An Efficient Algorithm for 3D Interconnect Capacitance Extraction Considering Floating Conductors

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***Abstract* – With the inclusion of floating conductors in integrated circuits, conventional simulation tools exhibit prohibitive calculation times. A new simulation tool, called ICARE, was developed to extract efficiently 3D capacitance matrix of interconnect structures embedded in a multi-layered dielectric environment. Using the so-called fictitious domain method, it leads to a coupled linear system, with the potential on a regular 3D grid of a simple-shaped domain, including the dielectric media, and the charge on a mesh of the conductor surfaces as unknowns. A specific adaptation of this numerical method is introduced, giving the possibility of taking into account the floating conductors in an efficient way. As a result, realistic structures, consisting of one or two conducting lines surrounded by a great number of floating conductors, are simulated.**

I. INTRODUCTION

The insertion of floating dummy metals contribute to reduce the pattern-dependent variations of the dielectric and metal thickness in interconnect realization processes; however, such metallic dummies have a strong impact on the electrical characteristics of interconnects, such as signal delay or crosstalk [1,2]. Most of the available parasitic extraction tools do not have specific treatment for floating conductors: a global calculation is realized including floating conductors, at the cost of CPU time when there is a lot of floating dummies, and a reduced capacitance matrix is extracted from the global capacitance matrix by considering that the global charge on the surface of a floating conductor is equal to zero.

In this paper, our numerical method is presented with specific adaptations dealing with floating conductors; it is particularly efficient in the case of a great number of floating conductors, as in modern integrated circuits.

In chapter II., the numerical solver from which our method was derived is firstly introduced. Then, the so-called fictitious domain method with Lagrange multipliers, which is the basis of this solver, is developed with corresponding approximations of potential and charge. Finally, the particular adjustment of the discrete formulation is presented, extending capability to cases where some of the conductors are floating.

In chapter III., numerical results are shown and analysed, confirming the efficiency and speed of our algorithm compared to a global calculation and reduction method.

II. AN EFFICIENT ALGORITHM TO TAKE FLOATING CONDUCTORS INTO ACCOUNT

II.1. FORMULATION AND DISCRETIZATION WITHOUT FLOATING CONDUCTORS

Static capacitances are calculated from the distribution of charge density on the surfaces of conductors:

$$Q_i = C_{ii} V_i + \sum_{j=1}^{nb\_cond} C_{ij}(V_i - V_j)\ \forall i = 1,.., nb\_cond$$

Charge density on the surface of conductors is calculated from the normal derivative of the potential. The potential u in the dielectric media ω is the solution of the Laplace equation with boundary conditions on the surfaces of the n conductors $\gamma = \bigcup_{1 \le i \le n} \gamma_i$:

$$(P)\begin{cases} \nabla(\varepsilon \nabla u) = 0 \text{ in } \omega \\ u = g \text{ on } \gamma \end{cases}$$

Extracting the static capacitances of a set of n conductors is equivalent to calculate the potential solution of (P) successively with n different sets of boundary conditions (condition n° i: $g = 1$ on γ_i and $g = 0$ on $\bigcup_{1 \le i \le n} \gamma_i - \{\gamma_i\}$).

The main idea of the Fictitious Domain Method is to replace a problem on a domain of complex geometry by another one on a simple shape domain, the fictitious domain (Figure 1), thus allowing the use of a regular grid of that domain and of fast solvers.

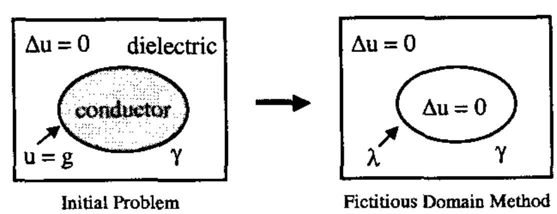

Figure 1: principle of Fictitious Domain Methods

Using the Fictitious Domain Formulation for solving (P) [3], the potential computation is artificially extended inside the conductors, and the boundary condition is taken into account via the introduction of a Lagrange multiplier λ. Defined on the surface mesh, it can be interpreted as a surface charge. A variational formulation is finally obtained with following unknowns: $\tilde{u}$ is the potential in Ω (union of the dielectric region ω with permittivity ε and of the conductor regions), and q_c is the charge on the surface of conductors.

$$(PV)\begin{cases}\int_\Omega \varepsilon \nabla \tilde{u} \nabla v dx = \int_\gamma q_c v d\gamma & \forall v \in X \\ \int_\gamma \mu(\tilde{u} - g) d\gamma = 0 & \forall \mu \in M\end{cases}$$

where X and M are proper spaces.

The potential is the restriction of $\tilde{u}$ to the dielectric region. The variational problem (PV) is discretized and an amelioration to the standard approximation is proposed to properly take into account the discontinuity of the potential on the surface conductors [3]: the potential on the fictitious domain is approached by $\tilde{u} = u^r + u^{nr}$, where u^r is the regular part, which is approached using Q1-finite elements basis on a regular grid of parallelepipeds, and u^{nr} is the irregular part, due to the jump of the normal derivative of the potential on the conductor surfaces γ. This part is approached using non-regular P0-functions Φ_{K_i}, defined on parallelepipeds, which have a non-zero intersection with γ. The surface charge q_c is approached by a vector Q, using P0-finite elements on a mesh of the surface γ. Finally, the vectors U and Q are solutions of the discretized problem (P_h): to find $(U,Q) \in X_h \times M_h$ solution of

$$\begin{cases} AU + B^T Q = 0 \\ BU - CQ = -G \end{cases}$$

where X_h (respectively M_h) is a N_v (resp. N_s)-dimension vectorial space included in X (resp. M). The function basis of M_h used for decomposition is noted $\{\varphi_k\}_{1 \le k \le Ns}$.

II.2. Modifications to take floating conductors into account

Considering now that m of the n conductors are floating: the potential is an unknown constant and the global surface charge is equal to zero for a floating conductor.

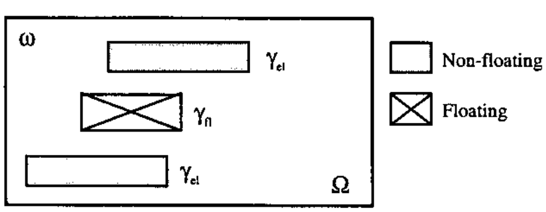

Figure 2: notations

According to figure 2, $\gamma = \gamma_{el} \cup \gamma_{fl}$ where γ_{el} is the part of the boundary with potential boundary condition and γ_{fl} is the part of the boundary with integral condition on the charge. V is the set of the m unknown potentials on the surface of each floating conductor.

The discretized problem P_h is modified in order to take into account m additional unknowns for the potential on the surface of each floating conductor and m additional equations to traduce the nullity of the global charge on the surface of each floating conductor.

With these m additional unknowns, the equation $BU - CQ = -G$ becomes $BU - CQ = -G' - PV$ where P is a $N_s \times m$ matrix defined by :

$$P_{i,j} = \begin{cases} \int_\gamma \varphi_i d\gamma = |\gamma_i| \text{ si } \gamma_i \in \gamma_{fl} \\ 0 \text{ si } \gamma_i \notin \gamma_{fl} \end{cases}$$

$$G_k = \int_\gamma g \varphi_k d\gamma \quad \text{becomes} \quad G'_k = \int_{\gamma_{fl}} g \varphi_k d\gamma$$

The m additional equations $\int_{\gamma_i} q_c d\gamma = 0 \ \forall i / \gamma_i \in \gamma_{fl}$ can be written with the matrix equation $P^T Q = 0$. Finally the following system is obtained:

$$(Q_h)\begin{cases} AU + B^T Q = 0 \\ BU - CQ + PV = -G' \\ P^T Q = 0 \end{cases}$$

and solved this way:

$$\begin{cases} U = -A^{-1} B^T Q \\ \begin{pmatrix} BA^{-1}B^T + C & -P \\ -P^T & 0 \end{pmatrix}\begin{pmatrix} Q \\ V \end{pmatrix} = \begin{pmatrix} G' \\ 0 \end{pmatrix} \end{cases}$$

With $M = \begin{pmatrix} BA^{-1}B^T + C & -P \\ -P^T & 0 \end{pmatrix}$, $X = \begin{pmatrix} Q \\ V \end{pmatrix}$ and $Y = \begin{pmatrix} G' \\ 0 \end{pmatrix}$, a symmetric system is obtained: $MX = Y$

In the case without floating conductors, the matrix M reduces to $BA^{-1}B^T$ and problem Q_h is identical to problem P_h. This case can be seen as a particular one from a more general case with floating conductors.

The system MX = Y is solved by the iterative Conjugate Gradient or GMRES algorithms; this resolution requires inversion of matrix A. Thanks to the regularity of the 3D mesh, A does not need to be stored, and its inversion is computed with a fast solver using FFT. It is noticed that a fast Poisson solver [4] takes advantage of the regularity of the equation and grid only in 2 directions. This allows to adapt the solver to the case of dielectric plane layers without extra cost. Moreover, C is diagonal and B sparse (B represents the coupling between the surface mesh and the grid).

III. NUMERICAL RESULTS

Our method was introduced in the software ICARE and was applied to a first numerical application, presented below, which demonstrates ICARE efficiency.

Figure 3 and 4 illustrate a silicon interconnect technology, with two conductor lines (1 μm wide and 300 μm long) situated at metallic level M2 and surrounded with square floating dummies located at both metallic levels M1 and M2. These conductors are embedded in a multi-layered dielectric with SiO_2 (thickness h = 2.8 μm), SiN (h=0.6 μm), and air (h=4 μm).

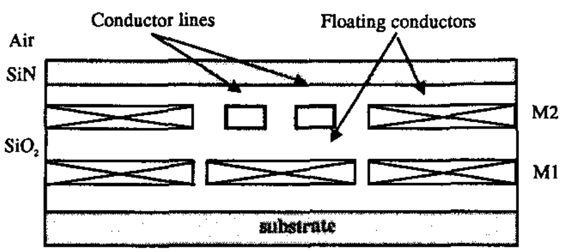

Figure 3 : cross section

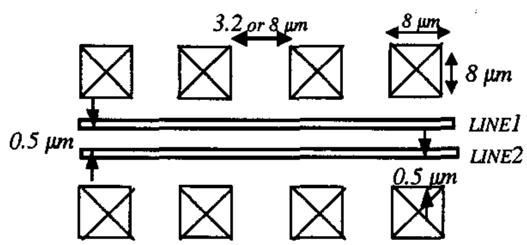

Figure 4 : horizontal section

Both the coupling capacitance between line1 and line2 and the capacitance of both lines in relation to substrate are determined as a function of dummy number and for three different configurations: two lines a) alone, b) with floating conductors all situated in M2 level and c) floating dummies distributed among both M1, and M2 levels (figure 3).

The dummies are considered either as floating or as non-floating conductors. The capacitance matrices corresponding to our method are validated by comparison to matrices obtained by reduction from global matrices.

Calculation with ICARE for 132 conductors including 130 floating	CPU time on a Sun Ultra 60
Global calculation and reduction	2.7 10^4 seconds ($\approx$ 7.5 hours)
Calculation with our method	4.2 10^3 seconds ($\approx$ 1 hour)

Table 1: calculation time comparison

The comparison of the CPU times between the two methods shows that our method is seven times as fast as global calculation methods (table 1). So, it may be used for heavy calculations, as required by designers for simulation of complex patterns.

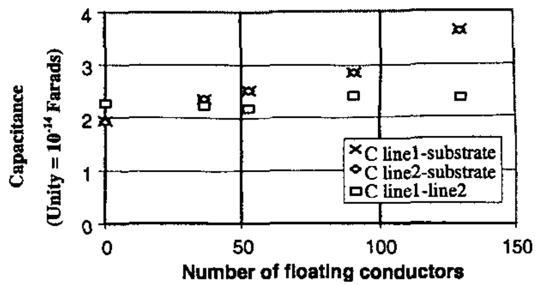

Figure 5: results of ICARE calculations

As expected, numerical results respect the symmetry of the structure: the capacitance of line1 in relation to substrate is equal to the capacitance between line2 and substrate. The presence of floating conductors raises the capacitance of line1 and line2 in relation to the substrate; this effect clearly increases with the number of floating conductors in M1 level.

Considering the case with no more than two lines as a reference, line1 to line2 coupling capacitance is reduced when all the floating conductors are located at level M2, whereas it increases when the floating conductors are in both M1 and M2 levels. Thus, by accurately taking into account the electric field dependence on the floating conductor patterns, ICARE greatly enhances the accuracy of designer's work for sensitive circuits.

Thanks to its speed, parametric analyses were conducted with ICARE at STMicroelectronics in order to optimise design rules (size and geometry) for floating conductors, omnipresent in actual integrated circuits. A structure composed of a conductive line surrounded by forty square dummies, all located in the same technological layer M2 as illustrated in figure 6, is simulated: the capacitance between line and substrate is extracted as a function of dummies size, density and distance between the line and the dummies.

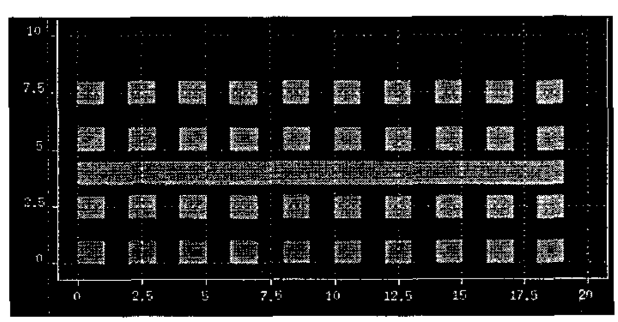

Figure 6: top view of a line and forty square surrounding dummies

First, calculations were performed with square dummies characterized by different sizes and densities and for three different values of the distance line-substrate: 0.5, 1 and 2 μm. As expected, the capacitance line-substrate decreases as the distance line-dummies increases whatever the size and density of the dummies may be.

As shown in figures 7 and 8, the line to substrate capacitance is greatly increased by wide floating conductor proximity, thus degrading overall circuit characteristics. Reducing the size of dummies seems to be an effective way to optimise or conciliate technology requirements, in terms of conductor densities and spacing, and electrical performances.

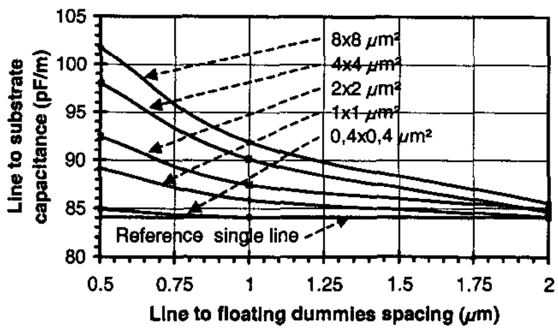

Figure 7: results of parametric calculations with a 25 % dummies density

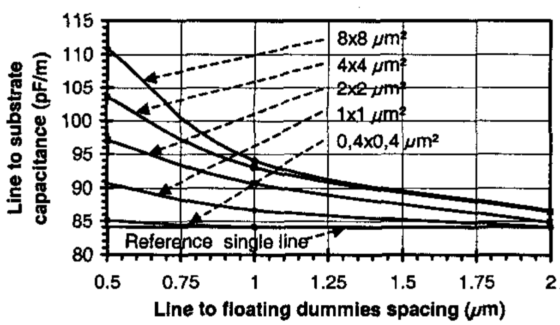

Figure 8: results of parametric calculations with a 50 % dummies density

IV. CONCLUSION

Most of the current parasitic extraction tools do not have specific treatments for floating conductors and conduct a global calculation including all conductors, before calculating a reduced capacitance matrix without the floating conductors, by extraction from the global one. This is very expensive in CPU time when there is a great number of a floating dummies. Our method is based on a specific adaptation of an existing numerical solver to the case with floating conductors. The algorithm thus obtained is particularly efficient in the case of a great number of floating conductors and can be used for parametric analysis to improve designer's work for sensitive circuits, as demonstrated through some representative examples.

IV.1. ACKNOWLEDGMENTS

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IV.2. REFERENCES

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