DELAY TIME FOR GENERAL DISTRIBUTED NETWORKS WITH APPLICATION TO TIMING ANALYSIS OF DIGITAL MOS INTEGRATED CIRCUITS

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SUMMARY

In this work we consider a general network with distributed parameter elements resistively coupled and additionally containing nonlinear capacitors. This network is a better model for the timing analysis of MOS integrated circuits than those previously considered. We define a global parameter "λ-delay time" which expresses the speed of signal propagation in this circuit. A computationally simple formula for an upper bound of this parameter is derived and compared with the bound obtained by numerical calculus.

1. INTRODUCTION

The major performance criterion for digital circuits is the speed of signal propagation. This performance is expressed by the "delay time", which is defined (and denoted) in many different ways by different authors, but having a single intuitive meaning: it is the time between the moment of changing the input variable and the moment when the associated output variable reaches a steady state value.

Of course, at various stages of MOS LSI design one has to verify this performance. This can be done by CAD circuit programs, so called "circuit simulators" as SPICE, ASTAP and SCEPTRE. This kind of programs are very slow for usual handling in the initial stages of design due to the use of complete models of components and especially due to computing method (the integration of differential equations describing the network). This is why faster programs so called "timing simulators" were developed , [1-9], which use simpler models of components and approximation and easily computable formulae for the delay time. In this respect, modelling MOS chips by RC networks has become a well accepted practice [1-6]. The transistor is usually replaced by a resistor, between drain and source, which has a pullup or a pulldown value in ON state and ∞ value in OFF state. Capacitances associated with the pullup source diffusion, contact cuts and the gates being driven are included, connecting respective nodes to the ground. In addition, transmission lines made of series resistors and shunt capacitors are models for connecting wires in MOS integrated chips. With this kind of models, the evolution of a MOS circuit is approximated by a sequence of RC networks corresponding to various states of each transistor.

It results from the above that the problem of estimating the delay of an MOS circuit reduces to that of an RC lumped network. For RC fan out ("RC trees") networks, easily computable bounds of the delay time associated with every node (the input being the same) are given in [1] and are incorporated into timing analysis programs [1,4,5]. The extension of these bounds to mesh networks was given in [10]. Also, in [2] computationally efficient formulae for the delay of any node were developed and included in a simulator.

The present paper tries to improve the evaluations of the delay time, beginning from the following remarks:

1. In all quoted papers the transistor interconnections are approximated by RC ladder networks. The accuracy of this approximation (from the delay time point of view) is not clear, being studied only for one interconnection line [12]. The problem is too important to be neglected, as the advances in technology, the integrated circuit chip size, complexity and device packing density are continuosly increasing (e.g. the prediction of a 0.5 μm feature size and 200 mm^2 chip size by the late 80's appears to be reasonable [13]. As the minimum feature size is made smaller, the cross-section area of the interconnection also reduces. At the same time, a higher integration level allows the chip area to increase, causing the length of the interconnection to increase. That is why, for a very large chip with extremely small geometries, the delay time associated with interconnections becomes an appreciable part of the total time delay, and in certain cases dominates the chip performance [13]. Consequently an increased attention is focused either to modelling [14] or towards the technological advance [15] of the interconnections.

All these considerations justify our intention to consider more elaborate models for the connecting wires, described by telegraph equations. These equations take into account the distributed resistive - capacitive character of the lines and the supplementary dielectric losses neglected in previous studies. Despite mathematical difficulties involved, this more exact modelling gives very easily computed final evaluations for the delay time.

2. Let us consider the example presented in Fig 1.1

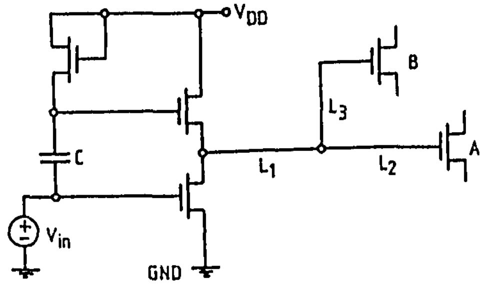

Fig 1.1

Here an enhancement load nMOS bootstraped inverter drives the gates A and B through the three lines L_1, L_2, L_3 (implemented in polycrystalline silicon, metal silicide or metal).

In Fig 1.2 we have drawn a possible equivalent circuit for Fig 1.1 network (where the lines are represented by using the Ghausi symbol)

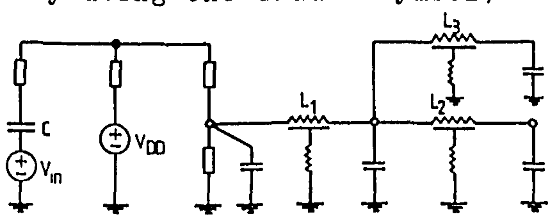

Fig 1.2

Most RC models considered in the quoted works have the capacitors exclusively connected between nodes and ground. As we see from the above example the capacitor C (boatstrap) does not fulfill this condition and so the Rubinstein evaluation [1], for example, does not hold. The same happens if we consider the Miller internodal capacitances. In fact the "floating" capacitors cause many difficulties in simulators [16]. Below we shall deal with a very general network, including the "RC tree" from [1,4,5] or the "RC meshes" from [2,10], (see Fig 2.1), which eliminates the above constraints.

3. The RC networks considered in [1,2,10,11] (with which our work is especially related) have a single

input source V_{DD} (or V_{GND}), and the authors evaluate a delay time associated to each node. Even the above example shows that it is possible that two inputs V_{in} and V_{DD} are simultaneously connected. The ability to handle this case rests on a clear definition of a global delay time, which expresses the rate of evolution of the whole network from the initial condition towards the steady state, when all (or a part of) inputs have step variations.

An asymptotic stability property of our general network allows us to obtain an upper bound of this delay time. The tightness of this bound is then verified by numerical calculus.

Thelast remark: the simplicity of the RC model of the transistors considered in all above quoted works gives obvious errors in the delay time of devices. Consequently, most authors consider linear RC models giving only the time associated with interconnections [1,10]. Of course many efforts must be made to unify the devices and the wiring delay. Our aim is to give computationally simple evaluations taking into consideration more realistic models for transistors and is therefore difficult to achieve. Yet it seems that some steps have been made, especially to include nonlinearities [3,11]. To the same goal we consider here the capacitors as nonlinear elements.

2. STATEMENT OF THE PROBLEM

Let us consider a linear resistive structure with 2n+m pairs of terminals, described by a constant matrix G of conductances and a vector B depending on sources. Because our interest is in step sources, the vector B may be supposed constant in time. Hence, we shall assume that the resistive multiport introduces the constraint

$$(2.1) \qquad i = -Gu + B,$$

where i and u are 2n+m vectors of terminals currents and voltages.

As we see in Fig 2.1., at first 2n terminals of the multiport are connected by n distributed parameter elements.

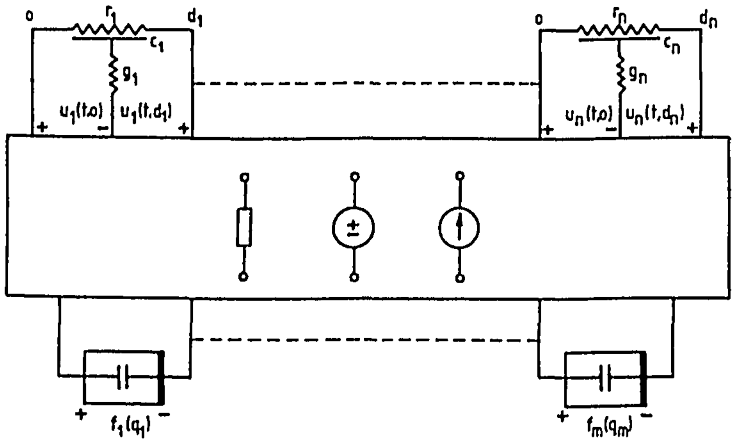

Fig 2.1

For each of these "r-c-g lines" we can write the well-known telegraph equations:

$$(2.2)\quad \begin{cases} \dfrac{\partial u_k(t,x)}{\partial x} = -r_k i_k(t,x) \\ \dfrac{\partial i_k(t,x)}{\partial x} = c_k \dfrac{\partial u_k(t,x)}{\partial t} - g_k u_k(t,x) \\ k = 1,2,\dots n. \end{cases}$$

Everywhere in the following we shall consider constant r_k, c_k and g_k parameters and $r_k, c_k > 0$, $g_k \geq 0$. Also, at last, m pairs of terminals are connected nonlinear capacitors, charge controlled by functions $f_k : \mathbb{R} \to \mathbb{R}$, $k = 1,\dots m$.

From Fig 2.1 and by using (2.1) and (2.2) we easily derive the following mathematical model of our problem:

- a system of parabolic linear equations

$$(2.3)\quad \begin{cases} \dfrac{\partial u_k}{\partial t} = \dfrac{1}{r_k c_k} \dfrac{\partial^2 u_k}{\partial x^2} - \dfrac{g_k}{c_k} u_k \\ x \in [0,d_k],\ t \geq 0,\ k = 1,\dots,n \end{cases}$$

- a set of nonlinear boundary conditions

$$(2.4)\qquad \begin{bmatrix} -\frac{1}{r_1}\frac{\partial u_1}{\partial x}(t,0) \\ \frac{1}{r_1}\frac{\partial u_1}{\partial x}(t,d_1) \\ \vdots \\ -\frac{1}{r_n}\frac{\partial u_n}{\partial x}(t,0) \\ \frac{1}{r_n}\frac{\partial u_n}{\partial x}(t,d_n) \\ \frac{dq_1}{dt} \\ \vdots \\ \frac{dq_m}{dt} \end{bmatrix} = -G\cdot \begin{bmatrix} u_1(t,0) \\ u_1(t,d_1) \\ \vdots \\ u_n(t,0) \\ u_n(t,d_n) \\ f_1(q_1) \\ \vdots \\ f_m(q_m) \end{bmatrix} + B$$

and, of course, the initial conditions

$$(2.5)\qquad \begin{cases} u_k(0,x) = u_{k0}(x) & , k = 1,\dots n \\ q_k(0) = q_0 & , k = 1,\dots m. \end{cases}$$

The following assumptions will be taken into consideration:

A_1: The functions $f_k:\mathbb{R}\to\mathbb{R}$ $k = 1,\dots,m$ are continuous and there exist strictly positive real numbers $\underline{M}_k$ and $\bar{M}_k$ such that for every $x,y \in \mathbb{R}$ we have

$$(2.6)\qquad \underline{M}_k|x-y| \le |f_k(x) - f_k(y)| \le \bar{M}_k|x-y|.$$

A_2: There exist positive constants $C_1,\dots C_m$, such that for each $i = 1,\dots 2n$ we have

$$(2.7)\qquad -G_{ii} + \sum_{j=1, j\neq i}^{2n} |G_{ij}| + \sum_{j=2n+1}^{2n+m} |G_{ij}| C_{j-2n}\bar{M}_{j-2n} < 0$$

and for each $i = 2n+1,\dots,2n+m$ it holds

$$(2.8)\qquad -G_{ii}\underline{M}_{i-2n}C_{i-2n} + \sum_{j=1}^{2n} |G_{ij}| + \sum_{j=2n+1, j\neq i}^{2n+m} |G_{ij}|\bar{M}_{j-2n}C_{j-2n} < 0.$$

In addition, we shall suppose that the initial conditions (2.5) are so smooth that our problem (2.3)+ +(2.4)+(2.5) has a unique solution in the classical sense, i.e. there are n functions $u_k:[0,\infty)\times[0,d_k]\to \mathbb{R}$, C^1 in time and C^2 in space, and m functions $q_k:[0,\infty)\to \mathbb{R}$, C^1 in time, satisfying the above equations. Also, we shall suppose that there exist twice differentiable functions $\bar{u}_k:[0,d_k]\to \mathbb{R}$, $k = 1,\dots,n$ and the constants $\bar{q}_1,\dots,\bar{q}_m$ which satisfy the time independent (steady state) problem corresponding to (2.3)+(2.4)+(2.5). In the case m=0 we considered the existence and unicity problem in [17] and [18]. Some remarks about our hypotheses:

- The hypothesis A_1, allows us to consider a very large class of nonlinear characteristics of capacitors, among which the important engineering cases of continuous differentiable (with bounded derivatives) and piecewise continuous functions.
- The constants $\bar{M}_k$ and $\underline{M}_k$ in (2.6) have to the dimensions of $[\text{capacitance}]^{-1}$ while the constants C_k from (2.7) and (2.8) have to the dimensions of capacitance.
- The relations (2.7) and (2.8) seem to be cumbersome. In fact they are very natural because, in the case of linear capacitors, we have $\underline{M}_k = \bar{M}_k = [\text{the capacitance}]^{-1}$ and if we choose C_k = the capacitance, then these relations become

$$(2.9)\qquad -G_{ii}+\sum_{j=1,j\neq i}^{2n+m}|G_{ij}|<0 .$$

This condition is the very well known "diagonally row-sum dominant" property that can assure the existence and unicity [17], [18] and is frequently considered in circuit theory.
- As our examples and our experience show, the above hypotheses are not very restrictive, they are satisfied in most practical circuits cases.

3. THE TRANSIENT BEHAVIOR. DELAY TIME EVALUATION

The global behavior of our network can be described by a fuction $D:[0,\infty]\to \mathbb{R}$ named "delay" and defined by

$$(3.1)\quad D(t)=\frac{\max\left[\max\limits_{1\le i\le n}\max\limits_{0\le x\le d_i}\left|u_i(t,x)-\bar{u}_i(x)\right|;\max\limits_{1\le i\le m}\frac{q_i(t)-\bar{q}_i}{C_i}\right]}{\max\left[\max\limits_{1\le i\le n}\max\limits_{0\le x\le d_i}\left|u_{i0}(x)-\bar{u}_i(x)\right|;\max\limits_{1\le i\le m}\frac{q_{i0}-\bar{q}_i}{C_i}\right]} .$$

The delay begins from 1 (corresponding to the initial conditions) and tends to 0 (corresponding to the steady state). If we choose $\lambda\in(0,1)$ then, the speed of this evolution can be expressed by the last moment when the delay passes through the λ value. In this way, we are conducted to define the "λ delay time" as

$$(3.2) \qquad T_\lambda = \sup\left\{t; D(t) = \lambda\right\}.$$

In the following we shall find an upper bound $\bar{T}_\lambda$ for this T_λ. We shall give our arguments only in outline. The reader can find the details in [19-23]. First of all a useful remark. By intuition we can say that the delay does not change if we invert the terminals of any line. This fact can be rigorously proved [22]. The initial notation of the terminals of the lines corresponds to the matrix G and to the sequence $\delta = (\delta_1,\ldots,\delta_n)$ where $\delta_j = 0$ for all j. The inversion of the terminals of the k-th line will be marked by $\delta_k = 1$, and will correspond to a matrix $G^\delta = G^{(0,1,0,0)}$ which has the 2k-1-th and 2k-th rows interchanged as well as 2k-1-th and 2k-th columns. In [22] we have proved that the initial problem (2.3)+(2.4)+(2.5) and the similar problem constructed with G^δ, where $\delta = (\delta_1,\ldots\delta_n)$ and $\delta_k = 0$ or 1, have the same delay. But the upper bound given below of the λ-delay time will depend on G^δ and will be denoted by $\bar{T}^\delta_\lambda$. It is clear now that we can take for $\bar{T}_\lambda$ the minimum value of these upper bounds, namely

$$(3.3) \qquad \bar{T}_\lambda = \min_\delta \bar{T}^\delta_\lambda ,$$

where the minimum with respect to δ means the minimum with respect to all 2^n cases appearing when we change the matrix G by the indicated manner. To find $\bar{T}^\delta_\lambda$ we shall use in the following the matrix G^δ instead of G. It is easily seen that the relations (2.7) and (2.8) are equivalent with the same relations written with the elements of G^δ.

For an arbitrary compact space K, we shall denote $Z = \{h:\prod_{i=1}^n [0,d_i]\times K\to \mathbb{R}^{n+m}$ with components $h_i:[0,d_i]\to\mathbb{R}$, $i=1,\ldots,n$ continuous and $h_i:K\to\mathbb{R}$ constants$\}$ a Banach space with the norm

$$(3.4) \qquad \|h\| = \max_{1\le i\le n+m} \max_{x_i} |h_i(x_i)|$$

We shall make a change of variables in our problem (2.3)+(2.4)+(2.5), namely

$$(3.5)\quad \begin{cases} u_k(t,x) = v_k(t,x)\cos\frac{\alpha_k x}{d_k}, x\in[0,d_k], k=1,\dots n \\ q_k(t) = v_{k+n}(t)\cdot C_k,\ k=1,\dots m \end{cases},$$

where the constants α_k will be chosen below. Let us take the following subset of Z:

$\mathcal{D}(A) = \{h\in Z; h_1, h_2, \dots h_n$ twice differentiable and

$$\begin{bmatrix} -\frac{1}{r_1}\frac{\partial h_1(0)}{\partial x} \\ +\frac{1}{r_1}\frac{\partial h_1(d_1)}{\partial x}\cos\alpha_1 \\ \vdots \\ -\frac{1}{r_n}\frac{\partial h_n(0)}{\partial x} \\ +\frac{1}{r_n}\frac{\partial h_n(d_n)}{\partial x}\cos\alpha_n \end{bmatrix} = -\bar{G}^\delta \begin{bmatrix} h_1(0) \\ h_1(d_1)\cos\alpha_1 \\ \vdots \\ h_n(0) \\ h_n(\alpha_n)\cos\alpha_n \\ f_1(h_{n+1}C_1) \\ \vdots \\ f_m(h_{n+m}C_m) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{\alpha_1 h_1(d_1)}{r_1 d_1}\sin\alpha_1 \\ \vdots \\ 0 \\ \frac{\alpha_n h_n(d_n)}{r_n d_n}\sin\alpha_n \end{bmatrix} + \bar{B}^\delta\}\ .$$

Here we have denoted by $\bar{G}^\delta$ the $2n\times(2n+m)$ matrix extracted from upper part of G^δ, and by $\bar{B}^\delta$ the vector with first (δ inverted) $2n$ elements from B. Let us take the operator $A:\mathcal{D}(A)\to Z$, defined by:

$$A\begin{pmatrix} h_1 \\ \vdots \\ h_{n+m}\end{pmatrix}(x_1,\dots,x_n) = \text{an } n+m \text{ vector with components:}$$

$$\frac{1}{r_k c_k}\frac{\partial^2 h_k(x_k)}{\partial x_k^2} - \left(\frac{2}{r_k c_k}\frac{\alpha_k}{d_k}\operatorname{tg}\frac{\alpha_k x_k}{d_k}\right)\frac{\partial h_k(x_k)}{\partial x_k} - \left(\frac{\alpha_k^2}{d_k^2 r_k c_k} + \frac{g_k}{c_k}\right)h_k(x_k)$$

for $k = 1,\dots n$ and

$$-\frac{1}{C_{k-n}}\left[\sum_{j=1}^{n} G^\delta_{n+k,2j-1}h_j(0) + \sum_{j=1}^{n} G^\delta_{n+k,2j}h_j(d_j)\cos\alpha_j + \right.$$
$$\left. + \sum_{j=1}^{m} G^\delta_{n+k,2n+j}\ f_j(h_{n+j}C_j)\right] \quad \text{for } k = n+1,\dots,n+m.$$

With these, we can easily show that our problem is equivalent with the following abstract Cauchy problem

$$(3.6)\quad \begin{cases} \frac{d}{dt}v(t,\cdot) = Av(t,\cdot) + B^* \\ v(0,\cdot) = \text{a function with components} \\ u_{k0}(\cdot)/\cos\alpha_k \frac{\cdot}{d_k} \quad \text{for } k = 1,\dots,n \text{ and} \\ q_{k-n,0}/C_{k-n} \qquad \text{for } k = n+1,\dots,n+m \end{cases},$$

where B* is a vector with zero for the first n components and b_{2n+k}/C_k, k = 1,...m for the following ones.

Let us further denote

$$(3.7)\quad S_i^\delta = \begin{cases} \sum_{j=1, j\neq i}^{2n} |G_{ij}^\delta| + \sum_{j=2n+1}^{2n+m} |G_{ij}^\delta| C_{j-2n}\bar{M}_{j-2n}, & \text{for } i = 1,\ldots,2n \\ \sum_{j=1}^{2n} |G_{ij}^\delta| + \sum_{j=2n+1, j\neq i}^{2n+m} |G_{ij}^\delta| C_{j-2n}\bar{M}_{j-2n}, & \text{for } i = 2n+1,\ldots,2n+m. \end{cases}$$

The following lemma is from the mathematical point of view our basic result.

Lemma 3.1 Under the hypotheses A_1 and A_2 let γ_j^δ, j = 1,...,n be the unique solution in $(0,\frac{\pi}{2})$ of the equation:

$$(3.8)\quad \cos\gamma_j^\delta = \left[S_{2j}^\delta + \sqrt{(S_{2j}^\delta)^2 + \frac{4}{r_j d_j}\left(G_{2j,2j}^\delta + \frac{1}{r_j d_j}\right)}\right] / 2\left(G_{2j,2j}^\delta + \frac{1}{r_j d_j}\right).$$

If we take

$$(3.9)\quad \omega_\varepsilon^\delta = \max\left\{\max_{1\leq j\leq n}\left[-\frac{(\gamma_j^\delta-\varepsilon)^2}{d_j^2 r_j c_j} - \frac{g_j}{c_j}\right]; \max_{n+1\leq j\leq n+m}\left(\frac{-G_{n+j,n+j}^\delta + S_{n+j}^\delta}{C_{j-n}}\right)\right\}$$

then, the operator $A - \omega_\varepsilon^\delta I$, constructed with $\alpha_j = \gamma_j^\delta - \varepsilon \in (0,\frac{\pi}{2})$. is dissipative in the space $\mathcal{Z}$ □

For the proof we refer to [21,23].

Theorem 3.1. If A_1 and A_2 hold, then for all $t\geq 0$

$$D(t) \leq e^{\omega_0^\delta t} / \min_{1\leq i\leq n} \cos\gamma_i^\delta \quad \text{and then}$$

$$(3.10)\quad \bar{T}_\lambda^\delta = \frac{\ln(\lambda \min_{1<i<n} \cos\gamma_i^\delta)}{\omega_C^\delta} \qquad □$$

This theorem gives a global asymptotic stability property for the steady state solution, i.e. the dynamic state tends to the same steady state for any initial conditions.

For the sake of clarity we shall resume the procedure to obtain $\bar{T}_\lambda$, namely in the linear case where the writing is shorter.

Procedure 3.1 (for computing $\bar{T}_\lambda$)

Step 1. Given the matrix G. Compute for each i = 1,. ...,2n+m, the size

$$S_i = \sum_{\substack{k=1 \\ k\neq j}}^{2n+m} |G_{ik}|$$

Step 2. For each $j = 1,\ldots,n$ calculate

$$(3.11)\quad \cos\gamma_j^\delta = S_{2j}+\sqrt{S_{2j}^2+\frac{4}{r_jd_j}(G_{2j,2j}+\frac{1}{r_jd_j})}/2(G_{2j,2j}+\frac{1}{r_jd_j})$$

or

$$S_{2j-1}+\sqrt{S_{2j-1}^2+\frac{4}{r_jd_j}(G_{2j-1,2j-1}+\frac{1}{r_jd_j})}\ /2(G_{2j-1,2j-1}+\frac{1}{r_jd_j}).$$

Step 3. For γ_j^δ from above, with $G_{n+j,n+j}^\delta=G_{n+j,n+j}$ and $S_{n+j}^\delta= S_{n+j}$ for $j=n+1,\ldots,n+m$ and with $\varepsilon=0$ compute the 2^n values of ω_0^δ from (3.9)
Step 4. The relations (3.10) and (3.3) give the desired value of $\bar{T}_\lambda$.

As we see we found a very simple and fast method to compute the upper bound of the λ-delay time.

4. THE EXAMPLES

The following example is given for the purpose to verify the tightness of $\bar{T}_\lambda$ to T_λ.

Let us consider the fan-out circuit from Fig 4.1, with two inputs $E_1=E_2=1v$, with lumped elements having values: $R_1=\frac{1}{2}\Omega$, $R_2=1\Omega$, $C_1=3F$, $C_2=3/2F$, $C_3=3F$, and with four r-c-g lines.

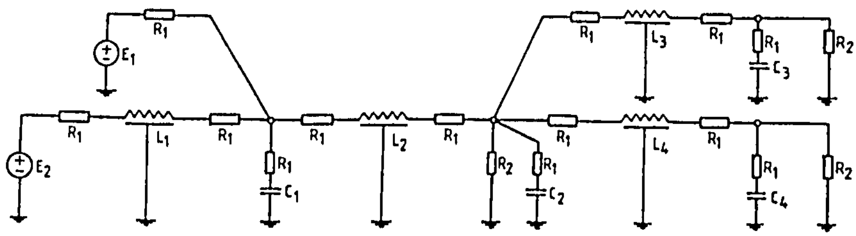

Fig 4.1

The lines have the parameters: $r_id_i=1\Omega$ for $i=1,\ldots,4$, $c_1d_1=3/2F$, $c_2d_2=1F$, $c_3d_3=5/4F$, $c_4d_4=4/3F$. This results a 12×12 G matrix whose nonzero elements are: $G_{11}=2$, $G_{22}=G_{33}=G_{99}=3/2$, $G_{44}=G_{55}=G_{77}=G_{10,10}=14/9$, $G_{66}=G_{88}=G_{11,11}=G_{12,12}=6/5$, and the nondiagonal ones: $G_{23}=G_{32}==G_{29}=G_{92}=G_{39}=G_{93}= -1/2$, $G_{45}=G_{54}=G_{47}=G_{74}=G_{4,10}=G_{10,4}==G_{57}=G_{75}=G_{5,10}=G_{10,5}=G_{7,10}=G_{10,7}=-4/9$, $G_{6,12}=G_{12,6}==G_{8,11}=G_{11,8}=-4/5$. The nonzero elements of B are $b_1=2$, $b_2=b_3=b_9=1/2$.

The following table summarizes the calculus for delay time in Procedure 3.1.

Table 4.1

δ	$\cos\gamma_1$	$\cos\gamma_2$	$\cos\gamma_3$	$\cos\gamma_4$	ω_0^δ	$\bar{T}_{0.1}$	$\bar{T}_{0.5}$
0000	0.863	0.938	0.880	0.880	-0.124	19.75	6.78
1000	0.577	0.938	0.880	0.880	-1.124	23.00	10.02
0100	0.863	0.863	0.880	0.880	-0.133	18.42	6.32
0010	0.863	0.938	0.938	0.880	-0.099	24.74	8.49
0001	0.863	0.938	0.880	0.938	-0.0929	26.37	9.04
1100	0.577	0.863	0.880	0.880	-0.133	21.45	9.35
0110	0.863	0.863	0.938	0.880	-0.099	24.74	8.49
0011	0.863	0.938	0.938	0.938	-0.0929	26.37	9.04
1010	0.577	0.938	0.938	0.880	-0.124	23.00	10.02
1001	0.577	0.938	0.880	0.938	-0.0929	30.70	13.38
0101	0.863	0.863	0.880	0.938	-0.0929	26.37	9.04
1110	0.577	0.863	0.938	0.880	-0.099	28.81	12.55
1101	0.577	0.863	0.880	0.938	-0.0929	30.70	13.38
0111	0.863	0.863	0.938	0.938	-0.0929	26.37	9.04
1011	0.577	0.938	0.938	0.938	-0.0929	30.70	13.38
1111	0.577	0.863	0.938	0.938	-0.0929	30.70	13.38

As we see, $\bar{T}_{0,1}=18.42$ s and $\bar{T}_{0,5}=6.32$ s. The values of delay time numerically computed (Crank - Nicolson method in time and finite element method in space (linear elements) - see detail in [22]) are $\bar{T}_{0,1}=7.1$ s and $T_{0.5}=2.11$. (see Fig 4.4).

Figure 4.2 shows the approximation for $u_i(t,\cdot)$ for different time moments t_k (indicated by —) as well as the steady state solutions $\bar{u}_i(\cdot)$ i=1,...,4 (indicated by ---)

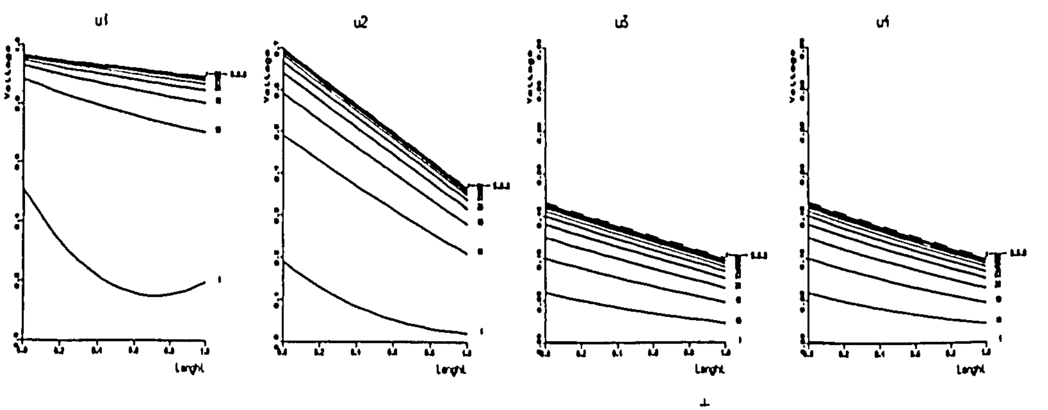

Fig 4.2

In Fig 4.3 we see approximation for $u_i(t)$, i=5,...,8 for $t\in(0,40)$.

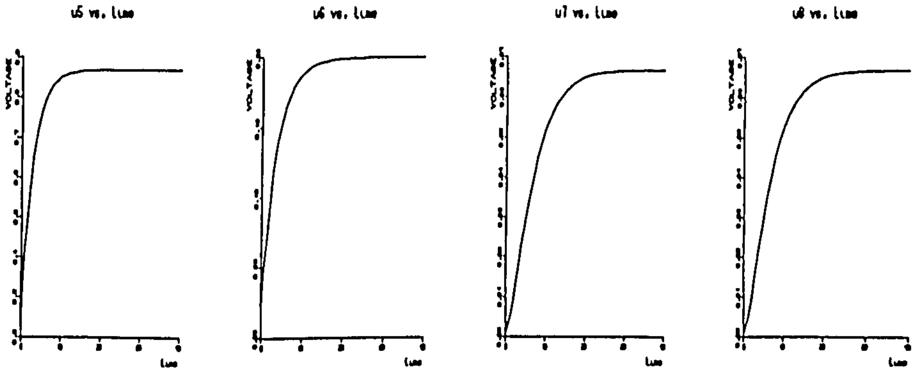

Fig 4.3

Figure 4.4 shows the a-priori upper bound for the delay time (obtained by Theorem 3.1, $\delta=(0,1,0,0)$) as well the delay time obtained from the discrete model (computed delay).

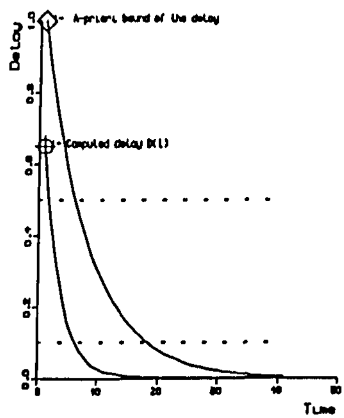

Fig 4.4

For further numerical examples (linear and non-linear cases) we refer to [17 - 23].

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