

## HOT ELECTRON TRANSIENT BEHAVIOUR

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**ABSTRACT :** The numerical model proposed take into account the hot carrier behavior. The usual transport equation set is completed by the energy conservation equation. The variables used to solve the equation set are the electrostatic and electrochemical potentials and an energy potential. Transient solutions are obtained from an implicit method within an uncoupled scheme of resolution.

### 1 - INTRODUCTION

In previous papers [1, 2] we showed that for cold carriers, the electrostatic and electrochemical potentials are an adequate set of computational unknowns to simulate steady or transient behaviors of heterostructures with excellent accuracy.

Hot carriers have already been studied by Monte Carlo simulations and electrokinetic models expressed in term of carrier densities, electrostatic potential and carrier energy as main unknowns [3]. We have already shown it was possible to establish a model involving electrochemical potentials rather than carrier densities for hot carriers, as it was done for cold carriers. The hot carriers set of equations of this model has already been compared to Monte Carlo simulations [4] and results showed good agreement for steady state simulations.

On the other hand, we found a new method to solve transient cold carriers problems. This method is characterized by its capacity to use incremental time steps far larger than the dielectric relaxation time usual limitation. This is achieved by using a variable implicitness method through out the transient [5].

The field effect transistor is one of the critical devices of integrated circuits and a time dependant description of the field effect transistors is of major interest especially for large signal simulation and for the low signal [s] parameters determination : but FET's are hot carrier devices to that hot carrier behaviors has to be include in the model.

So the purpose of this paper is to show how the hot carrier time dependant general set of equations can be solved, with the same advantage as for cold carriers, that is to say without limitation on time steps. The general set of equation is written for heterostructures as well as for homostructures. For the clarity of demonstration a simple example will be described consisting in a  $n^+ n n^+$  unidimensional GaAs structure. But in fact this time resolution method has been performed to be included in III.V FET general model which also describe bipolar effects and semi-insulating substrate [6].

## 2 - GENERAL SET OF EQUATIONS

The usual set of equations describing transport in semiconductors is :

$$\text{div } \epsilon_d \vec{\text{grad}} \phi = q(n-p-C) \quad (1)$$

$$\frac{\partial n}{\partial t} + \text{div } n \langle \vec{v}_n \rangle = -U_n \quad (2)$$

$$\frac{\partial p}{\partial t} + \text{div } p \langle \vec{v}_p \rangle = -U_p \quad (3)$$

$$\vec{J}_n = -qn \langle \vec{v}_n \rangle ; \vec{J}_p = qp \langle \vec{v}_p \rangle$$

From now on, let us consider only electrons as hot carriers. Let  $T$  be the electron temperature which is different from  $T_0$ , the lattice temperature. The electron energy conservation equation has to be added to the previous set of equations.

$$\frac{\partial n \epsilon}{\partial t} + \text{div } n \epsilon \langle \vec{v}_n \rangle = n \langle \vec{v}_n \rangle \cdot \vec{F}_n - n \frac{(\epsilon - \epsilon_0)}{\tau} - \epsilon U_n \quad (4)$$

-  $n \langle \vec{v}_n \rangle \cdot \vec{F}_n$  expresses the energy variation owed to the total force  $\vec{F}_n$  experienced by electrons.

-  $n[\epsilon - \epsilon_0]/\tau$  expresses the energy exchange with the cristal,  $\tau$  the energy relaxation time which is energy dependant -  $\epsilon_0 = 3/2 kT_0$ .

$-\epsilon U_n$  is the loss of energy in carrier emission capture processes.

By substitution of (2) into (4), it becomes :

$$\frac{\partial \epsilon}{\partial t} + \langle \vec{v}_n \rangle \cdot \vec{g} \text{grad } \epsilon = \langle \vec{v}_n \rangle \cdot \vec{F}_n - \frac{\epsilon - \epsilon_0}{\tau} \quad (5)$$

We have established [4] that the average electron velocity has to be expressed by :

$$\langle \vec{v}_n \rangle = - \frac{\mu_n}{q} \left[ \vec{g} \text{grad } E_{fn} + \frac{E_c - E_{fn} + kT}{kT} \vec{g} \text{grad } kT \right] \quad (6)$$

and the total force by

$$\vec{F}_n = - \vec{g} \text{grad} [ E_c + kT ] \quad (7)$$

We need now to remark that expressing the total force as a function of grade  $E_c$  may lead to problems for abrupt heterostructures. The gradient of energy may lead to the same difficulties, due to band structure discontinuities. These seemingly difficulties disappear by choosing the computational unknowns  $\phi$ ,  $\phi_n$ ,  $\psi$  defined as

$$-q\phi - \chi = E_c$$

$$-q\phi_n = E_{fn}$$

$$-q\psi = \epsilon - \chi ; -q\psi_0 = 3/2 kT_0 - \chi$$

where  $\chi$  is the electron affinity.

The unknowns  $\phi$ ,  $\phi_n$ ,  $\psi$  are continuous at hetero-interfaces.

The set of equations becomes :

$$\text{div } \epsilon_d \vec{g} \text{grad } \phi = q(n-p-C) \quad (8)$$

$$\frac{\partial n}{\partial t} + \text{div } n \langle \vec{v}_n \rangle = -U_n \quad (9)$$

$$\frac{\partial \psi}{\partial t} + \langle \vec{v}_n \rangle \cdot \vec{g} \text{grad } \psi + \frac{\psi - \psi_0}{\tau} = \langle \vec{v}_n \rangle \cdot \vec{g} \text{grad} (-\phi + kT/q) \quad (10)$$

$$\langle \vec{v}_n \rangle = \mu_n \left[ \vec{g} \text{grad } \phi_n + \frac{\phi - \phi_n + \chi/q - kT/q}{kT} \vec{g} \text{grad } kT \right] \quad (11)$$

In the cas of Maxwell Boltzmann statistic :

$$n = N_c(T_0) \exp \left[ \frac{q(\phi - \phi_n) + \chi}{kT} \right] \quad (12)$$

$-\mu_n(E)$ ,  $\tau(E)$ ,  $T(E)$  are only energy dependant and are given by analytical laws extracted from Monte Carlo simulations.

### 3 - NUMERICAL METHOD

The time dependant equations are solved within an implicit method. They can be expressed as :

$$\frac{\partial f}{\partial t} = H(x, t) \quad (13)$$

time discretisation gives

$$\frac{f(t+\Delta t) - f(t)}{\Delta t} = C H(x, t+\Delta t) + (1-C)H(x, t) \quad (14)$$

with a degree of implicitness  $C$  :  $0,5 < C < 1$ . Knowing the solution at time  $t$ , the solution at  $t + \Delta t$  is given by solving :

$$\frac{f(t+\Delta t)}{\Delta t} - C H(x, t+\Delta t) = \frac{f(t)}{\Delta t} + (1-C)H(x, t) \quad (15)$$

Equation (10) can be directly solved with that method. But equation (9) first must be transformed to be adapted to the method. Equation (9) can be written as

$$\frac{\partial \eta}{\partial t} = \frac{1}{\frac{dn}{d\eta}} \{-\text{div } n \langle \vec{v}_n \rangle - U_n\} \quad (16)$$

$$\text{where } \eta = \frac{q\phi - q\phi_n + \chi}{kT}$$

Let  $H(t)$  be the right side of equation (16), it comes

$$\frac{q(\phi - \phi_n)}{kT \Delta t} \Big|_{t+\Delta t} - C H(t+\Delta t) = \frac{q(\phi - \phi_n)}{kT \Delta t} \Big|_t + (1-C)H(t) \quad (17)$$

a similar equation is obtained for hole conservation equation.

At this point the uncoupled resolution sheme is quite similar to that already described for cold carriers [5].

Nevertheless we would like to point out that Poisson equation describes the electrostatic potential distribution for a given charge density distribution. The charge evolution is only described by the continuity equations, and

is a function of the incremental time steps. It means that a mere change of the electrostatic potential does not change charge densities. This remark has two consequences. First, Poisson equation is linear on  $\phi$ . Second, as charge densities are not meant to change when solving Poisson equation, a variation of the electrostatic potential  $\delta\phi$  induces the same variation on the electrochemical potentials.

The numerical resolution scheme can be resumed as follows.

For a given solution at time  $t$ , limit conditions at time  $t + \Delta t$  are changed and then the set of equations is solved iteratively. The Poisson equation is solved on  $\phi$ , equation (9) on  $\phi_n$ , equation (10) on  $\psi$ .

The iterations are carried on up to convergence on all potentials for time  $t + \Delta t$ . The precision we allow is of about  $10^{-6}$  V on each potential.

#### 4 - SIMULATION OF A $n^+ n n^+$ GaAs STRUCTURE

We performed the simulation of  $n^+ n n^+$  GaAs unidimensional structure, the doping profile is describe on figure 1. First consider the steady state behavior [4]. On figure 1 is plotted the electron density profile when a 2V bias voltage is applied, which corresponds to the saturation region of the current voltage characteristic, plotted on figure 4. On figure 2 are plotted the energy profiles. This figure shows the potential drop at the end of the  $n$  material and the corresponding increase of the electron energy which relaxes into the  $n^+$  material. To the potential drop corresponds a large increase of the electric field as it can be seen on figure 3. The same figure shows the electron average velocity profile. The maximum of velocity is obtained for cold carriers and decreases when carrier energy increases..

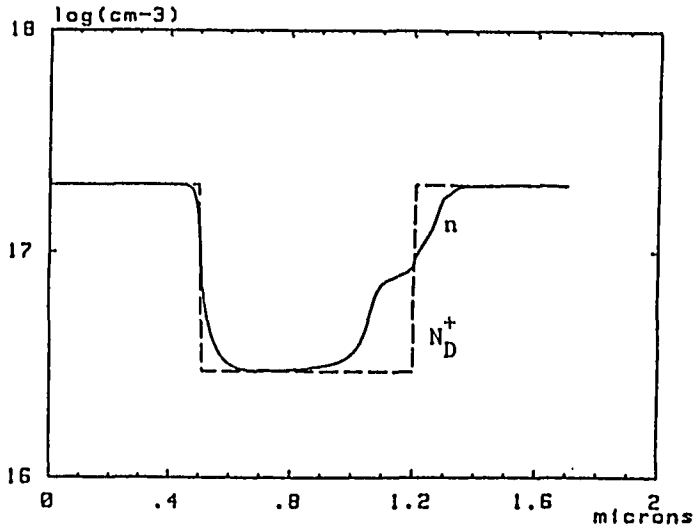


Fig. 1 : Doping profile and electron density profile for 2 V bias voltage.

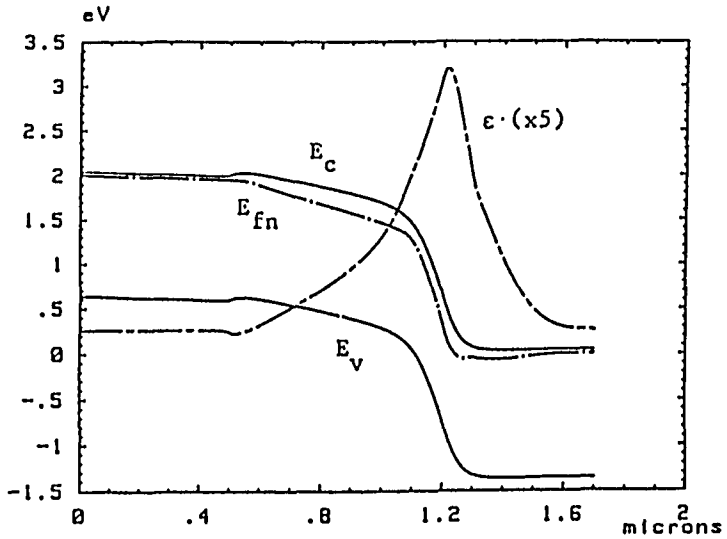


Fig 2 : Energy profile for 2 V bias voltage.

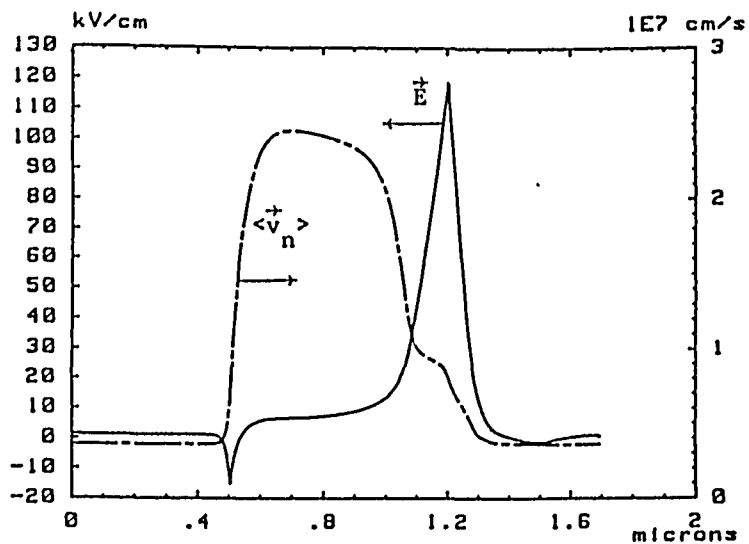


Fig. 3 : Electric field and average velocity profiles for 2 V bias voltage.

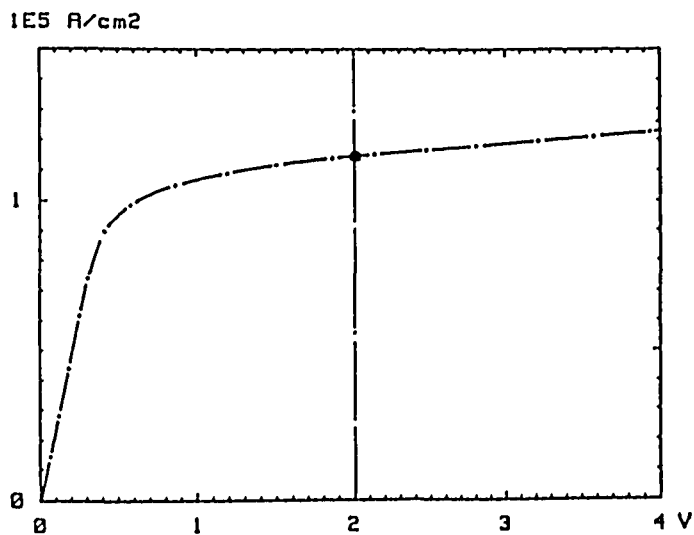


Fig. 4 : Current voltage characteristic.

Now a high frequency signal is applied on the structure. The bias time function is  $V(t) = V_0 + V_{HF} \sin(\omega t)$   $V_0 = 2V$  and  $V_{HF} = 1.5V$  chosen in order to have a large signal behavior of the structure to simulate this behavior the transient method is used. The method needs no limitation on the time step so that the choice of the time step has to be related to the number of steps needed to describe correctly a period. For a sinus, a choice of 20 steps to describe a period gives a good approximation of the function and leads to reasonable calculation time. The frequencies simulated are 100 GHz and 200 GHz which correspond respectively to  $10^{-11}$  and  $5 \cdot 10^{-12}$  s periods. The choice of 20 steps per period gives  $5 \cdot 10^{-13}$  and  $2.5 \cdot 10^{-13}$  s time steps.

To obtain the simulation of the frequency response of the device it is necessary to first obtain a permanent state: that is to say, starting from a steady state (2V bias), to carry on the simulation over the number of periods needed by the device to give the same response at each period. Due to the response time of such a structure, a few periods are needed. Practically the presented results have been obtained after 5 periods for 100 GHz and 10 periods for 200 GHz. They correspond to 100 time steps for 100 GHz and 200 time steps for 200 GHz. If the maximum of the time by step had been limited to the dielectric relaxation time it would have needed more than  $5 \cdot 10^3$  time iteration...

On figure 5 to 8 are plotted the electron density, electron average velocity, electric field and electron energy profiles over a period for a 100 GHz signal.



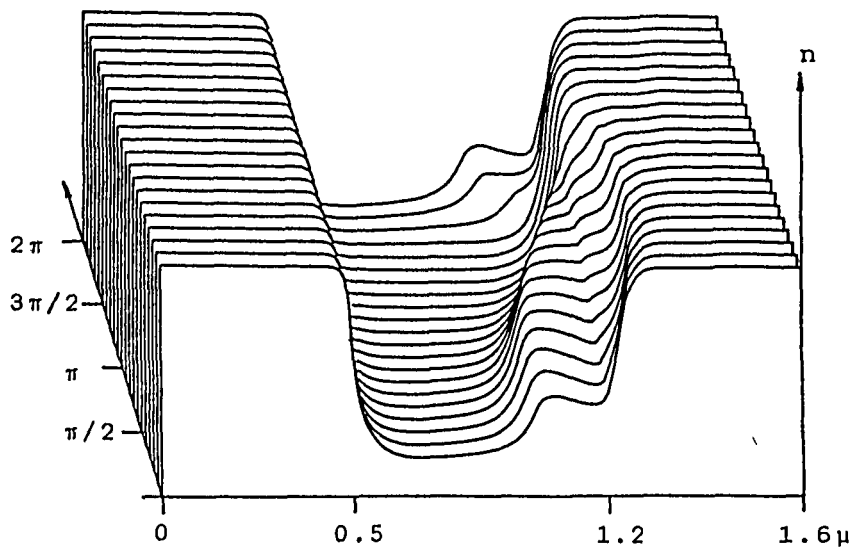


Fig. 5 : Electron density profile over a period at 100 GHz.

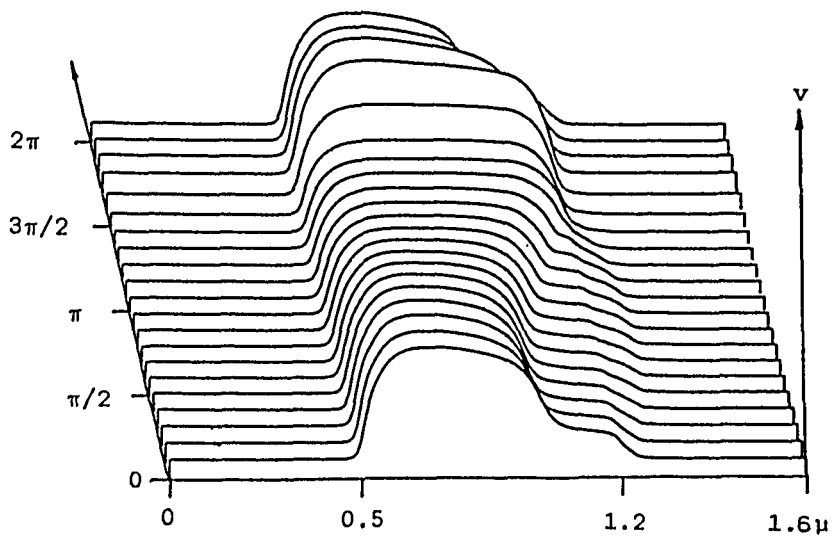


Fig. 6 : Electron average velocity profile over a period at 100 GHz.

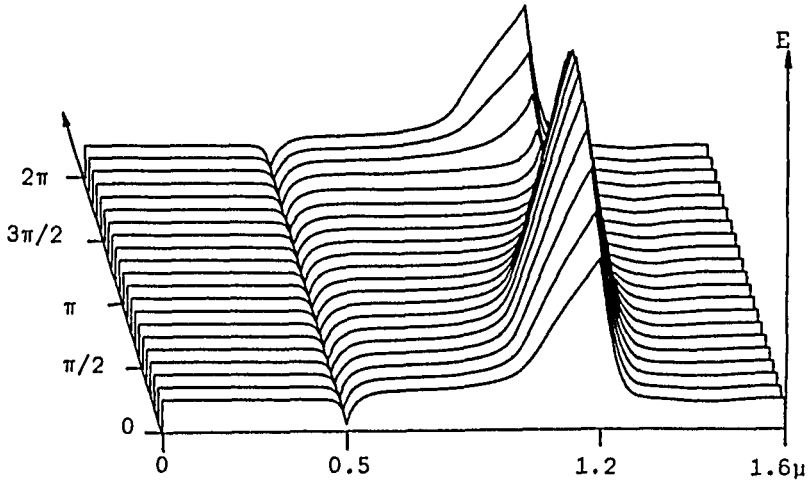


Fig. 7 : Electric field profile over a period  
at 100 GHz.

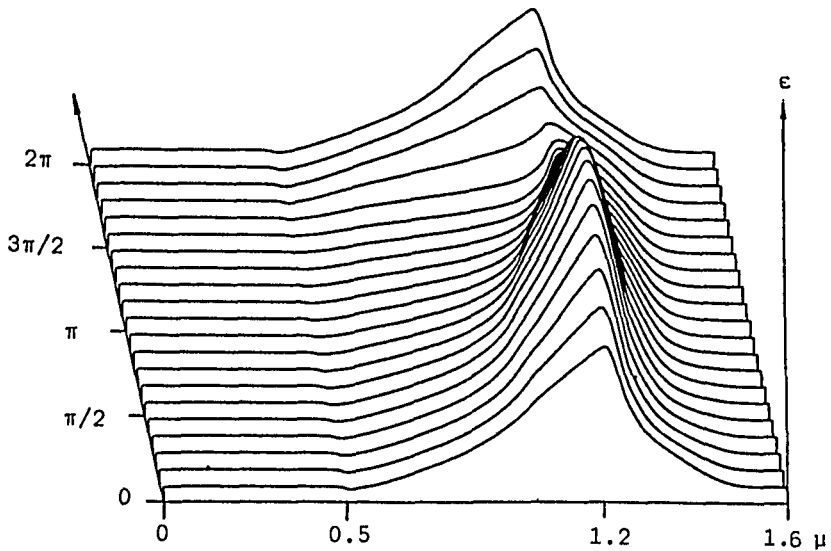


Fig. 8 : Electron energy profile over a period  
at 100 GHz.

The carriers are injected by the cathod ohmic contact on the left  $n^+$  layer with an energy close to  $3/2 kT_0$ . When incoming into the  $n$  layer they are accelerated and the increase in energy leads to a decrease of their velocity. This induces the formation of a carrier accumulation layer which itself induces the increase of the electric field. As the energy depends on the local fields into the sample, the accumulation layer position depends on the instantaneous bias which is responsible of the accumulation layer position evolution as a function of time into the signal period. The transit of this accumulation layer is sensitively achieved at the same velocity than the limit velocity. The space charge region inhomogeneities which transit into the sample are related to the excess carrier accumulation.

On figure 9 to 12 are plotted the same physical parameter for a 200 GHz signal. As the frequency is increased the time response of the device affects its behavior more clearly than for 100 GHz. The difference of behavior appears significantly on the velocity maps (fig. 6 and 10).

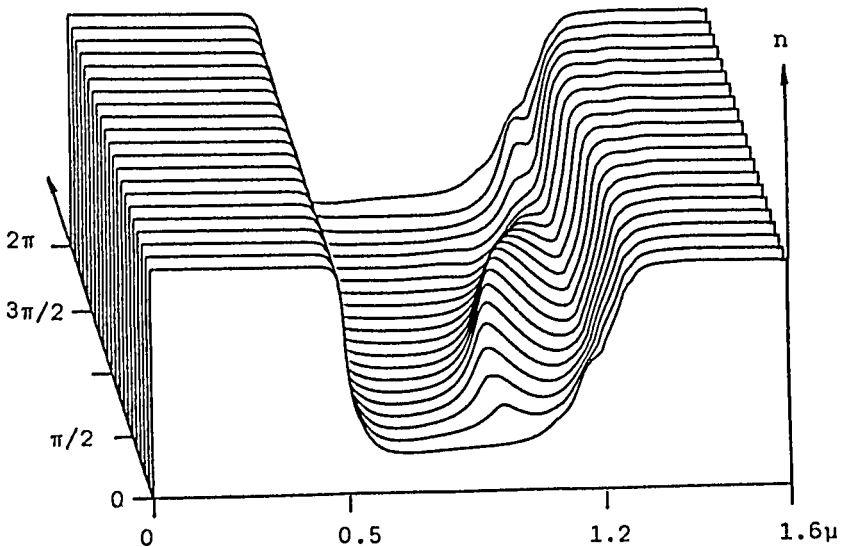


Fig. 9 : Electron density profile over a period at 200 GHz.

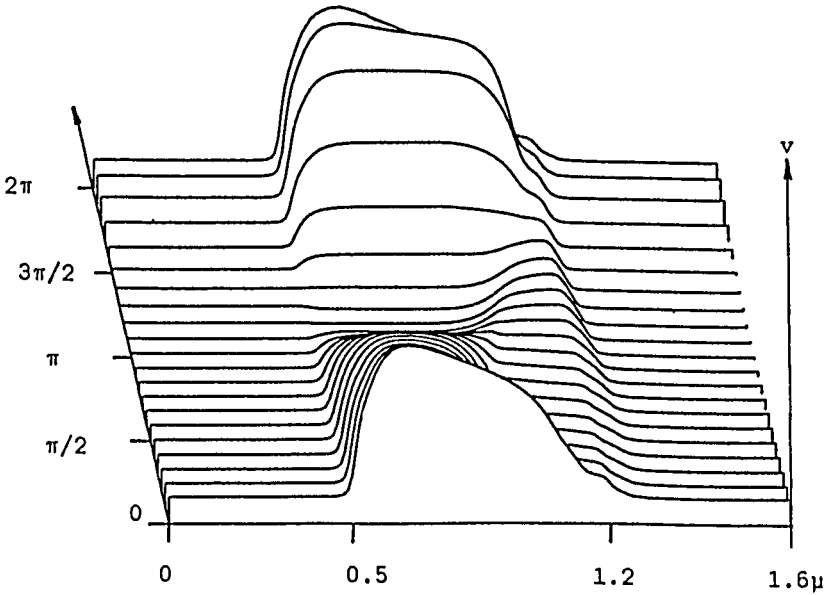


Fig. 10 : Electron average velocity over a period at 200 GHz.

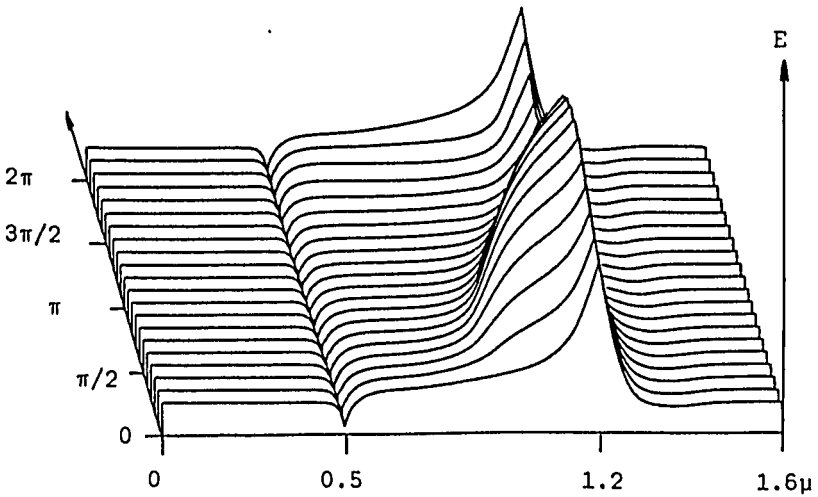


Fig. 11 : Electric field profile over a period at 200 GHz.

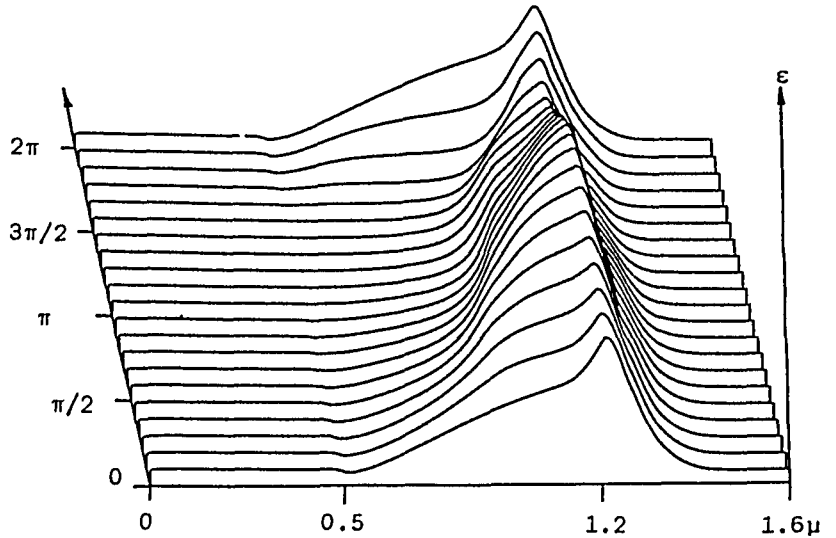


Fig. 12 : Electron energy profile over a period at 200 GHz.

### 5 - CONCLUSION

In this paper we have shown it is necessary to take into account the hot electron behavior when simulating III V devices such as FET's specially when high frequency large signal response has to be simulated.

For this purpose we have developed a model using potentials as main unknown which is consistent with the models we use to describe heterostructures and SI materials. The transient method we proposed allows to describe the transient behavior with a small number of time steps which permits high frequency large signal response simulation in renable calculation times.

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