Is there a mesoscopic Braess paradox?

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INTRODUCTION

In a recent paper Pala *et al.* present an experiment and a numerical simulation of an interesting phenomenon that is interpreted as a mesoscopic analogous of the Braess paradox. They consider the structure reported in Fig. 1 and observe that, in "congestion" conditions (i.e., if the total number of modes propagating in the parallel channels is less than that in the leads), addition of the channel in the middle may, somewhat counterintuitively, lead to a decrease in conductance instead of an increase. This effect is interpreted with reference to the Braess paradox [2] in transportation theory, in which opening a new path in a particular road network used by noncooperative players may lengthen the average travel time between two locations.

Here we perform an analysis showing that this situation is actually a particular case of destructive quantum interference. In other words, addition of the third channel leads to suppression of transmission for some of the modes in the vertical channels, due to a "stub" effect (see, e.g. Ref. [3]), and this suppression prevails over the conductance increase due to the additional channel.

MODEL AND RESULTS

We consider the same material parameters (for InGaAs) and device geometry as in Ref. [1], and use a recursive Green's function approach [3] to compute the trasmission matrix, from which conductance is obtained via the Büttiker-Landauer formalism.

Let us first analyze the dependence of conductance through the structure as a function of the position of the exit lead: results are reported in Fig. 2 for the case of only two channels (solid line), adding a third channel in the middle (dashed line), and with a third channel shifted 500 nm away from the center (dotted line). We immediately notice that the paradoxical behavior is observed only when the entrance and exit lead are on opposite sides with respect to the third channel (otherwise the addition of the third channel leads to an increase of the conductance)

We observe that the third channel not only acts as an additional conducting path, but also as a "stub" for the vertical channels to which it is connected, thereby letting only part of the propagating modes pass without being attenuated. If the third channel is between the leads, such an attenuation will occur along both paths: the clockwise [Fig. 3(a)] and the counterclockwise one [Fig. 3(b)], otherwise the attenuation will affect only one [Fig. 3(d)] of the paths (although connected in two locations, the same modes will be affected). Thus, in the former case the attenuation prevails over the increase in conductance resulting from the new open path, while in the latter case the reverse happens.

To confirm this interpretation, we have performed a simulation adding a third channel with a barrier in the middle: in this way we still have a stub action leading to the conductance decrease, but there is no new path for conduction. In Fig. 4 we plot the overall conductance as a function of the position of the exit lead in the absence of the third channel (solid line), and in the presence of the third channel obstructed with a 20 nm thick and 0.1 eV high barrier (the Fermi energy is 60 meV) placed in the middle. It is apparent that conductance is suppressed more than when the third channel is unobstructed, although there is no new open path.

REFERENCES

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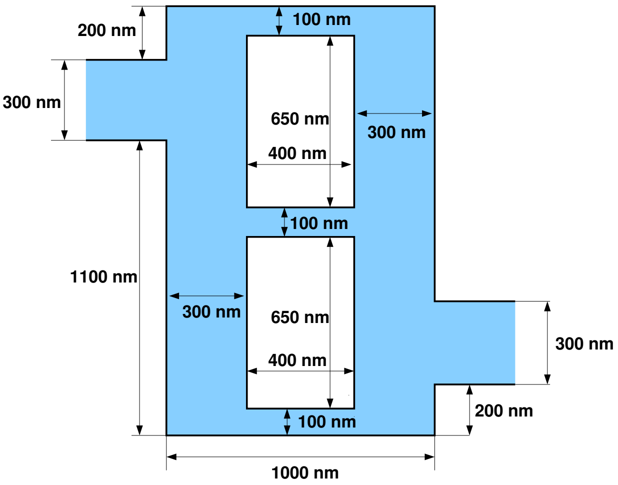

Fig. 1. Sketch of the considered structure

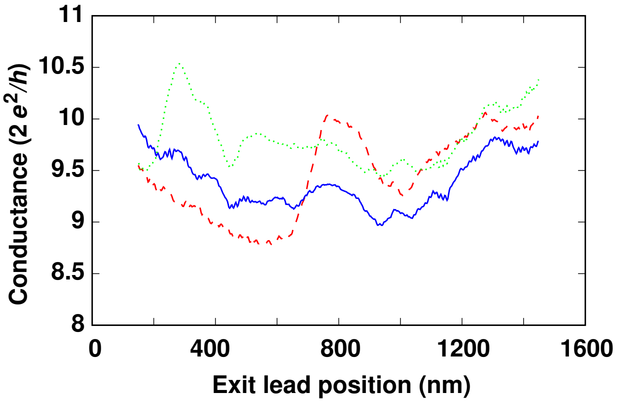

Fig. 2. Conductance as a function of the position of the center of the exit lead (measured from the bottom): for 2 channels (solid line), with the addition of the third channel in the middle (dashed line), and with the addition of the third channel shifted down by 500 nm (dotted line).

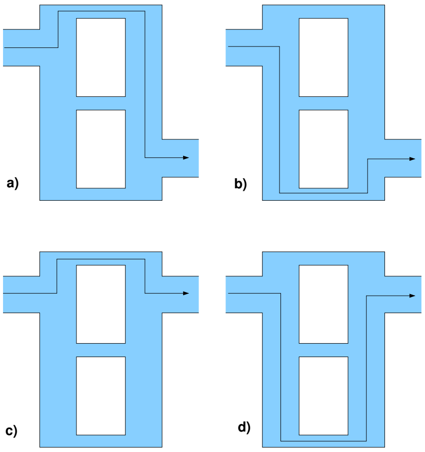

Fig. 3. Possible paths in the case of leads on opposite sides (a,b) and of leads on the same side (c,d) with respect to the third channel.

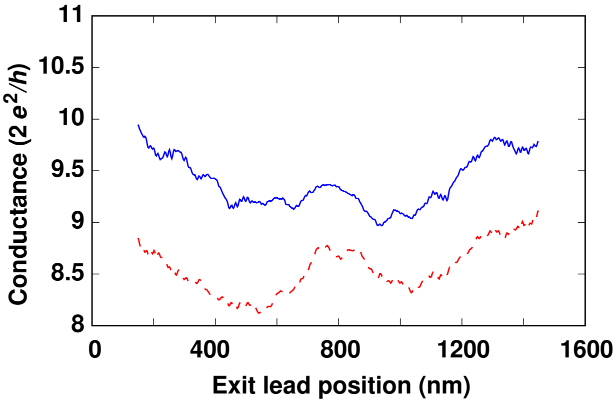

Fig. 4. Conductance as a function of the position of the exit lead for 2 channels (solid line) and for 3 channels (dashed line), but with an opaque barrier in the middle of the third channel.

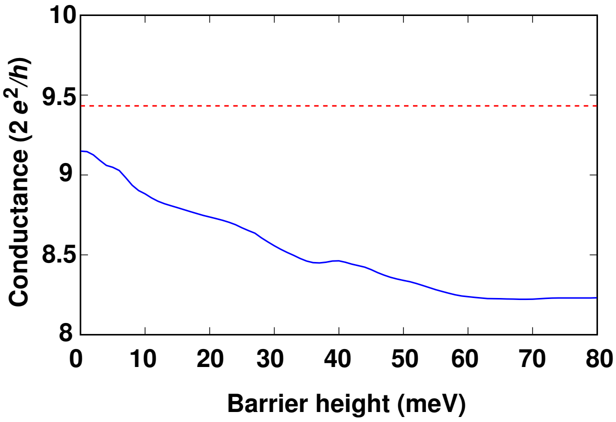

Fig. 5. Conductance as a function of the height of the barrier in the middle of the third channel, for the lead configuration of Fig. 1; the dashed line represents the conductance for the case of only 2 channels.