\sigma_i = \sqrt{\lambda_i(P \cdot Q)} \tag{5}$$

Once these values are known, there are a number of model reduction methods with guaranteed error bounds for the difference between the transfer function of an original n-dimensional system and its reduced r-dimensional system, as follows:

$$\|G - \hat{G}\|_\infty \le 2(\sigma_{r+1} + \ldots + \sigma_n) \tag{6}$$

provided that the Hankel singular values have been sorted in descending order. The basic idea behind the order reduction using a balanced truncation approximation (BTA) after B. C. Moore [5] is to transform the state vector in such a way that $P = Q$, so that all the states which are simultaneously weekly controllable and weekly observable can be excluded. Practically, this means to exclude those equations from the system (2) which correspond to the last (and smallest) $n - r$ Hankel singular values. The fact that the computation of those singular values doesn't depend on the input function(s), makes model reduction based on this method fully automatic. However, the time required to solve the Lyapunov equations, as well as to perform a singular value decomposition grows as the cubic power of the number of equations $O(n^3)$. Hence, for computational reasons, these order reduction methods are limited to relatively small systems, typically with state vector dimensions in the range of 1000 to 2000.

Large Linear Systems

By performing model reduction on equation (2), the goal is to effectively describe the behavior of the vector T in time, through some low-dimensional subspace as:

$$= V \cdot T_r + \varepsilon, T_r \in R^r, r \ll \tag{7}$$

Equation (7) states that, with the exception of a small error described by vector $\varepsilon \in R^n$, the possible movement of the n-dimensional vector T belongs, for all times, to a r-dimensional subspace, and is determined by an $n \times r$ transformation matrix V. The matrix V is composed from r n-dimensional vectors that form a basis for the reduced subspace, and the r-dimensional vector T_r represents a new low order set of coordinates for the given basis. When the subspace is found equation (2) is projected onto it, and this projection process produces a reduced order system (3).
It has been shown that, in the case of large-scale systems, very good candidates for the required low-order subspace of the equation (7) are Krylov subspaces [6].
The basic idea behind the Krylov-subspace-based Arnoldi algorithm is to write down the transfer function of (2) in the frequency domain using a Taylor series in the Laplace variable $s_0 = 0$:

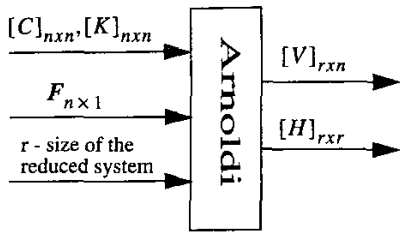
$$\{G(s)\} = -\sum_{i=0}^{\infty} \{m\}_i s \tag{8}$$

where $\{m\}_i = E^T(-C^{-1}K)^{-(i+1)}C^{-1}F$ is called the *ith* moment, and then to find a much lower order system (3) whose transfer function $\{G_r(s)\}$ has the same moments as $\{G(s)\}$ up to the degree r.
The moments are not computed explicitly. Instead, a Krylov subspace of the dimension r:

$$K_r\{A, b\} = span\{b, A^2 b, \ldots, A^{r-1} b\}$$
$$with\ A = -[K]^{-1}[C],\ and\ b = -[K]^{-1}F \tag{9}$$

is used, and through the computation of an orthogonal basis for this subspace, the matrices $[C]_r$ and $[K]_r$ and the load vector F_r of the reduced system are computed without tak-

ing into account the output matrix E. All the inputs and outputs of the Arnoldi algorithm are shown in Fig. 3.



$$C]_r = H = V^T(-K^{-1}C)V, [K]_r = -I, F_r = \left\|-K^{-1}F\right\|\vec{e}_1$$

Fig. 3 Model reduction by the Arnoldi process.

The property of the Krylov subspace (9) is such that the first r moments of $\{G_r(s)\}$ and $\{G(s)\}$ match, as required.

III. RESULTS

The decay of the Hankel singular values for the microthruster model is shown in Fig. 4.

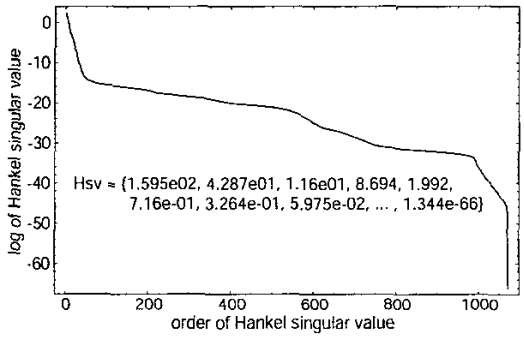


Fig. 4 Decay of the Hankel singular values for the microthruster model.

For the specified error bound (equation (6)) of 0.1 a BTA algorithm has estimated the reduced system to be of the order 7. An equation system (2) containing 1071 ODEs was reduced to 7 ODEs using each of the two presented algorithms (Fig. 5). A maximal relative error by Arnoldi-based reduction doesn't exceed 4% during the initial phase, whereas this error by an optimal BTA-based reduction amounts 0.3% within the steady-state phase (Fig. 6). The corresponding transfer functions of the full-scale and both optimal and non-optimal reduced systems are shown in Fig. 7.
For the thermo-electric model, the simple SISO setup for the Arnoldi algorithm was sufficient to approximate not only a single output response but also the transient thermal response in all the finite element nodes of the microthruster. Fig. 8 shows the mean relative difference for all the nodes between the full-scale and the reduced different order models. Already for the reduced system of order 20 a maximal mean relative

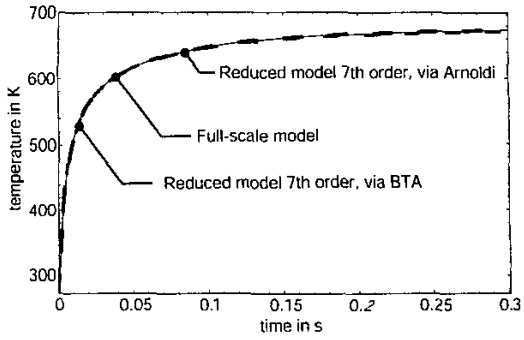


Fig. 5 Solution of the full system and of the 7th order reduced system for a single node (node 1 in Fig. 1).

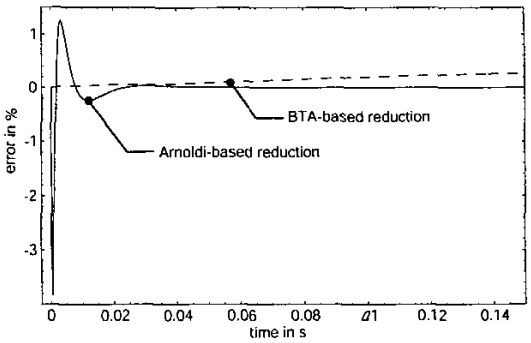


Fig. 6 Relative error corresponding to plots in Fig. 5 during the initial 0.15s.

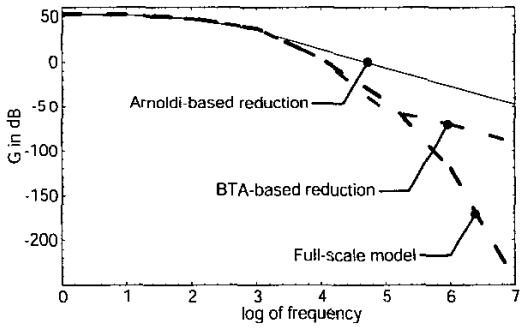


Fig. 7 SISO transfer function of the full-scale and of the reduced 7th order system corresponding to the node 1 in Fig. 1.

error amounts only 0.14%. Hence it was possible, after the simulation of the reduced model, to recover the solution for all the 1071 nodes by using the equation (7). In this case, the Arnoldi reduction algorithm can be viewed as a projection (equation (7)) from the full space to the reduced Krylov space (9), with an identity output matrix (SICO).
Currently, a software package is under development which generates reduced-order linear models directly from ANSYS data files containing more then 30 000 degrees of freedom. It forms a netlist suitable for the behavioral simulator SABER

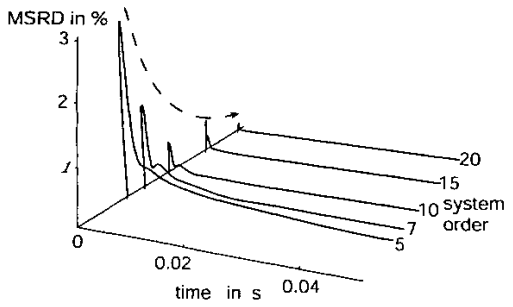


Fig. 8 Mean square relative difference (MSRD) for all the nodes during the initial 0.05s, for an Arnoldi-based reduction from order 1071 to 20, 15 10 and 5.

(Fig. 9) from the three-dimensional geometry and partial differential equations of the model.

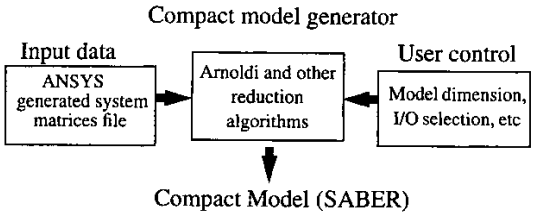


Fig. 9 Software block diagram.

The structure of the SABER input file is shown in Fig. 10.

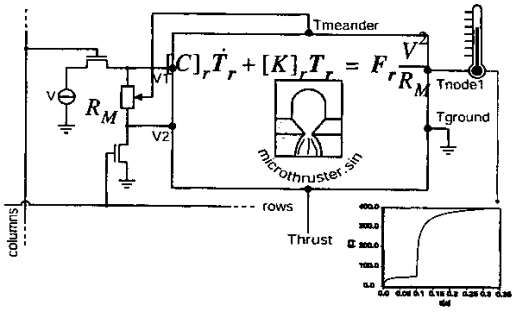


Fig. 10 Structure of the SABER model.

The template microthruster.sin is implemented in the form of ODEs (equation (3)). The SABER plot shows the temperature development in node 1 (Fig. 1) for the case when the preheating was performed, in order to improve the subsequent sustained combustion. Through back coupling of the meander resistor an approximation is made that the heat power $I(t)^2 R$ is uniformly distributed over the meander. This has the advantage that the monitoring of the temperature through a change of resistance is possible, and certain changes in design (such as the change of the meander resistivity) are still possible after the model reduction phase.

IV. DISCUSSION

Both algorithms presented offer the possibility for the automatic order reduction of ODE systems. The BTA method additionally offers reduced system order estimation for a given error. However, due to the computational cost of $O(n^3)$ for the BTA-method it is impractical for large-scale systems. The computational cost for Arnoldi-based reduction, which is less than $O(n^2)$, makes it suitable for MEMS models with more than 10 000 degrees of freedom. For the first time a Single-Input-Single-Output setup for the Arnoldi reduction algorithm was used to approximate a linear Single-Input-Complete-Output thermo-electric system. Model reduction is automatic and based on the original system matrices only. In this way, a time-consuming simulation of the full-scale model is circumvented with no loss of spatial distribution information. Furthermore, the model reduction is not limited to this particular device, but can be applied to a wide spectrum of important thermo-electric MEMS devices, as well as for applications in bonding, IC temperature control, packaging and so on.

V. ACKNOWLEDGMENTS

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