

# A Comparative Study of Two Numerical Techniques for Inductance Calculation in Interconnect Structures

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## Abstract

We present an advanced algorithm for an extraction tool that computes inductances of interconnect structures. As already pointed out in [1] the pursued energy concept leads to a 6-fold integral which can also be evaluated by use of the Monte Carlo method. Classical implementation of the Monte Carlo method, where the whole geometry has to be hunted for the associated element loses efficiency. Our approach is applied without time consuming element location for the random point coordinates to compute this integral.

## 1 Introduction

One consequence of technology scaling by shrinking feature sizes and increasing clock frequencies is the growing importance of interconnect lines. The performance of interconnects is limited by various parasitic effects (eg. signal delay, capacitive and inductive crosstalk, attenuation). The utilization of new materials (Copper and low-k dielectrics) reduces the RC time constant. Thereby decreased resistance and capacitance bring out inductive effects more intensively, requiring consideration in circuit simulation. Thus, inductance extraction becomes necessary for critical nets.

## 2 Physical approach

We compare two stationary inductance calculation methods both based on a numerical solution of Neumann's formula [2] for a precalculated current density distribution:

$$L_{ik} = \frac{\mu}{4\pi} \frac{1}{I_i I_k} \int_{V_i} \int_{V'_k} \frac{\vec{J}_i(\vec{r}) \cdot \vec{J}_k(\vec{r}')}{|\vec{r} - \vec{r}'|} dV dV' . \quad (1)$$

The integration is carried out numerically, where special attention has to be paid on the singularities of the integrand, or with the Monte Carlo method. For both methods the stationary current density is calculated with the finite element method. The first method [1] employs a summation of the contributions of all pairs of finite elements to solve the integral (1), where different kinds of approximation are used, depending on the term  $|\vec{r} - \vec{r}'|$ . For a large distance (compared to the tetrahedron diameter) simple integration formulae are sufficient. The evaluation for small distances demand special formulae with certain integration points, published by Stroud [3] who presented various integration formulae which are applicable for various n-simplexes (e.g. the unit triangle, the unit tetrahedron) as integration region. If  $\vec{r}$  and  $\vec{r}'$  are in the same tetrahedron, a partially analytic integration scheme is used to increase the accuracy of the integration.

### 3 The Program Package

The SMART ANALYSIS PROGRAMS [4] use the finite element method, because of its advantages, as numerical robustness, the ability to solve nonlinear systems, high obtained accuracy, and general applicability.

The geometry can be defined either directly from the layout by specifying layer thicknesses, or by a rigorous topography simulation [5, 6]. The layout of the interconnect structure can also be imported from CIF or GDSII files, or created interactively with a graphical layout editor [7]. Furthermore, the program package includes three preprocessors, one for two-dimensional applications (CUTGRID) the other for three-dimensional applications. The preprocessor LAYGRID allows a layer-based input of the simulation geometry and the specification of the boundary conditions on the borders of each subdomain. The fully unstructured three-dimensional Delaunay grid generator DELINK [8] utilize an advanced-front algorithm, whereby the mesh generation starts from the initial front to fill up the solids with tetrahedrons.

A preconditioned conjugate gradient solver (ICCG), which has been optimized specifically for the discretized Laplace operator, is used to solve the linear systems for domains of conducting materials [9]. By applying Ohm's law to the derivative of the electrostatic potential the distribution of the electric current density is obtained. The simulation is performed with the module STAP (Smart Thermal Analysis Program), where both inductance extraction methods have been implemented.

Two postprocessors complete the program package, whereby the visualization tool SV is based on VTK [10], a flexible and powerful visualization library. Both postprocessors can be used to verify the grid quality, and for the visualization of several distributions (e.g. electric potential, temperature, current density), whereby SV provides numerous features, as eg. cutting planes, volume rendering and contour faces representation of distributions. Fig. 1 gives an overview about the SMART ANALYSIS PROGRAMS.

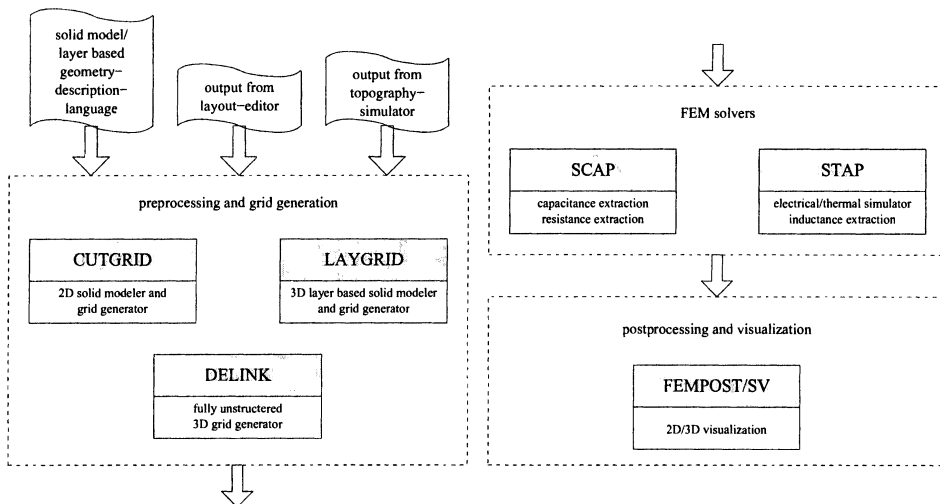


Fig. 1: The SMART ANALYSIS PROGRAMS: tools and dataflow

## 4 The Monte Carlo Implementation

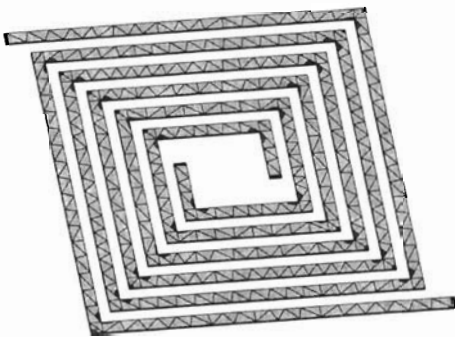
A well-known choice for the evaluation of multiple integrals is to apply the Monte Carlo method. Associated with this method, where by random the point coordinates are chosen, is a fairly high effort on CPU-time, because of the time consuming search for the associated element of the random point coordinates. To reduce the error a high number of function evaluations has to be carried out, whereby for each evaluation the aligned element with the precalculated current density must be found. To improve the convergence during the Monte Carlo sampling several variance reduction schemes (e.g. importance sampling, control variates) are known to accelerate the computation procedure [11].

One big advantage of our implementation is to bypass the high computational effort for the element location. We first determine the associated element to the evaluated probability function, and then locate the point inside the tetrahedron. For this purpose we take two arrays for every conductive segment. In the first one is the volume of each element, whereby the sum of all entries is scaled to one. In the second one is the probability function already evaluated for each conductor element by adding up all entries from the beginning to the current index of the first array. Then the random generator chooses a number between zero and one. The associated element complying to the probability function is found by a binary search.

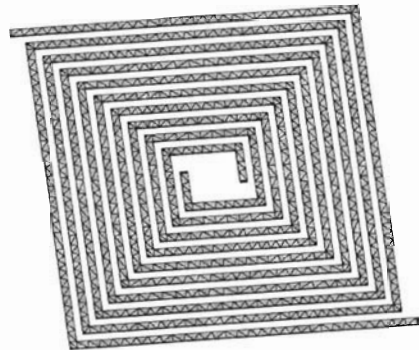
To ensure a uniform probability the local coordinates of the integration points are found by shooting into the unit cube. The first point inside the registered unit tetrahedron is taken. For the interpolation of the current density inside each element quadratic shape functions are used.

## 5 Application Example

Figure 2 and Fig. 3 show the current density of two planar transformers. These transformers are build of two interwound spirals each of 3, respectively, 5-turns metal with  $5 \mu\text{m}$  width, a spacing of  $15 \mu\text{m}$ , and an inner length of  $54 \mu\text{m}$ .



**Fig. 2:** Current density distribution of the planar transformer with 3-turns metal



**Fig. 3:** Current density distribution of the planar transformer with 5-turns metal

By utilizing the preprocessor LAYGRID three different grids were made. In Table 1 the simulation times for the current density and the Monte Carlo method, respectively, the first numeric method as accomplished above, and the calculated inductances are listed. The simulations were performed on a Digital Alpha workstation (DEC600/333 MHz).

The number of samples for the Monte Carlo method was 1 million. The first column of Table 1 implies all elements of the conductive segments, whereby tetrahedral grid elements with quadratic shape functions were used. The analysis time for the Monte Carlo method is not so strongly influenced by the number of elements ( $n$ ), because the computational effort for the binary search grows with  $\ln(n)$ . The simple integration formulae for the mutual inductances demand with increasing  $n$  almost the same time. Table 1 emphasize the advantages of the Monte Carlo method explicitly.

|         | Number of Elements | Time [s] |    |            |      | Results [nH] |      |            |      |
|---------|--------------------|----------|----|------------|------|--------------|------|------------|------|
|         |                    | MC       |    | Method [1] |      | MC           |      | Method [1] |      |
|         |                    | M        | L  | M          | L    | M            | L    | M          | L    |
| 3-turns | 1800               | 17       | 33 | 1          | 327  | 0.67         | 1.04 | 0.67       | 1.06 |
|         | 1968               | 17       | 34 | 1          | 627  | 0.67         | 1.05 | 0.67       | 1.06 |
|         | 2648               | 18       | 34 | 3          | 1764 | 0.67         | 1.06 | 0.67       | 1.06 |
| 5-turns | 4383               | 18       | 35 | 7          | 1945 | 2.71         | 3.58 | 2.70       | 3.62 |
|         | 4653               | 19       | 35 | 8          | 2088 | 2.71         | 3.60 | 2.70       | 3.62 |
|         | 5697               | 19       | 36 | 15         | 7885 | 2.70         | 3.60 | 2.69       | 3.63 |

**Table 1:** Analysis time and results of the planar transformers

## 6 Conclusion

We have presented a comparative study of two numerical techniques for inductance calculation in interconnect structures. Both methods are implemented into the package SMART ANALYSIS PROGRAMS, which allows simultaneous extraction of three-dimensional effective parameters of VLSI circuits.

## Acknowledgment

Our work is partly supported by Compaq Corporation, Shrewsbury, USA and by Sony Corporation, Atsugi, Japan.

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