Discretization Scheme for Multi-Dimensional Current Continuity Equations

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1. Introduction

The two-dimensional extension of the Scharfetter-Gummel scheme [1] uses the control volume defined by perpendicular bisectors, as shown in Fig. 1. The *scalar* current density is one-dimensionally defined along each mesh line using the Scharfetter-Gummel scheme. Recently, other approaches for the discretization of the multi-dimensional current continuity equation have been proposed, e.g., [2], [3].

We have introduced the Baliga-Patankar discretization scheme [3] to a triangular mesh device simulator TRIMEDES [4], and compared this scheme with the popular Scharfetter-Gummel scheme by using silicon n^+-p diode current-voltage characteristics.

2. Baliga-Patankar scheme

The Baliga-Patankar scheme uses the control volume defined by the line segments joining the centroid of the elements to the adjacent side midpoints, as shown in Fig. 2. The essential feature of this scheme is that the current density *vector* is defined within the element. A new set of coordinates X and Y is defined, as shown in Fig. 3. Its origin is located at the centroid o, and the X axis is aligned with the velocity vector $\mathbf{v}$ ($= -\mu E$).

In this coordinate system, in the absence of generation-recombination terms, the steady-state drift-diffusion transport of electron in the element of interest is governed by the following equation.

$$|v|\frac{\partial n}{\partial X} - D\left(\frac{\partial^2 n}{\partial X^2} + \frac{\partial^2 n}{\partial Y^2}\right) = 0 \ . \qquad (1)$$

In this equation, Y does not appear in the drift term so that only diffusion contributes to flow in the Y direction. Hence, the following carrier distribution within the element satisfies (1) [3].

$$n(X,Y) = Af(x) + BY + C \qquad (2)$$

where

$$f(x) = \frac{D}{|v|}\left\{\exp\left[\frac{Pe_\Delta(X - X_{min})}{(X_{max} - X_{min})}\right] - 1\right\} . \qquad (3)$$

Here, Pe_Δ is the element Peclet number, $Pe_\Delta = \frac{|v|}{D}(X_{max} - X_{min})$ and $X_{max} = \max(X_i, X_j, X_k)$, $X_{min} = \min(X_i, X_j, X_k)$. The Baliga-Patankar scheme can be interpreted as an application of the one-dimensional Scharfetter-Gummel scheme in the drift velocity direction.

3. Test Computations

For the n^+-p diode structure shown in Fig. 4 (a), in which the two electrodes are located on opposite sides, the difference between the current calculated by the two schemes is less than 0.1 percent for less than 1.5 volts of forward bias. For the n^+-p diode structure shown in Fig. 4 (b), in which the p-silicon electrode is located on the upper part, the difference in the calculated currents between the two schemes is also less than 0.1 percent for up to 1.0 volt of forward bias (diffusion dominant flow). As seen in Fig. 5, however, the current values obtained with the two schemes deviate for a high forward bias condition (drift dominant flow). With 1.5 volts of forward bias, the difference in current reaches 16 percent for the 30 x 5 mesh case. The difference is decreased by an increase in mesh points, i.e., by a decrease in mesh interval. It should be noted that the Baliga-Patankar scheme is rather insensitive to mesh. These results are due to so-called *false* or crosswind diffusion [2], [3].

False diffusion originates from the multi-dimensional nature of the problem under consideration. False diffusion increases with a high drift velocity, large mesh intervals and when the mesh line is not along the flow direction. The Baliga-Patankar scheme is robust for a two-dimensional current flow, since this scheme defines the current density vector within each element.

References

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[3] B. R. Baliga and S. V. Patankar, Numer. Heat Trans., vol. 3, pp.393-409, 1980.

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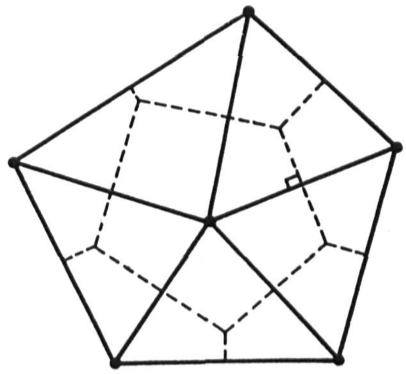

Fig. 1 Control volume defined by perpendicular bisectors, used for conventional Scharfetter-Gummel scheme.

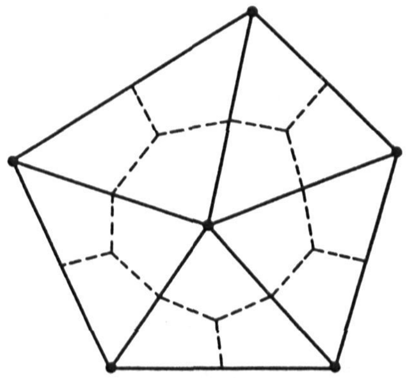

Fig. 2 Control volume defined by using centroid of element and midpoint of element edge, used for Baliga-Patankar scheme.

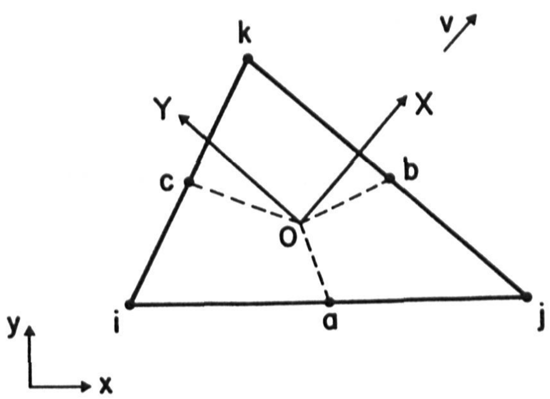

Fig. 3 Global (x, y) and local (X, Y) coordinate systems.

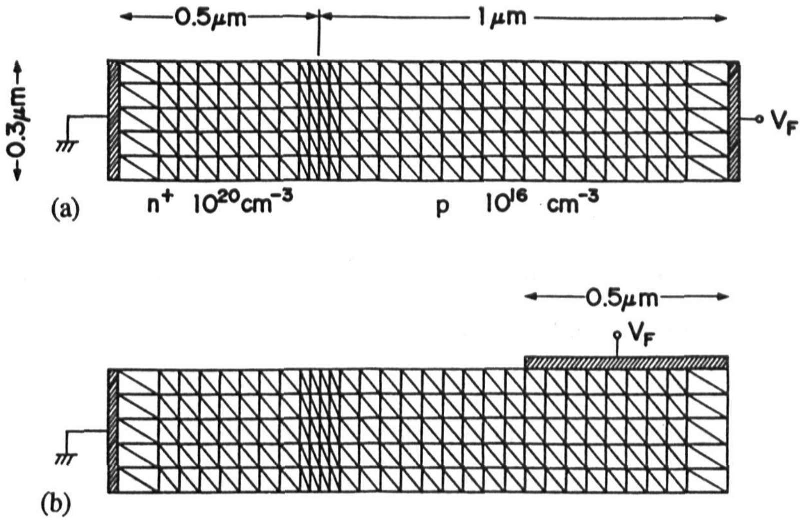

Fig. 4 Two n$^+$-p diode structures and its meshes. (a) one- and (b) two-dimensional diodes. Right-angled triangular mesh are used.

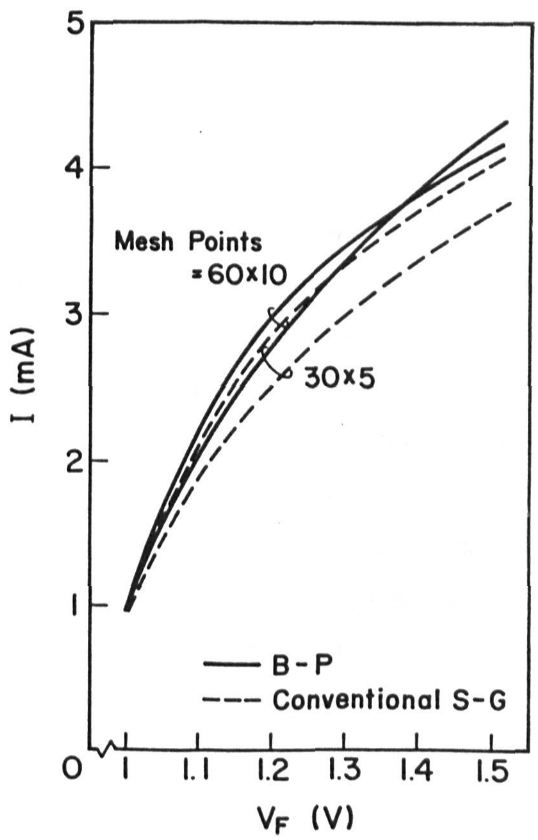

Fig. 5 Current-voltage characteristics for n$^+$-p diode shown in Fig. 4 (b).