Coherent Wigner Dynamics of a Superposition State in a Tunable Barrier Quantum Dot

M. Ballicchia, M. Nedjalkov, and J. Weinbub
Institute for Microelectronics, TU Wien, Gußhausstraße 27–29, 1040 Wien, Austria
e-mail: weinbub@iue.tuwien.ac.at

Tunable barrier quantum dots (QDs) are used in single electron sources [1], [2] which in turn are essential building blocks for electron quantum optics, quantum metrology, and quantum information processing [3]–[5]. The tunable barriers are realized by a two-gate structure along a nanowire [2]. The operational principle uses a pumping scheme of loading, capture, and ejection stages [6]: Understanding the coherent electron dynamics is vital for optimizing the overall performance.

In this work, we investigate the dynamics of the captured and ejected electron state by a Wigner simulation study [3], [7]–[9]. Extending previous work [10], here we consider the probability current and impact of different timings on the state evolution after ejection. Inspired by [6], we model a one-dimensional nanowire ($200\,\mathrm{nm}$) with a QD which is modeled by two Gaussian potential barriers placed at $x_1 = 25\,\mathrm{nm}$ and $x_2 = 75\,\mathrm{nm}$ (Fig. 1), where the first barrier has a $V_1 = 0.5\,\mathrm{eV}$ and the second barrier a time-dependent $V_2(t)$, i.e., driven by a gate voltage (barrier width $w_{bar} = 5\,\mathrm{nm}$). Up to $V_2(t) = V_1$, the energy is quantized in the spatial domain Ω bounded by the two Gaussian barriers and the electron state can be obtained in principle by the stationary Schrödinger equation as a superposition of the orthonormal set of eigenfunctions ψ_n. Limiting our analysis to the first two eigenfunctions of the QD, ψ_1 and ψ_2, as was done in [6], we get: $\Psi(x,t) = a_1\psi_1(x)e^{-\frac{i}{\hbar}\epsilon_1 t} + a_2\psi_2(x)e^{-\frac{i}{\hbar}\epsilon_2 t}$.

We use the wavefunction $\Psi(x,0)$ to determine the density matrix at time $t = 0$ and consequently obtain the initial Wigner function $f_w(x,p,0)$ which is then evolved by solving the Wigner equation [9]. The coherent dynamics of the superposition state are analyzed by considering the two first moments of the Wigner function, i.e., $n(x) = \int f_w(x,p)\mathrm{d}p$ (probability density) and $J(x) = (1/m_{eff})\int p\, f_w(x,p)\mathrm{d}p$ (probability current).

We studied two scenarios: (1) Capture stage: $V_2(t)$ is kept constant and equal to V_1. (2) Ejection stage: $V_2(t) = V_1 \cdot [1 - H(t - t_1)]$, where $H(t)$ is the Heaviside function and particularly consider $t_1 = 10\,\mathrm{fs}$ and $t_1 = 400\,\mathrm{fs}$. Initial condition: The electron at $t = 0$ is bounded inside the QD with $a_1 = a_2 = 1/\sqrt{2}$ and the effective mass is $m_{eff} = 0.19\,m_{el}$. Reg. (1), Fig. 1 shows the probability density (Wigner and verification with Schrödinger) after half a period of oscillation. The electron state oscillates from left to right and vice-versa with a period $T = 2\pi\hbar/(\epsilon_2 - \epsilon_1) \approx 500\,\mathrm{fs}$. Reg. (2), Fig. 2 shows the evolution of the probability density at $t = 500\,\mathrm{fs}$ and $t = 700\,\mathrm{fs}$ for $t_1 = 10\,\mathrm{fs}$, while Fig. 3 for $t_1 = 400\,\mathrm{fs}$. Different opening times t_1 clearly influence the shape of the probability density, potentially leading to separation or superimposition. Fig. 4 shows the probability current at $t = 500\,\mathrm{fs}$ for both $t_1 = 10\,\mathrm{fs}$ and $t_1 = 400\,\mathrm{fs}$. The first case shows clear peak separation, indicating the presence of significantly different velocities (energies) in the state. The latter case results in much more localization, higher magnitude, and almost no peak separation. We can thus show that a Wigner dynamics simulation is an attractive additional tool in designing the tunable barrier system for coherent single electron sources.

Acknowledgment: The financial support by the Austrian Science Fund (FWF): P33609 is gratefully acknowledged. The computational results have been achieved using the Vienna Scientific Cluster (VSC).

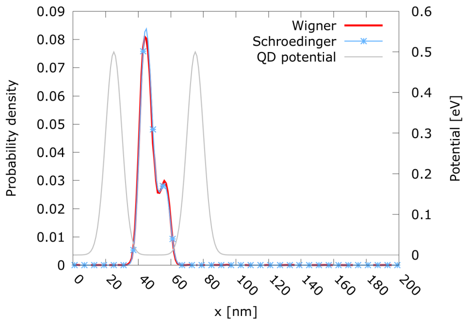

Fig. 1. Probability density of the superposition state at $t = 250\,\text{fs}$: Comparison between Wigner and Schrödinger. The two peaks of the captured electron state oscillate with $T \approx 500\,\text{fs}$.

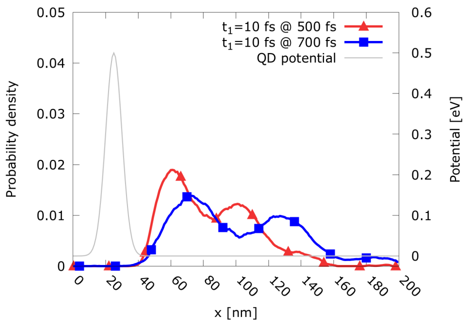

Fig. 2. Evolution along the channel of the electron superposition state ejected at $t_1 = 10\,\text{fs}$: Probability density at $t = 500\,\text{fs}$ and $t = 700\,\text{fs}$. The state's peak separation indicates the presence of significantly different velocities (energies) in the state (confirmed in Fig. 4).

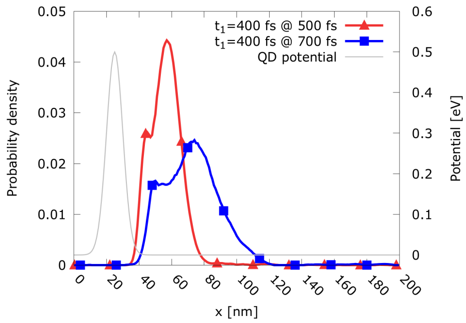

Fig. 3. Evolution along the channel of the electron superposition state ejected at $t_1 = 400\,\text{fs}$: Probability density at $t = 500\,\text{fs}$ and $t = 700\,\text{fs}$.

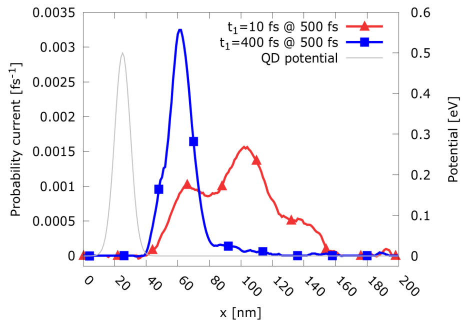

Fig. 4. Probability current at $t = 500\,\text{fs}$. The case $t_1 = 10\,\text{fs}$ (red) clearly shows the presence of significantly different velocities (energies) in the state.

References

[1] G. Yamahata *et al.*, Phys.Rev.B **103**, 245306 (2021).
[2] B. Kaestner *et al.*, Rep.Prog.Phys. **78**, 103901 (2015).
[3] J. Weinbub *et al.*, J.Phys.Condens.Mat. **34**, 163001 (2022).
[4] C. Bäuerle *et al.*, Rep.Prog.Phys. **81**, 056503 (2018).
[5] H. Edlbauer *et al.*, EPJ Quant.Techn. **9** (2022).
[6] G. Yamahata *et al.*, Nat.Nanotech. **14**, 1019 (2019).
[7] M. Nedjalkov *et al.*, *Wigner Function Approach*, Springer, 289 (2011).
[8] D.K. Ferry *et al.*, *The Wigner Function in Science and Technology*, IOP Publishing (2018).
[9] J. Weinbub *et al.*, Appl.Phys.Rev. **5**, 041104 (2018).
[10] M. Ballicchia *et al.*, Proc. IEEE NANO, 565 (2022).