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The Driven Two–Level System as an Inverse Problem

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We use direct approximate inversion of the time evolution of a dissipative two–level system to identify optimal control fields. The equations of motion are formulated within the density matrix formalism, assuming weak coupling to a phonon bath. We show that the problem can be solved exactly for the isolated driven two–level system. The solution is not unique. This strategy serves to identify approximate solutions for the coupled case which may subsequently be optimized by conventional techniques.

INTRODUCTION

In semiconductor physics recent research has concentrated on nanostructures like quantum dots (QD) or quantum wells. Electrons confined in such QD can be manipulated by external fields, as provided by lasers, but they also interact with their environment, in particular with phonons, leading to dissipation. This interaction is one of the crucial problems which prevent complete controllability of electronic quantum states and their temporal evolution in QD's [1].

In this paper we present an (approximate) solution to the inverse problem of finding an optimum control field to coherently steer a dissipative quantum system.

THE ISOLATED TWO–LEVEL SYSTEM

As a starting point for our investigations we consider an isolated 2-level quantum system, described by the von Neumann equation $i\hbar\dot{\rho}(t) = [H(t), \rho(t)]$. We assume a given $\rho(t)$ and $\dot{\rho}(t)$ and seek a time dependent Hamiltonian $H(t)$, that produce this $\rho(t)$. Setting

$$\rho(t) = \begin{pmatrix} \rho_{11}(t) & a(t) + ib(t) \\ a(t) - ib(t) & 1 - \rho_{11}(t) \end{pmatrix}, \quad (1)$$

and using $\frac{d}{dt}Tr\{\rho(t)^2\} = 0$, the Hamiltonian achieving this quantum evolution is

$$H(t) = \begin{pmatrix} \hbar\frac{\dot{a}(t)}{b(t)} & -\hbar\frac{\dot{\rho}_{11}(t)}{2b(t)} \\ -\hbar\frac{\dot{\rho}_{11}(t)}{2b(t)} & 0 \end{pmatrix}. \quad (2)$$

Here $b(t) = \sqrt{\rho_{11}(t) - \rho_{11}(t)^2 - a(t)^2 - C}$, where $0 \leq C \leq 1/4$ is a constant. In particular, $C = 0$ corresponds to pure states. $a(t)$ and $b(t)$ are real. Clearly the solution is not unique, as follows immediately from inspection of the homogeneous equation $[H(t), \rho(t)] = 0$, which gives $H = F(\rho)$, for arbitrary function F. We note that to completely control an isolated 2-level system two independent fields are needed.

DISSIPATIVE TWO–LEVEL SYSTEM

In this case where the situation is more complicated, we derive a non–Markovian density matrix equation. Our calculations are based on a perturbation theory in the electron–phonon–interaction up to second order [2]. We take the continuum limit for the phonon modes to obtain a realistic dissipative model. The density matrix equation has the structure

$$i\hbar\dot{\rho}(t) = [H(t), \rho(t)] + \int_0^t K(\rho(t'), U(t'), t')dt', \quad (3)$$

where the evolution operator $U(t)$ is obeys

$$i\hbar\dot{U}(t) = -H(t)U(t). \quad (4)$$

The dissipation part (the integral term in (3)) not only depends on the phonon spectral density and the temperature of the phonon bath [3], but also on the time dependent system Hamiltonian $H(t)$ containing the electric field in a dipole- interaction (DI).

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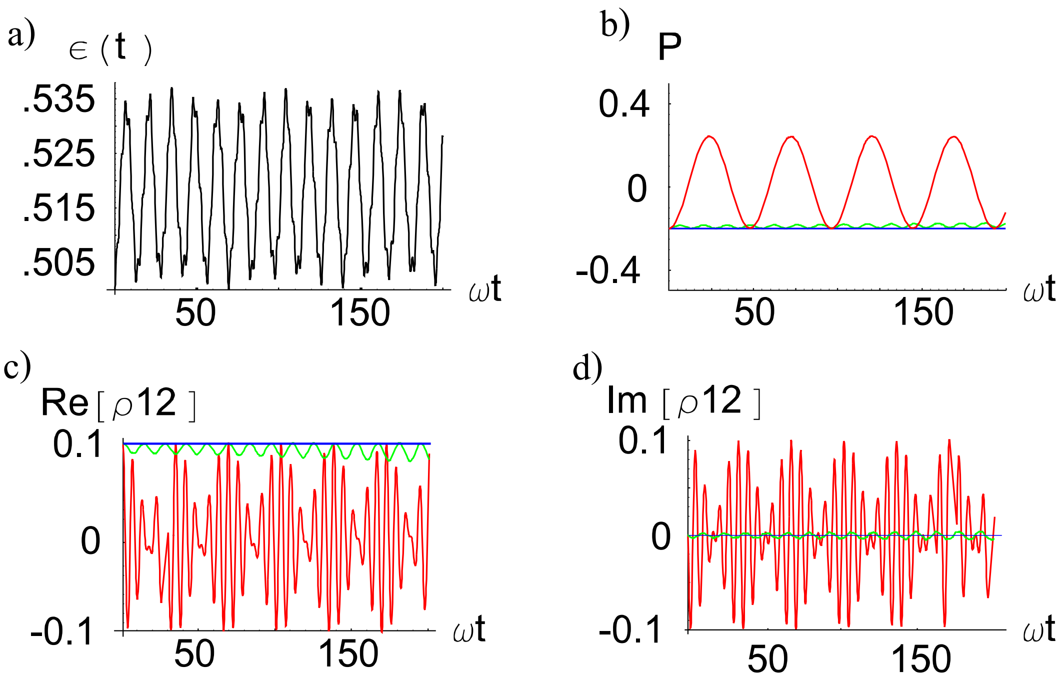

Fig. 1. Numerical results for state trapping for a simple phonon system, which contains only one resonant frequency ω. a) The optimal field founded by inversion of the density matrix equation, b) shows $P = \langle\sigma_z\rangle = \rho_{11} - \rho_{22}$ versus time, c) and d) the real- and imaginary part of the off–diagonal element of the system density operator. The red lines show the free evolution, the evolution with field is shown by the green lines, the target (trap) state is indicated by the blue lines. The occupation number of the phonons is $n = 10$, and the electron–phonon coupling strength is $M/\hbar\omega = 1/50$.

Since the inversion is performed analytically, we can identify more stable regions and less stable regions in the Hilbert space or, in this case equivalently, the Bloch sphere. For example, for an electron–phonon coupling which is proportional to σ_x, we find an optimal solution for a DI to trap the state in σ_x direction. This is done by an "inversion" of the density matrix equations. We set

$$H(t) = H_0 + \epsilon(t)\sigma_x, \quad (5)$$

where H_0 is the (diagonalised) system Hamiltonian, $\epsilon(t)$ is the control field, and "solve" (3) with

$$\epsilon(s) = -\frac{1}{4}Tr(\{C^{-1}, R\}). \quad (6)$$

Here C denotes the matrix $[\sigma_x, \rho]$, which we will assume as invertible. R contains all the remaining terms contained in (3). $\{.,.\}$ denotes the anti–commutator. This expression is put into (4), which we solve numerically.

A simple example is shown in Fig. 1 where we seek to trap the two–level system coupled resonantly to a single phonon mode in an excited state. It is seen that the control field can largely suppress the oscillation in the density matrix elements and stabilize them in the vicinity of the constant target state.

Further optimization of the control field obtained by direct inversion is performed by standard techniques, such as a conjugate gradient method (see [4]) and/or a genetic algorithm.

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