

# Hydrodynamic Modeling of Electronic Noise by the Transfer Impedance Method

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## Abstract

The transfer impedance method in the time-domain formulation is applied to calculate the impedance field of submicron  $n^+nn^+$  GaAs diodes in the framework of a closed hydrodynamic approach. The method enables us to determine the voltage noise-spectrum associated with velocity-fluctuations. The good agreement found with Monte Carlo simulations validates the proposed theoretical approach.

## 1. Introduction

The impedance-field method is widely used for noise modelling in the framework of the drift-diffusion approximation [1]. Within a one-dimensional geometry and considering single-carrier velocity-fluctuation as source of noise, the spectral density of voltage fluctuations between two terminals,  $S_U(\omega)$ , as measured under constant-current operation, takes the form:

$$S_U(\omega) = Ae^2 \int_0^L n(x) |\nabla Z(x; \omega)|^2 S_v(x; \omega) dx \quad (1)$$

where  $\omega$  is the cyclic frequency,  $e$  the electron charge,  $A$  the cross-section area of the device,  $L$  the length of the device between the probing electrodes taken along the  $x$  direction,  $n(x)$  the local carrier concentration,  $S_v(x; \omega)$  the local spectral density of single-carrier velocity fluctuations,  $\nabla Z(x; \omega)$  the local impedance field. This latter quantity relates a perturbation of the voltage drop  $U$  with the perturbation of the conduction current-density  $j_d$  in a point  $x_0$  through

$$\delta U(x_0; \omega) = \nabla Z(x_0; \omega) \delta j_d(x_0; \omega) \quad (2)$$

In this work we present a procedure for the numerical calculation of the impedance field spectrum  $\nabla Z(x_0; \omega)$  of a two-terminal semiconductor structure in the framework of a closed hydrodynamic approach [2,3]. The procedure is based on the modeling of a spatio-temporal response of the electrical characteristics to a local perturbation of  $j_d$ . The relevance of the approach is illustrated by calculations of  $S_U(\omega)$  in submicron  $n^+nn^+$  GaAs diodes.

## 2. Procedure

We describe carrier transport in the framework of the conservation equations for the carrier velocity  $v(x, t)$  and mean energy  $\varepsilon(x, t)$  [3]. To simulate total-current operation, we use the definition of the total current-density,  $J$ , taken to be constant both in time and space:

$$J = en(x, t)v(x, t) + \varepsilon\epsilon_0 \frac{\partial E(x, t)}{\partial t} = \text{const} \quad (3)$$

where  $\epsilon_0$  is the free-carrier permittivity,  $\varepsilon$  the static dielectric constant of the material and  $E(x, t)$  the instantaneous local electric-field. Then the Poisson equation writes:

$$n(x, t) = N_d + \frac{\varepsilon\epsilon_0}{e} \frac{\partial E(x, t)}{\partial x} \quad (4)$$

$N_d$  being the donor concentration. After the substitution of Eq. (4) in Eq. (3) we obtain an equation for the electric field  $E(x, t)$  in the form:

$$\frac{\partial E}{\partial t} + v \frac{\partial E}{\partial x} + \frac{e}{\varepsilon\epsilon_0} v N_d = \frac{1}{\varepsilon\epsilon_0} J \quad (5)$$

Equations (4) and (5) together with the velocity and energy conservation equations constitute the closed model which allows both to calculate the steady-state characteristics for a given value of  $J$  and to investigate the spatio-temporal evolution of various perturbations responsible for the electronic noise in the structure. Since  $J$  is taken to be constant, only the fluctuations of the conduction current-density  $j_d = env$  are responsible for the noise. Supposing that the initial perturbations are small enough to satisfy linearization, the difference between the time-dependent perturbed solution,  $E(x, x_0, t)$ , and the steady-state solution  $E_s(x)$  of the unperturbed system gives the Green-function which describes the spatio-temporal evolution of the electric field perturbation caused by the local perturbation of the conduction current at the point  $x = x_0$ . By definition [4], the impedance field is given by

$$\nabla Z(x_0; \omega) = \int_0^L \int_0^\infty \exp(-i\omega t) z(x, x_0; t) dx dt \quad (6)$$

## 3. Results and discussions

Numerical simulations are performed for a  $n^+nn^+$  GaAs diode with the following parameters: the doping levels are of  $n = 10^{16}$  and  $n^+ = 2 \times 10^{17} \text{ cm}^{-3}$ , the cathode,  $n$ -region and anode lengths are respectively of 0.2, 0.6 and 0.4  $\mu\text{m}$ . Abrupt homojunctions are assumed. The initial perturbation is taken of the Gaussian form:  $\delta(x - x_0) = \exp[(x - x_0)^2/\gamma^2]/(\gamma\pi^{1/2})$ , and  $S_v(x; \omega)$  is calculated using the hydrodynamic approach developed in [2]. Typical features of the spatio-temporal evolution of the electric field perturbation initiated at  $t = 0$  by the local perturbation of the conduction current in the  $n$ -region are illustrated in Figs. 1 and 2. All the results correspond to an applied voltage  $U_d = 2.3 \text{ V}$ . Figure 1 shows the spatial distribution of  $\delta E(x, t)$  at successive time moments for the initial perturbation placed in the point  $x = 0.4 \mu\text{m}$ . Figure 2 represents a time dependence of the voltage perturbation at the whole structure for the initial perturbations placed in points

$x = 0.3, 0.4, 0.5, 0.6, 0.7 \mu\text{m}$  (respectively, curves 1 to 5). At the initial stage of the perturbation evolution, there appear two shock-waves which move along the structure toward opposite sides (see Fig. 1, solid curve). Such a behavior is typical for physical systems with nonlinear convective and diffusion processes. A quick damping of the shock-waves corresponds to an initial fast decrease of  $\delta U(t)$  (see Fig. 2,  $t < 0.5 \text{ ps}$ ). In the general case, when electron heating in the  $n$ -region is small and the negative differential conductivity (NDC) is absent, the perturbation evolution ends when the shock-waves vanish entirely. If the plasma frequency is considerably higher than the damping rate of the shock-waves, the shock-wave propagation is accompanied by the plasma oscillations. Such a situation is typical for the perturbations in  $n^+$  regions. For the case considered here, the local NDC is present in the  $n$ -region and the time evolution of the perturbation is not finished with the disappearance of shock-waves. The transit through the diode of the right shock-wave creates a secondary perturbation of the electric field (caused by the dipole perturbation of carrier concentration) which grows by approaching the anode contact (see Fig. 1). The dipole-domain propagation leads to the bell-shaped evolution of the voltage perturbation (see Fig. 2,  $t > 0.5 \text{ ps}$ ). Figure 3 illustrates the spatial dependence of  $|\nabla Z(x, \omega)|^2$  at the frequency  $f = 10 \text{ GHz}$  (dashed curve). For comparison, the solid line represents the square module of the differential impedance field defined as [4]:

$$\nabla Z'(x, \omega) = \int_0^L \int_0^\infty \exp(-i\omega t) z(x, x_0, t) dx_0 dt \quad (7)$$

This quantity, which is complementary to the field impedance in Eq. (6), is used to determine the local electric field response to a perturbation of the total current flowing in the structure [3]. The spatial profile of both are reported in Fig. 3. The difference between the two shapes is due to transit-time effects associated with the formation of accumulation layers inside the  $n$ -region. Accordingly, the dashed curve, which represents the global response to a local current perturbation, exhibits its maximum around the cathode. In contrast, the continuous curve, which represents the local response to a global current-perturbation, exhibits its maximum around the anode where the accumulation-layer disappears. The spectral density of the voltage fluctuations calculated by using Eq. (1) is presented in Fig. 4 (dashed line). For comparison the solid line shows the result of the Monte Carlo simulation. The excellent agreement found between the noise spectra obtained in the framework of the hydrodynamic and Monte Carlo approaches fully supports the physical reliability of the proposed method.

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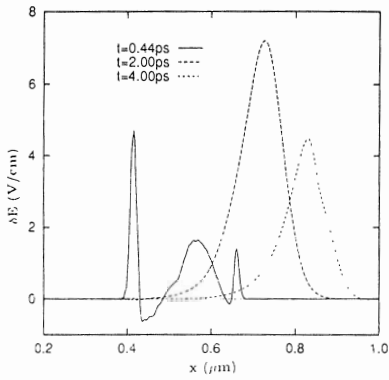


Fig. 1. Spatial profiles of the electric field perturbation at successive time moments for the initial perturbation placed at point  $x = 0.4\ \mu\text{m}$ .

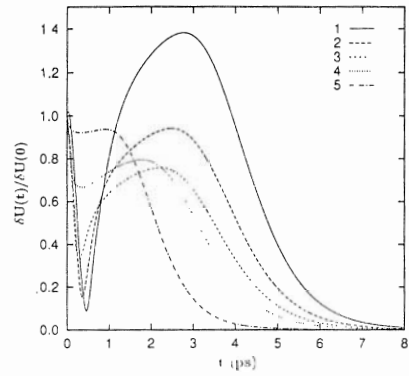


Fig. 2. Voltage perturbation evolution for initial perturbations placed at points  $x = 0.3, 0.4, 0.5, 0.6, 0.7\ \mu\text{m}$  (respectively, curves 1 to 5).

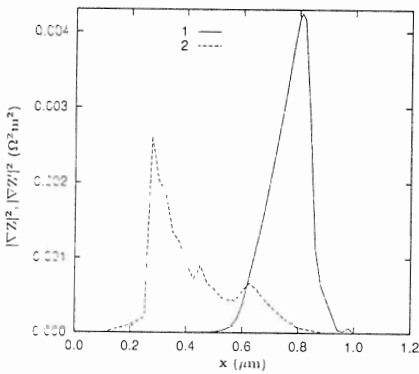


Fig. 3. Spatial profiles of the absolute value squares of the differential impedance field  $|\nabla Z'(x; \omega)|^2$  (solid line) and of the impedance field  $|\nabla Z(x; \omega)|^2$  (dashed line) at  $f = 10\ \text{GHz}$ .

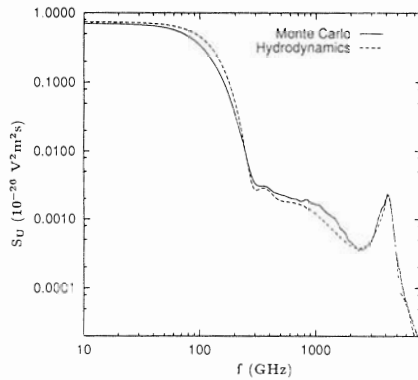


Fig. 4. Frequency dependence of the spectral density of voltage fluctuations calculated by Monte Carlo and hydrodynamic approaches (solid and dashed lines, respectively)