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# Extension of the Scharfetter-Gummel scheme for the case of avalanche generation


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*In this paper we present an extension of the wellknown Scharfetter- Gummel scheme to the case where avalanche generation is taken into account. The scheme reduces to the normal Scharfetter-Gummel scheme in case of no avalanche generation.*


## Introduction

The classical way of treating avalanche generation in the drift-diffusion equations consists of considering the generation terms similar to the recombination terms, i.e. using a one point quadrature rule and an evaluation of these terms only in the centre of the box (when using the box method). Mathematically, however, the character of the equations is quite different when taking avalanche generation into account, since the first order differential equations for the current densities then also contain zero-order terms. This means that the current densities may behave in an exponential manner, as opposed to the polynomial behaviour when no generation terms are present in the equations. The classical discretisation methods do not take this into account, and therefore the solutions of the resulting discrete systems of equations sometimes exhibit non-physical behaviour whenever the avalanche effects are dominant. Especially for coarse grids, problems can be expected when using the classical discretisation method.

In view of the above, a new discretisation method for the semiconductor equations has been developed, which takes the exponential behaviour of the current densities into account. The scheme is an extension of the wellknown Scharfetter-Gummel scheme, the assumption of piecewise constant current densities being replaced by the assumption of piecewise exponential. The scheme has attractive properties both from the mathematical and practical point of view: it is stable (in the sense that it allows a maximum principle) even for coarse grids, and it is easily implemented into existing codes.

In the following, we will describe the scheme in 1-d. Its extensions to 2-d and 3-d are straightforward, and will be discussed in the talk. Furthermore, we show that the scheme is easily implemented into existing codes. Finally, we present some examples.

Derivation of the scheme in 1-d

We start from the equations:

$$J_p' = -qR + \alpha_p|J_p| + \alpha_n|J_n| \tag{1}$$

$$J_n' = +qR - \alpha_p|J_p| - \alpha_n|J_n| \tag{2}$$

which we can write as:

$$\mathbf{J}' = q\mathbf{R} + A\mathbf{J} \tag{3}$$

where $\mathbf{J} = (J_p, J_n)^T$, $\mathbf{R} = (-R, R)^T$ and

$$A = \begin{pmatrix} +v_p\alpha_p & +v_n\alpha_n \\ -v_p\alpha_n & -v_n\alpha_n \end{pmatrix}$$

with $v_p = sign(J_p)$, $v_n = sign(J_n)$. We can consider $v_p$ and $v_n$ as the directions of $J_p$ and $J_n$. Remark that, because of the definition of the current densities, $v_p$ is the direction of $-\phi_p'$ and $v_n$ the direction of $-\phi_n'$. Therefore, equation (3) may be considered as a linear equation in $\mathbf{J}$. This is an important observation, especially for 2-d and 3-d problems.
We now discretise the system above, and assume that the device considered is divided into subintervals $[x_i, x_{i+1}]$, and that A is constant on each of these intervals (since $\alpha_p$ and $\alpha_n$ depend on gradients of $\psi$, $\phi_p$ and $\phi_n$, this is a plausible assumption). Multiplying (3) by $exp{-}(x - x_{i+1/2})A$ we obtain:

$$exp\{(x - x_{i+1/2})A\} \;\; exp\{-(x - x_{i+1/2})A\}\mathbf{J}' = q\mathbf{R} \tag{4}$$

Thus, the right assumption in this case is that

$$exp\{-(x - x_{i+1/2})A\}\mathbf{J} = \mathbf{C}$$

where $\mathbf{C}$ is a constant vector, as opposed to the assumption that $\mathbf{J}$ is constant for the normal Scharfetter-Gummel scheme.
Using this, an extension of the Scharfetter-Gummel scheme can be developed:

$$exp\{(x_i - x_{i+1/2})A_r\}\mathbf{J}_{i+1/2} - exp\{(x_i - x_{i-1/2})A_l\}\mathbf{J}_{i-1/2} = q(x_{i+1/2} - x_{i-1/2})\mathbf{R}(x_i) \tag{5}$$

where $A_r$ and $A_l$ are the matrices A in the intervals to the right and the left of $x_i$, respectively. Remark that if $A_r = A_l = 0$, then the scheme reduces to the ordinary Scharfetter-Gummel scheme.
An implementation into existing codes is easy, since the scheme can be written in the form:

$$\{I + \gamma_r(-h_r/2)A_r\}[I + E_r^{-1}F_rA_r]^{-1}\mathbf{J}_{ordSG,i+1/2}$$
$$- \{I + \gamma_l(h_l/2)A_l\}[I + E_l^{-1}F_lA_l]^{-1}\mathbf{J}_{ordSG,i-1/2} = q\frac{h_l + h_r}{2}\mathbf{R}(x_i) \tag{6}$$

i.e.: the ordinary Scharfetter-Gummel expressions are pre-multiplied by 2 x 2 matrices. The accurate calculation of the matrices $E^{-1}F$ is of vital importance (they are an extension of the wellknown Bernouilli function), and has to be performed with the utmost care. The $\gamma_r$ and $\gamma_l$ occurring in (6) stem from the calculation of the matrix-exponentials, which is straightforward for 2 x 2 matrices.