Simulation of Substrate Currents

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Abstract— **Recently a new approach was presented to determine the high-frequency response of on-chip passives and interconnects. The method solves the electric scalar and magnetic vector potentials in a prescribed gauge. The latter one is included by introducing an additional independent scalar field, whose field equation needs to be solved. This additional field is a mathematical aid that allows for the construction of a gauge-conditioned, regular matrix representation of the curl-curl operator acting on edge elements. This paper reports on the convergence properties of the new method and shows the first results of this new calculation scheme for VLSI-based structures at high frequencies. The high-frequent behavior of the substrate current, the skin effect and current crowding is evaluated.**

I. Introduction

One of the important simulation challenges in VLSI design is the adequate characterization of high-frequency interconnects and on-chip passives. Important effects are substrate currents, current crowding at the edges of the interconnects due to skin effect and the proximity effect. The characteristic electrical length at the frequencies under consideration (GHz range) is rather large (cm scale). However, the mesh scale required for an accurate field calculation is determined by the very fine geometrical details (sub-micron scale) of the metal lines.

Although the physics of these problems is understood for a long time, detailed and fast calculation schemes are still lacking. We recently introduced an approach to simulate high-frequency effects of on-chip interconnects, dedicated to the specific geometry of the problem. The detailed description of the method is presented elsewhere [1-5].

The frequency domain is addressed. The meshing is Cartesian, suitable to show the validity of the model and to simulate on-chip interconnects, because in a first approximation, interconnects can be regarded as parallel to the axes of a Cartesian frame. However, this is not an essential restriction and the technique can be extended to unstructured meshes.

This paper is organized as follows: In the second section we discuss the need for a gauge condition. In the next section the essential properties of the solution method are given, emphasizing the novel aspects of the approach. In the next section the skin effect is calculated for benchmarking purposes. and the substrate current is discussed. In the final section, we reach our conclusions.

II. The Need for a Gauge Condition

To describe the electrodynamical environment, different approaches can be pursued. The electric and magnetic field variables can be used as independent variables. Since these variables are gauge invariant, no gauge condition is required. This is the case in most finite-difference schemes. However, to comply with the needs of IC designers who work in the (quasi-) static regime, a formulation that uses the potentials V and $\mathbf{A}$ as independent variables simular to the FIT approach [6], [7], [8] is preferred. The electric potential V is associated to the nodes, while the magnetic potential ($\mathbf{A}$) is put on the links between the nodes of the mesh. The electrodynamic description results into a Poisson equation for the electric scalar potential and a curl-curl equation for magnetic vector potential respectively. These potentials are not uniquely defined which results in a singular matrix representation. In order to arrive at a unique solution for the potentials we need to introduce a gauge condition. The inclusion of a gauge condition, such as the Lorentz gauge or Coulomb gauge, is occasionally referred to as 'gauging'. The curl-curl equation can be regularized by eliminating the unknown vector potentials assigned to the edges of a spanning tree. However, this kind of gauging leads to a slow convergence of the Krylov-subspace iterative solvers [9]. But do we really need to carry out the extra work of fixing the gauge? Let us start with a matrix representation of a singular linear system: $M\,\mathbf{x} = \mathbf{b}$ and $\det(M) = 0$. It has been shown that if $\mathbf{b}$ has no component in the range of M, then the standard conjugate gradient like methods are successful [9], [11], [10]. However, if $\mathbf{b}$ contains a component outside the range of M, then the problem is ill-posed and no convergence is reached. The iterative solution methods without gauging is effective if the right-hand side of the curl-curl equation can be constructed in such a way that its divergence vanishes, i.e. if there is no component outside the range of the curl-curl operator. This can be understood by realizing that the Krylov space that is spanned by $\{\mathbf{x}, M\mathbf{x}, M^2\mathbf{x}, M^3\mathbf{x}, \ldots\}$ and that the search for the solution fully takes place in the range of M. However such a construction is not always possible. Whereas in magnetostatic calculations with metallic conductors one can easily realize that at the start of solving the equation $\nabla \times \nabla \times \mathbf{A} = \mu_0 \mathbf{J}$, the condition $\nabla \cdot \mathbf{J} = 0$ is satisfied, this is much less easy if a non-linear dependence of the current $\mathbf{J}$ on the vector potential exists. This is the case for non-linear media such as semiconductors as well as for time-dependent fields. Furthermore, numerical errors are inevitable and especially for very large systems or systems that need many iterations, a small component of $\mathbf{b}$ outside the range of M may be amplified and leading to lack of convergence. Therefore, a gauging may be preferred. Also for particular problems in time domain [12] or multigrid we must look for an adequate gauge construction. This might be the tree-cotree gauging [13], the grad-div gauging [7], or the ghost-field gauging [1].

III. Ghost-Field Solver

The method of [1-5] introduces an additional scalar field that needs to be obtained as part of the solution method. The solution for this additional field does not carry energy. Therefore, we have named it a 'ghost field', being a mathematical aid that allows for the construction of a gauge-fixed, regular matrix representation of the curl-curl operator acting on edge elements. With the use of the ghost-field gauging technique, the Maxwell problem also results into a Poisson problem for the scalar potential

$$-\nabla \cdot (\varepsilon \nabla V) = \rho, \tag{1}$$

and a curl-curl equation for the magnetic vector potential that is solved *together* with a gauge equation for the ghost field χ:

$$\nabla \times \nabla \times \mathbf{A} - \gamma \nabla \chi = \mu_0 \mathbf{J} - \mu_0 \varepsilon \frac{\partial}{\partial t}\left(\nabla V + \frac{\partial \mathbf{A}}{\partial t} + \frac{\partial \nabla \chi}{\partial t} \right)$$

$$\nabla \cdot \mathbf{A} + \nabla^2 \chi = 0 \,. \tag{2}$$

An extra parameter γ (with dimension m^{-2}) is introduced in order to account for the dimensions of the system. So instead of the curl-curl operator combined with the gauge condition

$$\begin{pmatrix} \nabla \times \nabla \times \\ \nabla \cdot \end{pmatrix}, \tag{3}$$

that lead to matrices M that are sparse, well-posed, yet *not* square, the operator

$$\begin{pmatrix} \nabla \times \nabla \times & -\gamma \nabla \\ \nabla \cdot & \nabla^2 \end{pmatrix}, \tag{4}$$

is considered. This operator leads to matrices M that are also sparse, regular *and* square. Moreover, the resulting matrices are semi-definite and therefore well suited for iterative solvers.

IV. Simulation Results

A. Skin effect

The quantitative description of skin effect in a cylindrical wire can be found in many text books. Although the discovery of the effect took place more than 125 year ago, nowadays still papers appear on resistance and inductance calculations [15], [17]. The internal impedance of a cylindrical wire with radius a and skin depth δ is given by [16],

$$Z_{int} = \frac{1+j}{2\pi a \delta \sigma} \frac{I_0\left[(1+j)a/\delta\right]}{I_1\left[(1+j)a/\delta\right]}. \tag{5}$$

with the use of the Bessel functions I_n. This is an excellent benchmark problem for high-frequency solvers. We start with a brick representation for the circular form as

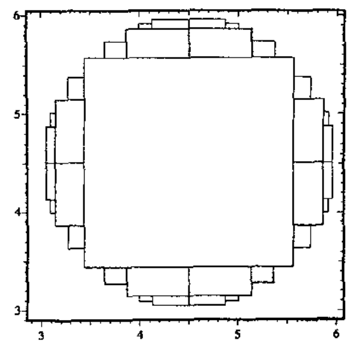

Fig. 1. Cartesian approximation of a circle.

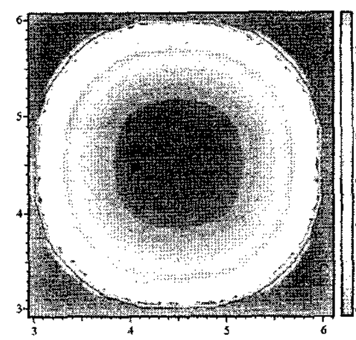

Fig. 2. Current density in a circular cross section at 100GHz.

shown in Fig. 1. The current density at 100 GHz is also shown, and for an aluminum cylinder, the skin depth becomes 0.28 μm as can be verified in Fig. 2.

The impedances calculated with this solver compared with the analytical solution, can be seen in Fig. 3. For the line resistance (upper curves), the relative error is less than 0.08, while for the reactance (lower curves), the results are much better, and the two curves match closely.

B. Ring Structure

The method is also capable of dealing with substrate or eddy currents. The complexity of the problem is illustrated in Fig. 5. A ring-shaped conductor that is separated a finite distance from a conducting substance (the 'substrate') carries an oscillating current. As a consequence, the magnetic flux enclosed by the ring varies in time. From the time-dependent Maxwell equations one can conclude that along closed paths, e.g. paths in the substrate, an electromotive force is induced. This electromotive force gives rise to currents in the substrate.
We show the results of an aluminum ring embedded in silicon oxide, on top of a moderately conducting substrate

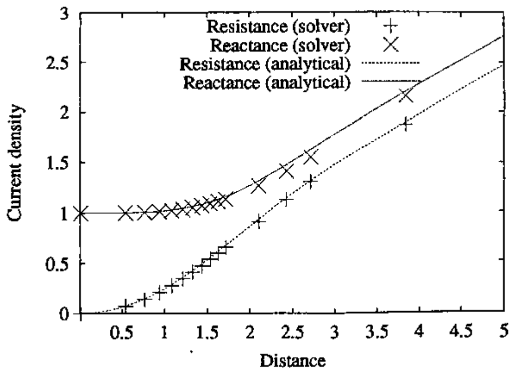

Fig. 3. The internal impedance of a cylindrical wire.

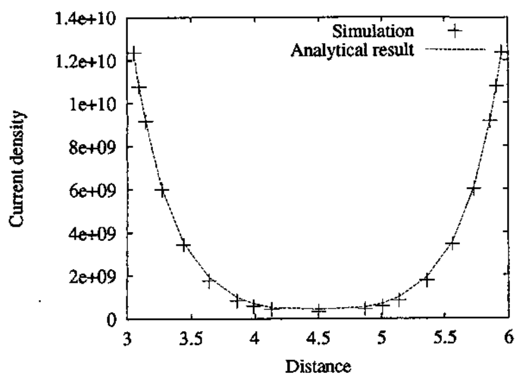

Fig. 4. The current density in the wire (analytic and numerical).

at 500 MHz as illustrated in Fig. 6. The ring dimensions are 90 x 50 μm, the substrate is modeled as a low conductive metal ((=0.01 (ρm) and we used a 20 x 20 x 20 mesh. An AC (electric) voltage is put on one of the output ports with an amplitude of 0.01 V, forcing a current in the ring.

- The skin effect of the currents in the lines that connect the output ports and the ring is shown in Fig. 7-(a). The currents is crowding at the inner edges of the conductor.
- Fig. 7-(b) shows the current density in the substrate, and the circular Eddy currents in the substrate. In a vector plot of the current density we observe that the current is flowing in the opposite direction of the current in the aluminum ring. A region of higher substrate current occurs under the input ports due to a higher magnetic field and hence a higher induced current.
- Fig. 7-(c) shows a cross-section of the current density of the ports and confirms the occurrence of the proximity effect. Because the currents are flowing in the opposite direction for the two ports, these currents will attract each other and the highest current density can be found in the inner corners. Fig. 7 also shows that the current density is the highest in the inner part of the ring, due to the proximity effect.

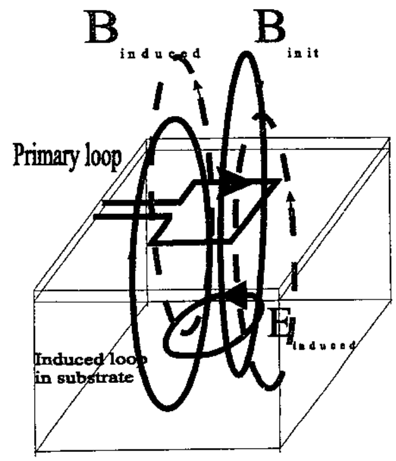

Fig. 5. Schematic view on the origin of substrate currents.

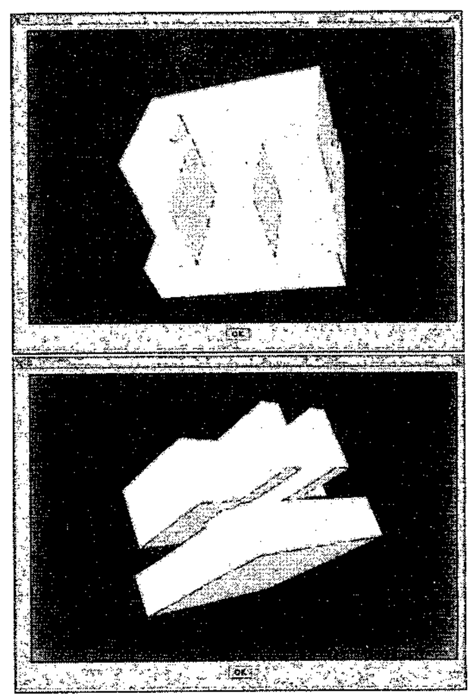

Fig. 6. Aluminum ring above a low conductive substrate

V. Conclusions

We showed that the new approach of exploiting a ghost-field for the implementation of a gauge condition, discretizes the curl-curl operator and provides square and regular as well as sparse matrices. This computational scheme can also be used also for time-dependent electromagnetic fields. The benchmark results for the skin effect demonstrate that the method is already accurate for

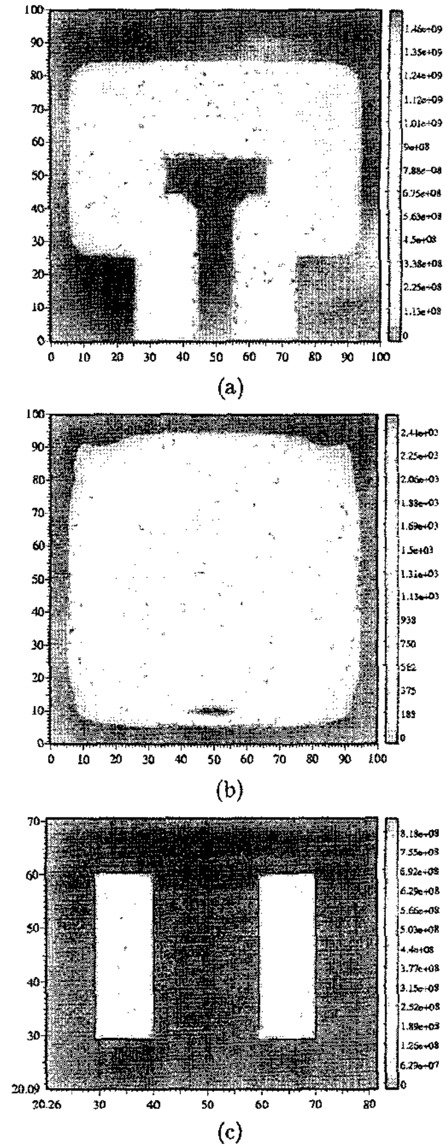

Fig. 7. Top view of the current densities in the ring (a), of the substrate (b) and side view of current densities of the ports (c) at 500 MHz.

rather coarse meshes. This result can be understood as a consequence of the correct assignment of physical variables to the computational grid. Just as the Scharfetter-Gummel discretization techniques respects the underlying principle of charge conservation by assigning the current variables to the links of the grid, we have assigned the vector potential also to the link of the grid. In the present work we have studied the substrate current induction in moderately conducting material. Future work consists of applying the present method to semiconducting substrates. Finally, we emphasize that the method introduces a parameter γ, that controls the 'degree of regularity' of the matrices. Whereas, $\gamma = 0$ corresponds to a singular matrix, non-zero values of γ shift the former zero eigenvalues of the matrix to the positive real axis and into the complex plane. Further work is required to find optimal preconditioning schemes for inverting the matrices. The parameter γ can be used to improve the speed and robustness of the solver.

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