# Space-Time Galerkin/Least-Squares Finite Element Formulation for the Hydrodynamic Device Equations

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## Introduction

Semiconductor device simulation has been an active area of research for over two decades. Continued device miniaturization has pushed geometry sizes to below 0.3 micron, leading to ever higher electrical fields. It is widely accepted that the assumption of a simple linear relationship between carrier velocity and local electric field is no longer reasonable. Instead, a model that can explicitly deal with the carrier-heating phenomenon is needed. One powerful model that treats the carrier heating phenomenon is the Hydrodynamic (HD) model [1, 2]. Simulations employing the HD model require numerical schemes that are stable, robust, and accurate. Most codes reported to date lack in at least one of these areas. In this study, a general space-time Galerkin/Least-Squares (GLS) finite element method is employed to simulate the HD model.

## Approach

A strong similarity has been observed between the HD model for a single carrier device and the Compressible Euler and Navier-Stokes equations of fluids. In its standard form, the system of HD equations is non-symmetric and nonlinear. The system can be symmetrized by employing a generalized entropy function. We developed a GLS finite element formulation based on the symmetrized HD equations, which automatically satisfies the Clausius-Duhem inequality, or the second law of thermodynamics, which is a basic stability requirement for the nonlinear system. Furthermore, the conditioning of the system can be improved by nondimensionalizing the parameters in the HD equations. Numerical simulations employing the HD model are computationally very intensive. In our work, the GLS finite element method has been implemented on a parallel Intel Hypercube computer. The results observed indicate that very good speedups can be obtained on the parallel computer.

## Example

Fig. 1 shows a 2-dimensional, single carrier MESFET example. The results shown in Figs. 2 - 8 demonstrate the effectiveness of the GLS finite element method for the HD model. In this example, we have shown one set of consistent boundary conditions. A different set of consistent boundary conditions may give rise to a different set of solutions. For instance, by specifying a homogeneous Neumann boundary condition for temperature at the drain contact, a temperature overshoot could result. We have developed a systematic approach to analyze consistency of a set of boundary conditions within our formulation.

## Summary

A general space-time Galerkin/Least-Squares finite element formulation has been developed for solving HD model of semiconductor devices. The method has been implemented on a parallel Intel Hypercube computer. Both theoretical and numerical results indicate that this method routinely leads to robust and accurate solutions with good convergence behavior. We plan to extend this work to include the simulation of 2 carrier device problems and complex 3-D device structures. Current efforts also involve the development of an adaptive finite element method for the HD model.

## Acknowledgment

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## References

[1] N. R. Aluru, A. Raefsky, P. M. Pinsky, K. H. Law, R. J. G. Goossens, R. W. Dutton, "A Finite Element Formulation for the Hydrodynamic Semiconductor Device Equations", to appear in Comp. Meth. Appl. Mech. Engg.

[2] C. L. Gardner, J. W. Jerome and D. J. Rose, "Numerical methods for the hydrodynamic device model: subsonic flow", IEEE Transactions on CAD, Vol. 8, pp. 501-507, 1989.

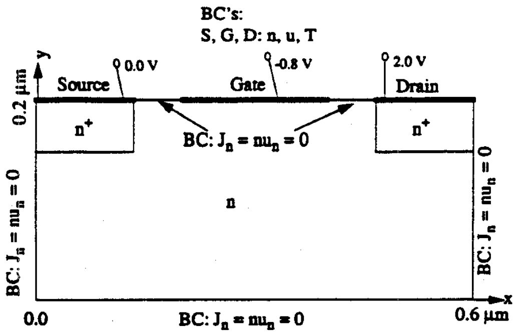


Fig 1. A two dimensional MESFET example

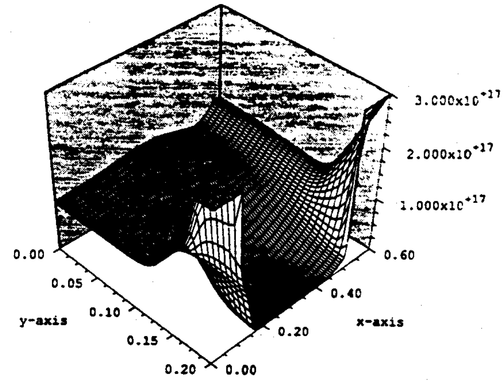


Fig 2. Electron Concentration ($cm^{-3}$)

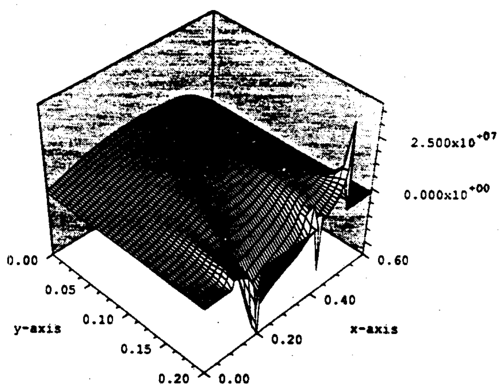


Fig 3. Horizontal Velocity of Electrons (cm/s)

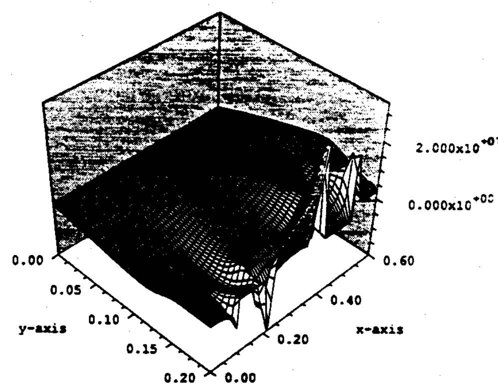


Fig 4. Vertical Velocity of Electrons (cm/s)

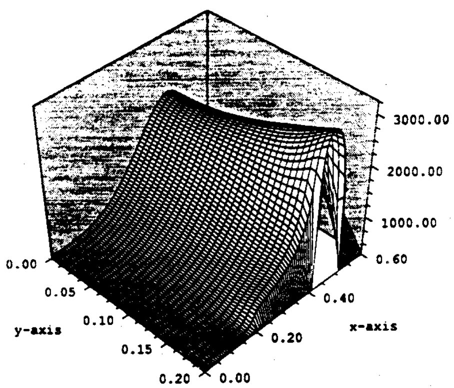


Fig 5. Electron Temperature (K)

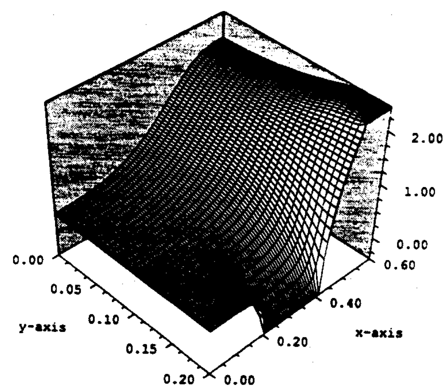


Fig 6. Potential (V)

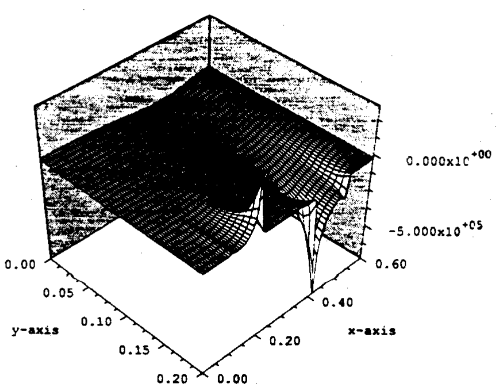


Fig 7. Horizontal component of Electric field (V/cm)

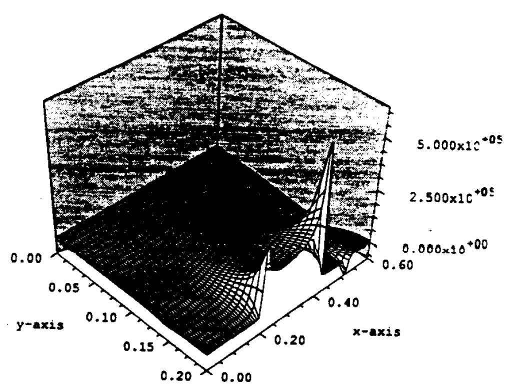


Fig 8. Vertical component of Electric Field (V/cm)