δ-Zone Triangulation: A Boundary Refinement Scheme For Quadtree Based Mesh

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Introduction

Quadtree meshing scheme has been employed to generate unstructured grid for device and process simulation. However, one of the drawbacks for such a scheme is its inability to guarantee well-shaped mesh elements along the boundary of the simulation domain. This paper presents a new boundary refinement scheme, which takes advantages of the quadtree mesh but overcomes its deficiency. Coupled with the level-control function[3], the hybrid method has shown good results in generating quality mesh for both internal and boundary regions.

Mesh Quality and δ-Zone Triangulation

Mesh quality directly affects the numerical results for device and process simulation. Much effort has been taken to produce optimal mesh[1]. One quantitative measure for triangular mesh elements is its minimum interior angle θ, as shown in Fig. 1. For most numerical methods, elements with θ less that 10° is considered ill-shaped. Ill-shaped elements are caused by two reasons: the bad distribution of the mesh points and non-optimal triangulation schemes. The latter problem is often resolved by the Delaunay triangulation scheme, which is adopted by most quadtree mesh schemes. However, quadtree based mesh schemes could produce bad mesh points along the boundary. This problem arises because a quadrant is always divided into 4 equal size sub-quadrants regardless the domain boundary. So quadrant corner points could be located very close to the boundary, which induce ill-shaped elements in the triangulation step. In the proposed δ-zone scheme, a δ-width zone along the boundary is created, as shown in Fig. 4. Inside this zone, any quadrant corner points are eliminated before the triangulation.

Fig. 3 outlines the procedure of the new mesh scheme. First the problem domain is decomposed based on a quadtree representation. It is followed by the creation of the δ-zone, which consists of all quadrants being within δ distance from the domain boundaries. Then the internal region and the δ-zone are triangulated. A detailed discussion of this new scheme can be found in [4].

Example

The new mesh scheme has been applied to a bipolar device with deep trench, shown in Fig. 2. Two results are reported in this paper. First, we compared two quadtree based mesh schemes, with and without the δ-zone boundary refinement scheme to illustrate the effectiveness of the δ-zone, The comparison data is shown in Table 1. In this comparison, we choose minimum and maximum element sizes to be 0.02 μm and 0.3 μm. The quadtree decomposition was done without considering the doping effect. In other words, the mesh density distribution is purely determined by the geometry structure. It can be seen that this new mesh refinement scheme produces not only smaller number of elements but also elements with better interior angles.

In the second case, we considered the effect of the doping profile on the mesh density distribution. The level control function is based on the gradient of the doping profile. Fig. 4 shows the quadtree decomposition and its δ-zone. The corresponding final mesh is shown in Fig. 5.

Conclusion

A new boundary refinement scheme is proposed for quadtree based mesh generation. This hybrid method takes advantages of the tree based mesh scheme while still generates quality meshes for the boundary region. We plan to extend this boundary refinement scheme to octree based mesh for 3d device simulation.

Acknowledgments

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References

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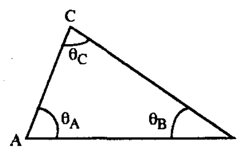

$\theta = \min(\theta_A, \theta_B, \theta_C)$

Fig. 1 Mesh quality measurement

number of elements with minimum angle θ between	0°~10°	10°~20°	20°~30°	>30°	total
Q-tree mesh without δ-zone	40	46	161	1913	2170
Q-tree mesh with δ-zone	0	9	17	1825	1851

Table 1. Comparison: with and without δ-zone.

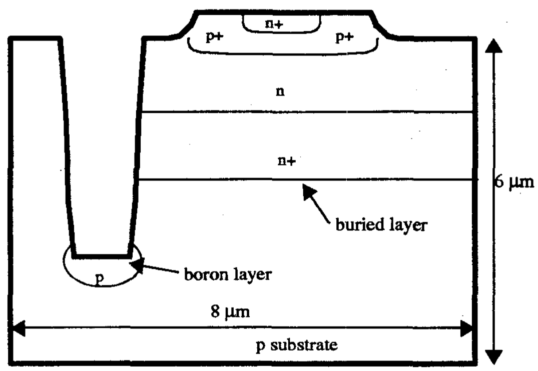

Fig. 2 Bipolar device with a deep trench

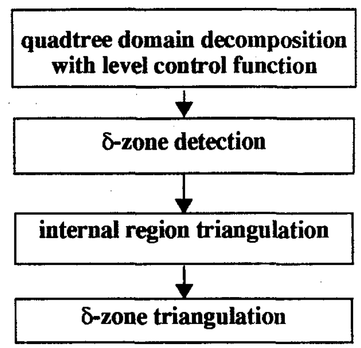

Fig. 3 Meshing procedure

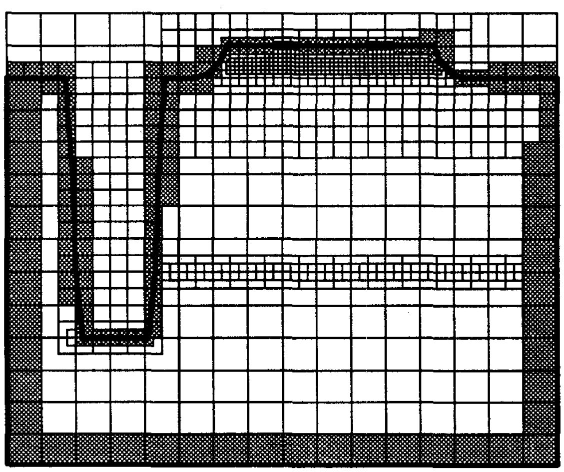

Fig. 4 Quadtree decomposition with δ-zone.

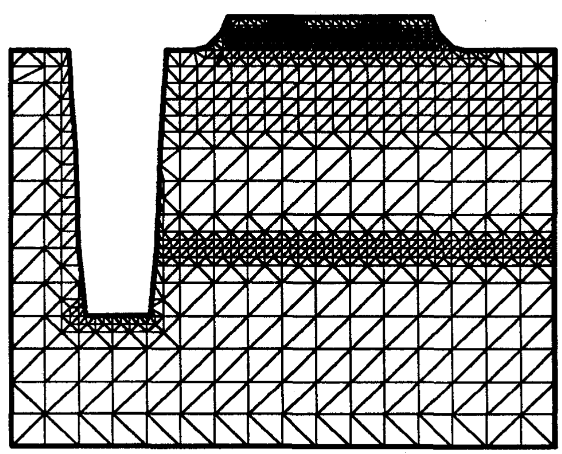

Fig. 5 Final quadtree mesh