

Hybrid method for electromagnetic modelling of coherent radiation in semiconductor lasers.

Mateusz Marek Kryszicki*, Bartłomiej Salski†, Paweł Kopyt‡

Warsaw University of Technology
Institute of Radioelectronics and Multimedia Technology

Nowowiejska 15/19, 00-665 Warsaw, Poland

*m.kryszicki@ire.pw.edu.pl, † b.salski@ire.pw.edu.pl, ‡ p.kopyt@ire.pw.edu.pl

Abstract—In this paper hybrid method for electromagnetic (EM) modelling of coherent radiation in semiconductor lasers is presented. Described approach consist of drift diffusion (DD) model and electromagnetic simulation. Four-level two-electron atomic system with Pauli Exclusion Principle (PEP) extended by electric pumping ratio has been used as lasing model.

Index Terms—simulation, device, FDTD, ADE, PEP, drift diffusion, nonlinear optics

I. INTRODUCTION

Nowadays laser action can be simulated using various methods, as Finite Difference Time Domain (FDTD) method with auxiliary differential equation (ADE) which represents classical dispersive Lorentzian gain [1] or with FDTD with rate equations ADE [2]. Another ADE-FDTD technique presented in [3] and extended in [4], uses the density-matrix method for solving Maxwell-Bloch equation.

In this paper ADE-FDTD method for electrically-pumped lasers is reported. Flowchart of algorithm is presented on figure 1. Initial point base on device simulation, computed using drift diffusion method [5] and recalculated in co-processing. Results are computed using electromagnetic simulation.

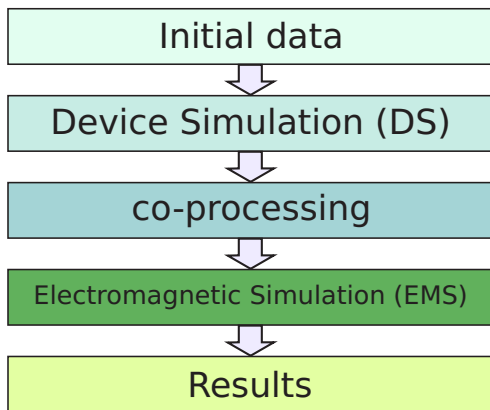


Fig. 1. Flowchart of hybrid method for EM modelling of coherent radiation in semiconductor lasers.

This work was partially supported by the Polish National Science Center within the SONATA project titled “Full-wave electromagnetic modelling of coherent radiation in electrically-pumped metal-clad semiconductor lasers with a folded cavity”.

II. DEVICE SIMULATION

First step in the proposed method is the computation of a drift-diffusion model of a device [5], which consists of the Poisson’s equation: (3):

$$\nabla \cdot \epsilon \nabla \Psi = -q(p - n + N_D^+ - N_A^-) \quad (1)$$

and continuity equations:

$$\nabla J_n - q \frac{\partial n}{\partial t} = +qR \quad (2)$$

$$\nabla J_p + q \frac{\partial p}{\partial t} = -qR \quad (3)$$

where ϵ is permittivity, Ψ is electrostatic potential, q is elementary charge, p, n, N_D^+, N_A^- are densities of electron, hole singly ionized donors and singly ionized acceptors, R is generation/recombination rate, and $J_n(p)$ is electron (hole) current density.

As a result of this calculation spatial distribution of electron and hole density (or Fermi-Dirac distributions for electron f_e and holes f_h) is obtained, which will be recalculated to quasi-chemical potentials (4) (5):

$$\mu_e = f_e - E_c \quad (4)$$

$$\mu_h = -f_h + E_v \quad (5)$$

where $E_{c(v)}$ is energy level of conduction (valence) band, and, subsequently, used in co-processing part of algorithm presented in the next Section.

III. LINKING ELECTROMAGNETIC AND DRIFT-DIFFUSION SIMULATIONS (CO-PROCESSING)

Electromagnetic simulation of a semiconductor laser with eq. (12) - (16) requires knowledge on material parameters, which can be determined with the aid of DD. All calculations presented in this section are performed for one selected voltage. For both type of computing processes the same mesh grid have to be used. In this paper, the study is limited to a 1D structure.

Once DD is computed, gain characteristic is calculated in each cell [6]:

$$g(z) = \frac{\nu}{4\pi^2 \hbar \gamma \epsilon_0 n c} \left(\frac{2m_r}{\hbar^2} \right)^{\frac{3}{2}} \dots \int_0^\infty d\epsilon |\mu_k|^2 [f_e(\epsilon) + f_h(\epsilon) - 1] \dots L(\omega_k - \nu) \left(1 - j \frac{\omega_k - \nu}{\gamma} \right) \quad (6)$$

where ν is angular frequency, \hbar is reduced Planck constant, γ is homogeneous linewidth factor, ϵ_w is vacuum permittivity, m_r is reduced mass given by:

$$m_r^{-1} = m_e^{-1} + m_h^{-1} \quad (7)$$

where $m_{e(h)}$ is electron (hole) mass, $d\epsilon$ is part of energy used for integral calculations, $|\mu_k|$ is dipole moment, $L(\omega_k - \nu)$ is Lorentzian lineshape function, ω_k is angular plasma frequency of Lorentz model, and f_e, f_h are Fermi-Dirac distributions for electron and holes respectively given by equation:

$$f_\alpha(\epsilon) = \frac{1}{\exp \left[\beta \left(\epsilon \frac{m_r}{m_\alpha} - \mu_\alpha \right) \right] + 1} \quad (8)$$

where α stands as $e(h)$ for electron (hole) distribution, β is coefficient equal to:

$$\beta = \frac{1}{k_B T} \quad (9)$$

where T is temperature and k_B is Boltzmann coefficient, and μ is quasi-chemical potential.

As an example of those calculation gain curve and imaginary part of electrical susceptibility, which be easy obtained using equation:

$$\chi'' = -2 \frac{g}{n K_0} \quad (10)$$

where K_0 is the wavenumber in vacuum and n is refractive index, is presented on figure 2.

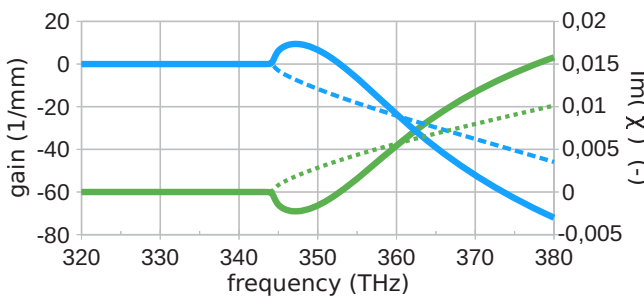


Fig. 2. An example of gain curve (blue lines, left scale) computed using equation (6) and corresponding to it imaginary part of electrical susceptibility (green lines, right scale) computed using equation (10). Solid (dashed) lines are obtained for active (lossy) material

If gain is negative in a given cell, it is assumed to be lossy, which is represented with electric conductivity (11):

$$\sigma = \omega \epsilon_0 \chi'' \quad (11)$$

where χ'' is imaginary part of electrical susceptibility.

On the contrary, when gain characteristic is positive at the frequency of interest, pumping rate P_R , is calculated using spatial distribution of current density, by integrating density of states between 0-3 energy levels.

IV. ELECTROMAGNETIC SIMULATION

In the next step, electrodynamic simulation will be undertaken with the aid of a finite-difference time-domain (FDTD) method with auxiliary differential equation (ADE). Rate equations are coupled with Ampère's circuital law (with Maxwell's addition) as density of electric polarization:

$$\nabla \times H = \frac{\partial D}{\partial t} + \frac{\partial P_a(t)}{\partial t} + \frac{\partial P_b(t)}{\partial t} \quad (12)$$

where i indicates a or b , $P_a(P_b)$ stands for the density of electric polarization between 0-3 (1-2) levels of a four-level atomic model, γ are linewidths, and ω are resonant frequencies, $\Delta N_{a(b)}$ is difference between population densities at 0-3 (1-2 levels) and κ is model parameter.

In addition to a typical form of rate equations [4] [2], electrical pumping rate, P_R , has been added:

$$\frac{\partial N_3}{\partial t} = -\frac{N_3(1-N_2)}{\tau_{32}} - \frac{N_3(1-N_0)}{\tau_{30}} + \frac{1}{\hbar \omega_a} E \frac{\partial P_a}{\partial t} + P_R \quad (13)$$

$$\frac{\partial N_2}{\partial t} = \frac{N_3(1-N_2)}{\tau_{32}} - \frac{N_2(1-N_1)}{\tau_{21}} + \frac{1}{\hbar \omega_b} E \frac{\partial P_b}{\partial t} \quad (14)$$

$$\frac{\partial N_1}{\partial t} = \frac{N_2(1-N_1)}{\tau_{21}} - \frac{N_1(1-N_0)}{\tau_{10}} - \frac{1}{\hbar \omega_b} E \frac{\partial P_b}{\partial t} \quad (15)$$

$$\frac{\partial N_0}{\partial t} = \frac{N_3(1-N_0)}{\tau_{30}} + \frac{N_1(1-N_0)}{\tau_{10}} - \frac{1}{\hbar \omega_a} E \frac{\partial P_a}{\partial t} - P_R \quad (16)$$

where N_i is population density at level $i \in \{0, 1, 2, 3\}$, τ_{ij} is the decay time constant between i and j levels and \hbar is reduced Planck constant

Due to chosen model, wide spectrum of results can be obtain, for example spectral characteristic of generated radiation.

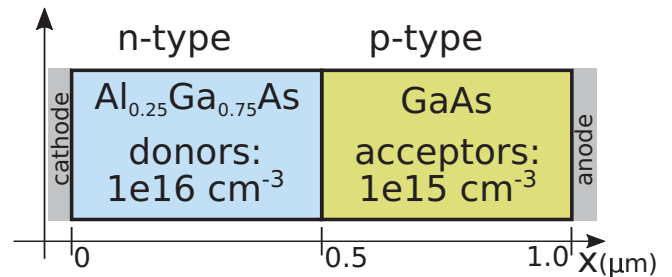


Fig. 3. Geometry of PN junction [7] used for calculations.

V. NUMERICAL RESULTS

As an example PN junction [7] (see fig. 3) has been used. It consist of $0.5\mu\text{m}$ n-type $\text{Al}_{0.25}\text{Ga}_{0.75}\text{As}$ and $0.5\mu\text{m}$ p-type GaAs blocks with refractive index $n = 3.6$. Firstly, DD simulation is used for computing IV-characteristic (see fig. 4). After that, for chosen voltage (1.4 V) parameters for novel model have been computed using quasi-chemical potentials (see fig. 5).

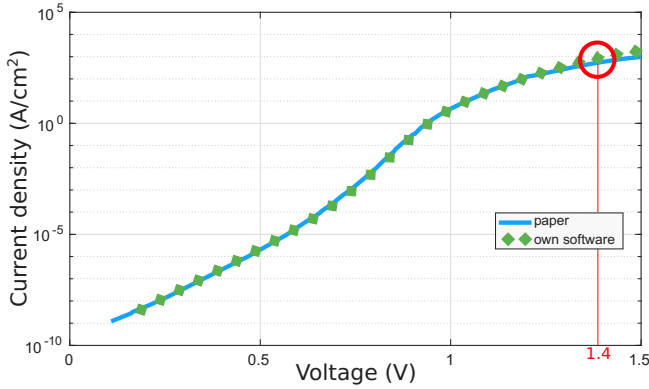


Fig. 4. IV characteristic of PN junction presented on fig. 3).

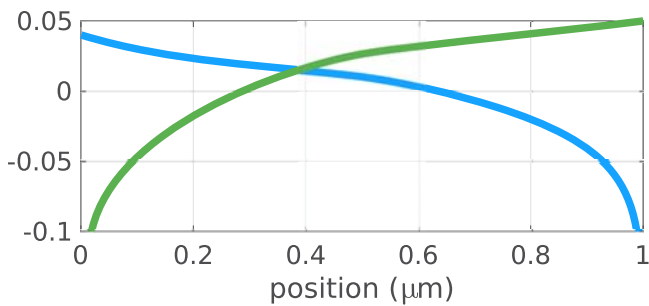


Fig. 5. Spatial distribution of electron (hole) quasi – chemical potential $\mu_n(\mu_p)$ marked by green (blue) line.

As an example of results evolution of population densities between populations densities at levels 2 and 1 (see fig. 6) and value of E-field (see fig. 7) have been presented.

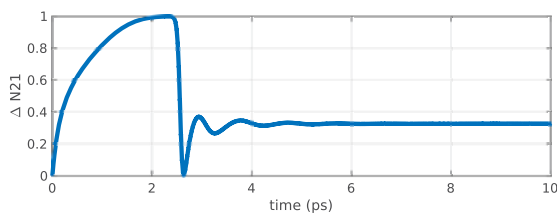


Fig. 6. Time evolution of difference between population densities at levels N_2 and N_1 .

For next example of results the simplest optical cavity (Fabry-Perot) of total length $20\mu\text{m}$ with similar PN junction as presented on fig. 3 has been used. Two parallel ends are cleaved, and ended with low- and high- reflectivity (LR/HR)

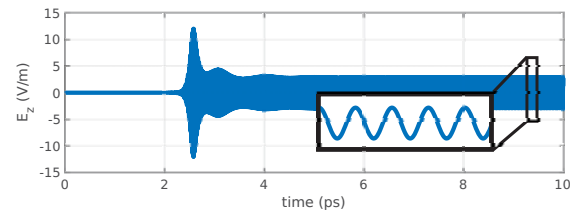


Fig. 7. An example of gain curve computed using equation (6).

facets. Spectral characteristic of radiation from LR facet has been calculated and presented on fig. 8.

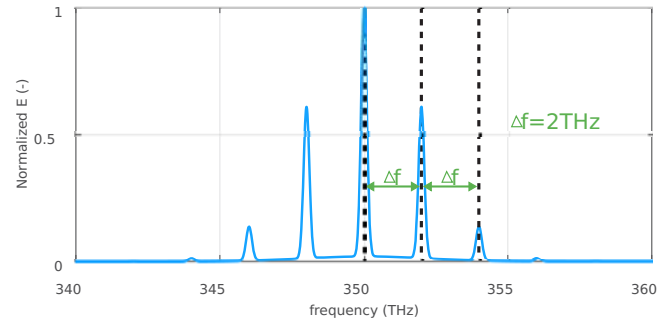


Fig. 8. Normalized E - field spectrum radiated from low-reflectivity facet of FP cavity.

In presented case it is possible to detect many longitudinal modes spaced by $\Delta f = 2THz$ which clearly corresponds to cavity length.

VI. CONCLUSIONS

Using novel method presented in this paper it is possible to calculate insensitivity of generated electromagnetic field and observe dynamics of difference between population densities. Presented methodology will be extended for two dimensional simulations.

REFERENCES

- [1] A. Taflove, S. C. Hagness., *Computational electrodynamics: the finite-difference time-domain method.*, Artech house, 2005.
- [2] S. H. Chang, A. Taflove, "Finite-difference time-domain model of lasing action in a four-level two-electron atomic system", *Optical and Quantum Electronics*, Optics express, vol. 12, no. 16, pp. 3827–3833, 2004.
- [3] A. S. Nagra and R. A. York, "FDTD analysis of wave propagation in nonlinear absorbing and gain media", *IEEE Trans. Antennas Propagat.*, 46, 334, (1998).
- [4] B. Salski, "Hybrid FDTD Analysis of Two- and Four-Level Atomic Systems", *Optical and Quantum Electronics*, vol. 47, pp. 1703–1712, 2014.
- [5] D. L. Scharfetter, H. K. Gummel, "Large-signal analysis of a silicon read diode oscillator", *IEEE Transactions on electron devices*, vol. 16, no. 1, pp. 64–77, 1969.
- [6] W. W. Chow, S. W. Koch, *Semiconductor-laser fundamentals: physics of the gain materials.*, Springer Science & Business Media, 2013.
- [7] K. Horio, H. Yanai, "Numerical modeling of heterojunctions including the thermionic emission mechanism at the heterojunction interface", *IEEE Transactions on electron devices*, vol. 37, no. 4, pp. 1093–1098, 1990.