# HYDRODYNAMIC MODELING OF SILICON BJT WITH MONTE CARLO CALIBRATED TRANSPORT COEFFICIENTS

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## Introduction

High electric fields and steep field gradients are routinely present in modern submicron devices. Hence, electrons are significantly heated and their transport behavior is highly nonstationary. The conventional Drift-Diffusion Equation(DDE), based on the thermal equilibrium assumption, is no longer valid in this regime. The set of Hydrodynamic Equations(HDE) has been shown to be a better model in that it solves an energy conservation equation in addition to the DDE and phenomena such as velocity overshoot and carrier heating can be predicted. In this study, the transport coefficients in the HDE are carefully examined by a Monte Carlo method and a new "calibrated" HDE model, which quantitatively reproduces the Monte Carlo calculation results of a BJT, is proposed.

## Monte Carlo Calibration

A Monte Carlo program for silicon has been set up as a calibration reference for the HDE. The code includes acoustic phonon scattering, f, g-type equivalent intervalley scattering, ionized impurity scattering, and a nonparabolic band model with a nonparabolicity factor of 0.5. An "experiment" is performed to study the nonstationary transport characteristics of electrons in an inhomogeneous electric field as shown in Fig.1. The calculated spatial variation of the mobility  $\mu$ , and the energy relaxation time  $\tau_w$ [1] are also shown in Fig.1.

One of the major assumption in the conventional HDE formulation is that the  $\mu$  and  $\tau_w$  are functions of the average carrier energy W[2], which implies that  $\mu(x)$  and  $\tau_w(x)$  follow W(x). This is examined by plotting the  $\tau_w$  data from Fig.1 against W, as shown in Fig.2. Since all the data follow the  $\tau_w - W$  relationship calculated by the homogeneous field MC, the validity of a general  $\tau_w(W)$  expression is confirmed. At high average energy, the value of  $\tau_w$  is approximately a constant of 0.3ps.

The  $\mu$  data are plotted against the electric field E, W, and the ratio between average energy flow S and average velocity V in Fig.3. The hysteresis loops in (a), (b) indicate that  $\mu$  cannot be described by any single-valued function of E or W. A better model

for  $\mu$  in a spatially varying electric field is proposed as 1/[a + b(S/V)], where a, b are constants related to the parameters  $\mu_0, v_s$  and  $\tau_w[3]$ . The impurity doping dependence is included empirically in low field mobility  $\mu_0$ . Additionally, a high-field correction is made by replacing  $\mu_0(N_I)$  with  $\mu_0(N_I/W^{1.5})$ , where  $N_I$  is the doping concentration. This  $N_I/W^{1.5}$  dependence reflects the fact that the impurity scattering plays a diminishing role to the total scattering rate as the field increases.

For one-dimensional electric field, only the longitudinal component of the energy tensor  $U_l$  is important. This term can be approximated by two third of the average energy, 2W/3. This energy formulation is found to be more accurate than the temperature formulation.

The average energy flow S is the sum of the convective energy flow ( $\approx (5/3)VW$ ) and the heat flow Q. The heat flow Q consists of two components. The convective heat flow  $Q_{conv}$  is proportional to -VW and the proportionality constant  $\tilde{\delta}_q$  is about 0.35. This convective component of the heat flow does not vanish even under an homogeneous electric field only. The diffusive component of the heat flow  $Q_{diff}$  is approximately proportional to dW/dx and the proportionality factor is assumed to be similar to the Wiedemann-Franz law, *i.e.*,  $\tilde{\Delta}\mu W$ . The parameter  $\tilde{\Delta}$  is determined to be approximately 0.2. This corresponds to an equivalent Lorenz number of 0.45. In summary, S can be expressed as  $(5/3 - \tilde{\delta}_q)VW - \tilde{\Delta}\mu W(dW/dx)$ .

#### **Device Calculations**

The calibrated models are implemented in the HDE for device simulation and a BJT device with an impurity doping profile shown in Fig.4 is calculated for two different bias conditions. The calculated electric field profiles are also shown in Fig.4. The numerical solutions from the HDE[4] are compared with the Monte Carlo calculation results. The average velocity and the average energy profiles shown in Figs.5 and 6 clearly demonstrate that this HDE model can quantitatively predict the Monte Carlo data over a wide range of conditions. It is worth mentioning that there is no second velocity peak at the subcollector junction for both calculations, in contrast to[5].

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#### Conclusion

An improved HDE model is constructed from the Monte Carlo calculation of a spatially varying electric field. The mobility and the energy relaxation time are empirically expressed as functions of S/V and W, respectively. The longitudinal component of the average energy tensor as well as the energy flow are also carefully modeled. This HDE model is then applied to an npn BJT. The results show excellent agreement with the Monte Carlo data.

# References

- [1] S.C. Lee, Ph.D. Dissertation, University of Massachusetts, Amherst, 1990.
- [2] M.S. Shur, Electron Lett., vol.12, pp.615-616, 1976.
- [3] W. Hänsch and M. Miura-Mattausch, J. Appl. Phys., vol.60, pp.650-656, 1986.
- [4] S.C. Lee, T.W. Tang and D.H. Navon, Proc. NASECODE VI Conf., Dublin, Ireland, pp.478-483, 1989.
- [5] R.K. Cook, IEEE Trans. Electron Devices, vol.ED-30, pp.1103-1110, 1983.



Fig.1 Electric field, mobility, and energy relaxation time profiles.



Fig.2 Energy relaxation time versus average energy.





1.5

10<sup>21</sup> =0.7 10 10<sup>20</sup> 8 N<sub>0</sub>+N<sub>A</sub> (cm<sup>-3</sup>) 10<sup>19</sup> V (10<sup>′</sup>cm/s) 1.0 E 10<sup>18</sup> 0 10<sup>17</sup> 0 10<sup>18</sup> -2 V<sub>CB</sub>=0.5∨ 10<sup>15</sup> 0.1 0.2 0.3 D.4 0.5 0.6 0.0 X (μm)





0 M.C.

HDE

