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A fully 2D, Analytical Model for the Geometry and Voltage Dependence of Threshold Voltage in Submicron MOSFET's

A. Klös, A. Kostka

Solid State Electronics Laboratory, Technical University of Darmstadt
Schloßgartenstraße 8, D-64289 Darmstadt, GERMANY

Abstract

In this paper we present a physics-based, compact model for the threshold voltage shift in short-channel MOSFET's, which is based upon a new theoretical approach in MOS modeling. This method uses conformal mapping techniques to solve the 2D Poisson equation in the space charge region underneath the gate and considers inhomogeneous doping profiles therein. The derived model equations appear in closed form and require only two physical fitting parameters related to a geometry and a doping approximation. A comparison with numerical device simulations reveals a high degree of accurateness down to channel lengths of $0.2\mu m$.

1. Introduction

For circuit development, accurate models to predict the layout dependence of the threshold voltage of short-channel devices are needed. Various quasi two-dimensional models have been published for this purpose. Most of them are based upon the unphysical charge-sharing principle [1, 2] to approximate the influence of the source/drain space charge regions on the threshold voltage. Better accuracy is achieved by more physics-based models, which solve an approximation of Poisson's equation in the device cross-section [3]. To arrive at analytical model equations suitable for circuit simulators, these models have to introduce process and layout dependent fitting parameters. Therefore, a great number of devices of different geometries are required for parameter extraction. Also, they have no forecasting ability to estimate the sensitivity of threshold voltage to process data or to investigate fictive processes at all.

The model presented in this paper follows a fully two-dimensional approach and from this, it does not need any fitting parameter to describe the two-dimensional effect of threshold voltage shift. The only fitting parameters used are one for simplification of the device structure and one for consideration of an implantation profile.

2. Solving Poisson's Equation

The device cross-section of a MOSFET with the definitions of process and geometry parameters is shown in Fig. 1a. To model the short-channel effects upon the threshold

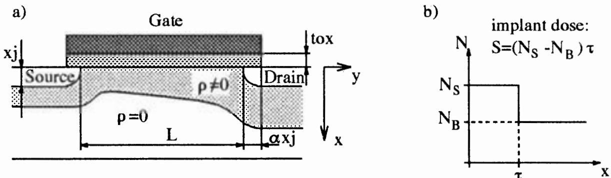

Figure 1: (a) cross-section of a MOSFET, (b) doping box approximation.

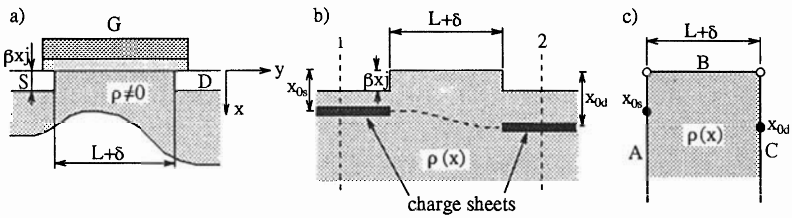

Figure 2: Simplification of the model structure.

voltage, the two-dimensional Poisson equation has to be solved. A surface potential $\phi_i + V_{sb}$ according to the onset of strong inversion is applied as boundary condition relevant for the threshold condition along the channel region. If mobile carriers are neglected, the dielectric flux density within the gate-oxide is maximal at the position y_0 where, by raising the gate voltage, the strong inversion sets on latest. Together with the work function difference Φ_{ms} and the oxide charge density q_{ox}, this flux density D_0 is used to determine the threshold voltage:

$$V_T = \Phi_{ms} - \frac{q_{ox}}{C'_{ox}} + \phi_i + \frac{D_0}{C'_{ox}} \qquad \text{where} \quad C'_{ox} = \frac{\varepsilon_{si}}{t_{ox}}. \tag{1}$$

In [4, 5] conformal mapping techniques have been shown to be useful in a strictly physical, 2D analysis of planar geometries in lateral bipolar transistors, leading to closed form solutions for the Laplacian DEQ. In an attempt to apply this method in MOS modeling, the solution Φ of Poisson's equation can be found by superposition of a homogeneous, two-dimensional solution φ and a non-homogeneous, one-dimensional solution Φ_p:

$$\Delta\Phi(x,y) = -\frac{\rho}{\varepsilon_{si}} = \Delta\varphi(x,y) + \Delta\Phi_p(x) \;\text{ where }\; \Delta\varphi(x,y) = 0,\; \Delta\Phi_p(x) = -\frac{\rho(x)}{\varepsilon_{si}}. \tag{2}$$

This is possible, as long as the space charge ρ is a function of only one coordinate. $\Phi_p(x)$ then is the one-dimensional potential solution of a long-channel MOSFET. The conformal mapping technique [4, 5] provides a method to get analytical solutions of the Laplacian DEQ $\Delta\varphi = 0$. For this, the boundary conditions of Poisson's equation have to be transformed by substracting Φ_p before solving the Laplacian DEQ.

3. Simplifying the Model Structure

In order to keep mathematics manageable, the following approximations are made. First, the doping profile of an implantation for threshold voltage shift is approximated by a dose equivalent box (Fig. 1b) to ensure closed form solutions. The depth τ of the box can be found from the best fit of the model to the body effect [6].

Secondly, to get a closed form solution for the two-dimensional potential problem outlined above, the geometry of the structure is simplified as shown in Fig. 2. In a first step the curved shape of the source/drain junctions will be replaced by a box approximation described by one shape-fitting parameter β (Fig. 2a). For homogenous substrate doping this is the only fitting parameter of the model at all. The influence of β is only of second order, and investigations using the device simulator ATLAS II for devices of different processes state the fact, that for realistic device parameters a process independent value of $\beta = 0.5$ gives a very good fit.

In a second step, a simplification is necessary for the one-dimensional description of the space charge ρ. If the space charge ρ is extended to the whole substrate (Fig. 2b), and a part of the charge underneath the source/drain regions is compressed into a charge sheet, an appropriate choice of the depths $x_{0s/d}$ of the charge sheets will result in nearly the same potential solution in the channel region as in the structure shown in Fig. 2a. A one-dimensional calculation of the potential along line 1 and 2 is used as a compatible approximation of the boundary conditions along lines A and C of Fig. 2c, respectively, which shows the shape of the area the conformal mapping is applied to at the end, and where the charge sheets do not appear. The error caused by introducing the charge sheets outside the critical area should be small due to the relatively large distance to the point at which D_0 is determined.

4. Model for NMOS-processes

Proceeding in the way described in section 3, after some mathematics, one arrives at a set of explicit, closed form equations for V_T. Due to the extent of the model equations, we concentrate on illustrative examples for NMOS-processes with $\tau > \beta x_j$, which should hold for most of them, visualizing the progress of the derivation.

(a) L, x_j, α, β, N_B, N_S, τ, V_{ds}, V_{sb} represent the input data of the model. V_{bi} is the built-in potential of the source/drain junctions, ϕ_i the surface potential according to the onset of strong inversion and δ a definition for convenience:

$$\phi_i = \frac{kT}{q}\ln\frac{N_B N_S}{n_i^2} \quad (3) \qquad V_{bi} = \frac{E_G}{2}+\frac{kT}{q}\ln\frac{N_B}{n_i} \quad (4) \qquad \delta = 2\alpha x_j(1-\sqrt{1-\beta^2}) \quad (5)$$

(b) From one-dimensional field calculations at the surface of a long channel device and at the edges of the depletion regions underneath source/drain the corresponding electrical fields E_0 and $E_{0s/d}$, respectively, are determined in closed form. From this, the depths of the charge sheets result in:

$$x_{0s/d} = \frac{\frac{qN_S}{2\varepsilon_{si}}(\beta x_j)^2 + \beta x_j E_{0s/d} + V_{bi} + \varphi_{s/d} - \phi_i}{\frac{qN_S}{\varepsilon_{si}}\beta x_j + E_{0s/d} - E_0} \quad \text{where } \varphi_s = 0 \text{ and } \varphi_d = V_{ds} \quad (6)$$

(c) The following parameters are used for conformal mapping:

$$b = \cosh\frac{\pi\beta x_j}{L+\delta} \quad (7) \qquad c_{s/d} = \cosh\frac{\pi x_{0s/d}}{L+\delta} \quad (8) \qquad d = \cosh\frac{\pi\tau}{L+\delta} \quad (9)$$

(d) The conformally mapped position of the surface potential minimum is given by:

$$u_0 = \frac{\sqrt{V_{bi}} - \sqrt{V_{ds}+V_{bi}}}{\sqrt{V_{bi}} + \sqrt{V_{ds}+V_{bi}}} \qquad (10)$$

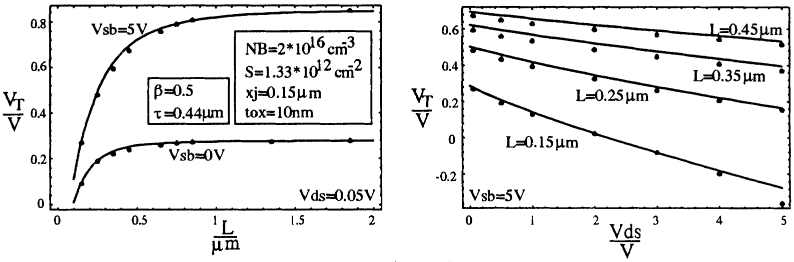

Figure 3: Comparison of the model (lines) with numerical results (dots).

(e) Solving the Laplacian DEQ results in source/drain related flux densities at u_0:

$$D_{s/d} = \frac{\varepsilon_{si} E_0}{\pi}\left(\arctan\frac{bu_0 - 1}{\sqrt{b^2-1}\sqrt{1-u_0^2}} + \frac{\pi}{2}\right) + 2\sqrt{2}\frac{L+\delta}{\pi^2}\sqrt{1-u_0}\,qN_S\,f(b,1)$$
$$+(\varepsilon_{si}(E_0 - E_{0s/d}) - qN_S\beta x_j)\frac{g(c_{s/d},b)}{\pi} + \frac{\varepsilon_{si}}{L+\delta}\sqrt{\frac{1+u_0}{1-u_0}}\,(V_{bi} + \varphi_{s/d} - \phi_i), \quad (11)$$

where f and g are abbreviations of analytical functions.

(f) Superposition of the one-dimensional solution and the results for the Laplacian DEQ gives the flux density D_0, and, hence V_T via (1):

$$D_0 = \varepsilon_{si} E_0 - D_s - D_d \quad (12)$$

Fig. 3 shows a comparison of the presented model with results of the numerical device simulator ATLAS II for a NMOS-process. Both are in very good agreement down to a channel length of about $0.2\mu m$. The only fitting parameters used are the flatband voltage and the depth τ of the implant doping box approximation.

5. Conclusion

The presented method for solving Poisson's equation in MOS devices based upon conformal mapping techniques made it possible to develop a physics-based, analytical model for the geometry and voltage dependence of the threshold voltage in short-channel devices in closed form. The small number of fitting parameters, which are all physically meaningful, qualifies the model to be useful in circuit simulators as well as in calculations of scaling behaviour. The method is a new theoretical approach in MOS modeling and not restricted to the effect of threshold voltage shift. It can be transferred on modeling other effects like the subthreshold behaviour, which is currently under work by the authors.

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