An Accurate Metric to Control Time Step of Transient Device Simulation by Matrix Exponential Method

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***Abstract*—An accurate metric for the time step control in the power device transient simulation is proposed. This metric contains an exponential term of the dominant time constant of the whole device structure obtained by matrix exponential method. The proposed metric allows larger time step widths than the conventional metric of the 2nd-order approximation of the local truncation error because it focuses on the dominant part of the transient response and its truncation error approximation is more accurate. Total CPU-time of the transient simulation of a silicon power DMOSFET by using the proposed method decreases down to 30% of that by the conventional metric with assuring the current accuracy of the dominant transient response.**

Keywords—power device; transient device simulation; time step control; local truncation error; matrix exponential method; Arnoldi method

I. INTRODUCTION

Device simulation is intensively used for power MOSFET design today because the uncertainties in impurity profile, geometry size and carrier transport physics of power MOSFET are all in acceptable level and therefore it can clearly elucidate the mechanism of the phenomenon resulting from the complex coupling of plural physical effects. On the other hand, its calculation time is problematic especially in transient analysis because a large number of meshes must be used to express its large device structure and a large number of time steps must be consumed to trace its relatively slow response. Therefore, optimum time step control is important for the calculation time reduction with assuring the accuracy of transient device simulation. In this paper, we propose an accurate metric for the time step control by using the dominant time constant information of the whole device structure obtained by matrix exponential method [1].

II. AN EXPONENTIAL BASED LTE METRIC

In the conventional device simulation, product of second time derivative of carrier density and squared time step width are usually used for the time step width control as a metric of the LTE (Local Truncation Error) as shown in Fig. 1. However, the accuracy of this metric is dubious because it only adopts the most dominant term in the Taylor expansion of the LTE and the contributions from the other higher order terms are ignored. On the other hand, if a dominant time constant (τc) which can well approximate the response of the whole device structure is known a priori, the LTE can be estimated as Exp-LTE-metric of (7) in Fig. 2 by putting the Taylor series into an exponential function. The time step width (Δt) dependence of the main Exp-LTE-metric term other than the first time derivative of the variable with respect to τc is shown in Fig. 3. This figure tells us two important features. The first one is that the main Exp-LTE-metric term exponentially decreases as Δt becomes smaller than τc. This feature is advantageous for adopting large time step width by filtering out the unimportant responses whose time constants are much smaller than the dominant one. On the contrary, as the conventional 2nd-order-LTE-metric watches local variations only, it cannot overlook any quick response even if it is unimportant. The second feature is that although the τc of the active device can be either positive or negative, the main Exp-LTE-metric term never exceeds 1 as long as the τc is positive. This means that if the first time derivative of the variable is smaller than the required LTE criterion, it is possible to take Δt as infinite. On the other hand, Δt in the 2nd-order-LTE-metric of (5) in Fig. 1 never fails to be restricted to some limited value unless the second time derivative of the variable is 0.

Differential equation: $\frac{\partial x(t)}{\partial t} = f(x(t))$ (1)

⇓ Discretization by Backward Euler method:

$\frac{x_{BE}(t_0+\Delta t)-x_{BE}(t_0)}{\Delta t} = f(x_{BE}(t_0+\Delta t))$ (2)

Taylor expansion of the exact solution $x(t)$ at $t = t_0 + \Delta t$:

$x(t_0) = x(t_0+\Delta t) - \dot{x}(t_0+\Delta t)\cdot\Delta t + \frac{1}{2!}\ddot{x}(t_0+\Delta t)\cdot(\Delta t)^2 + O((\Delta t)^3)$ (3)

LTE evaluation at $t = t_0$ by (2) - (3) assuming $x_{BE}(t_0+\Delta t) = x(t_0+\Delta t)$:

$LTE \equiv |x_{BE}(t_0) - x(t_0)| = \left|\frac{1}{2}\ddot{x}(t_0)\cdot(\Delta t)^2 + O((\Delta t)^3)\right|$ (4)

By omitting unknown higher order terms:

$2^{nd}-order-LTE-metric \equiv \left|\frac{1}{2}\ddot{x}(t_0)\cdot(\Delta t)^2\right|$ (5)

Fig. 1. Derivation of the conventional LTE metric.

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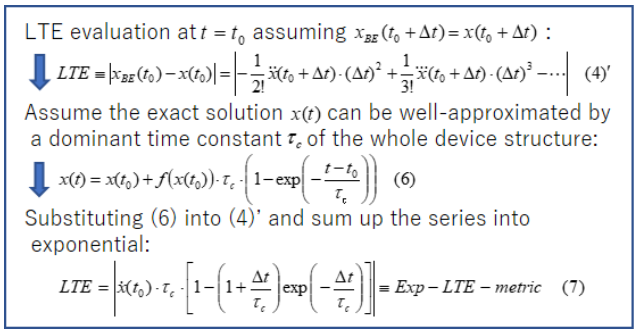

Fig. 2. A proposed exponential based LTE metric.

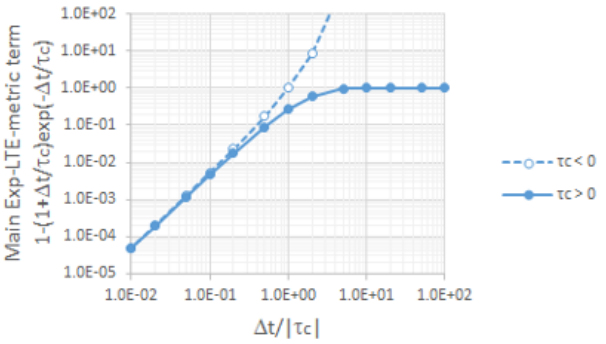

Fig. 3. Time step width dependence of the Exp-LTE-metric with respect to the dominant time constant.

III. Dominant Time Constant Extraction

Poisson and continuity equations are linearized and its formal solution [2] is derived as shown in Fig. 4. Since the device equations are nonlinear system, the formal solution is valid only within limited time duration and transient analysis with time discretization such as Backward Euler method is necessary to get long range solution [2]. The largest eigenvalue of $G^{-1}C$ corresponds to the smallest eigenvalue of $C^{-1}G$ in Fig. 4 which is also the inverse of the dominant time constant of the whole device structure that should be extracted. Here, Arnoldi algorithm [1] shown in Fig. 5 is used to embed the eigenvalues of $G^{-1}C$ into Hessenberg matrix from larger side. Then, the largest eigen value of the Hessenberg matrix is calculated. Since the device equations may result in complex conjugate eigen value pairs, the Hessenberg matrix size m in Fig. 5 should be at least 2.

Fig. 6 (a) shows a 1-D N+P- diode structure and Fig. 6 (b) shows an example of the variation of the dominant time constant with the progress of the transient analysis after forward step bias is applied to the 1-D diode. The blue and orange dots are in the case of moderate (0.8V) and strong (1.0V) forward biases, respectively. Open orange circles in Fig. 6 (b) are negative time constants which mean the device is in positive feedback state. This is due to conductivity modulation which continues to increase carrier density until SRH recombination becomes significant. In the case of negative time constant, the time step width is usually restricted by the convergence of Newton iteration rather than LTE.

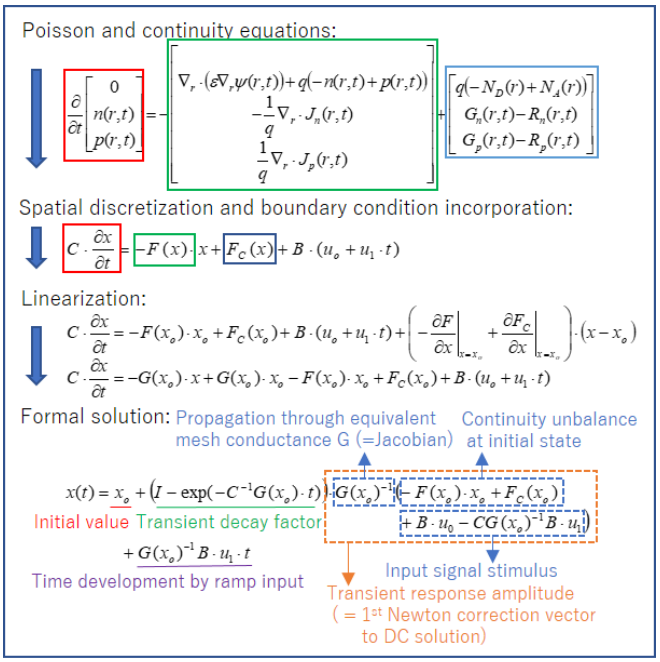

Fig. 4. Derivation of the formal solution of linearized Poisson and continuity equations.

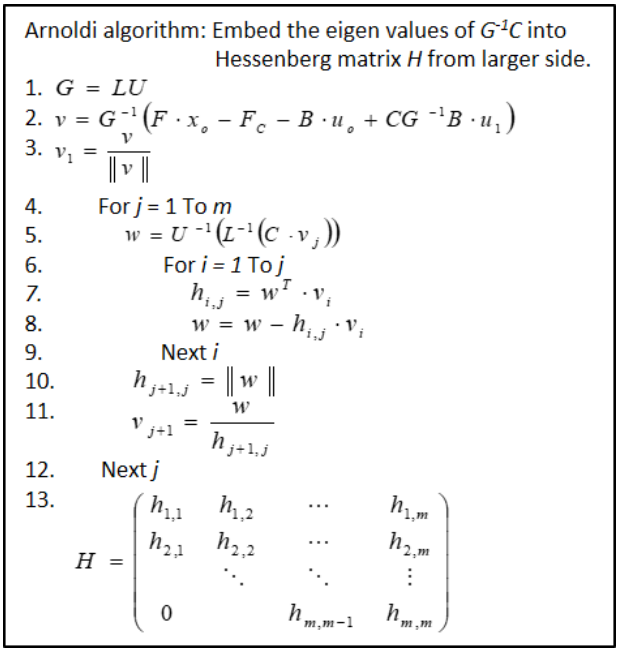

Fig. 5. Flow chart of Arnoldi algorithm.

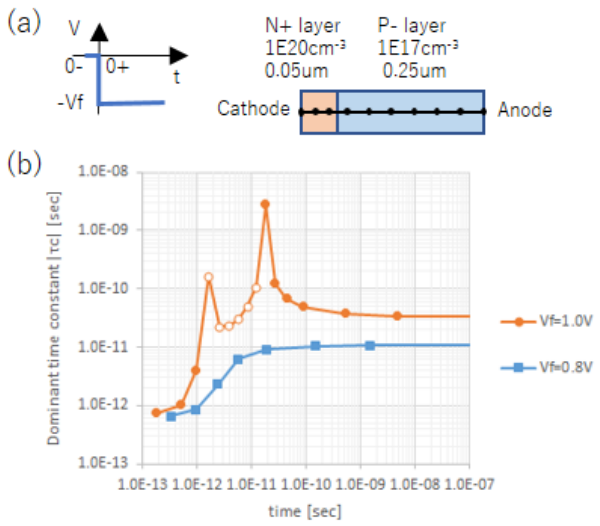

Fig. 6. (a) 1-D diode structure with forward step bias. (b) Time variation of dominant time constant.

IV. Performance Verification by Transient Simulation of 2-D Power DMOSFET

A 2-D power DMOSFET [3] with step-Vg application shown in Fig. 7 (a) is used for the performance verification of the proposed method. As shown in Fig. 7 (b), the dominant time constant (blue dots) stays around 1E-10 sec which is almost equal to the electron traveling time from source to drain after the channel is formed (1E-11 sec). The open blue circles in Fig. 7 (b) are negative time constants which appear at the onset of electron diffusion from the source while the current limiting feedback from the channel has not been established yet. Time step width distribution for Exp-LTE=1% (orange triangles) is also shown in the same figure. There are dense time steps between the channel formation and transient decay. Analysis time step progress with respect to the number of time steps is shown in Fig. 8 for LTE criteria of 1–50%. Exp-LTE-metric (broken lines) shows more rapid time progress than 2nd-order-LTE-metric (solid lines). This comes from the following two reasons. First, since 2nd-order-LTE-metric ignores higher order Taylor expansion terms, cancellation effect between Taylor expansion terms is not included and therefore it overestimates the actual LTE. Second, since Exp-LTE-metric focuses on the dominant time constant, shorter time responses which are less important for the actual device operation are not highly considered.

Fig. 9 compares the drain current between the solution with Exp-LTE=1% (orange dots) and the exact solution obtained by setting 2nd-order-LTE to 0.1% (blue line). Although the unrealistic step-Vg application assumed here brings into ultra-fast time response related to the displacement current coupling between the terminals, such a response is less meaningful from a view point of dubious physical correctness and negligible impact on the amount of total charge conservation. Exp-LTE-metric just ignores such an ultra-fast time response and this is the reason why drain current error (open green triangles) becomes large in this duration. After the analysis time reaches about 1/5 of the dominant time constant, the error becomes a few %.

CPU-times between 2nd-order-LTE-metric and Exp-LTE-metric are compared for 2–50% LTE criteria in Fig. 10. Although the CPU-time per time step of Exp-LTE-metric (orange broken lines) is about 40% longer than that of 2nd-order-LTE-metric (blue broken lines), the total CPU-time of Exp-LTE-metric (orange solid line) is as small as about 30% of 2nd-order-LTE-metric (blue solid line).

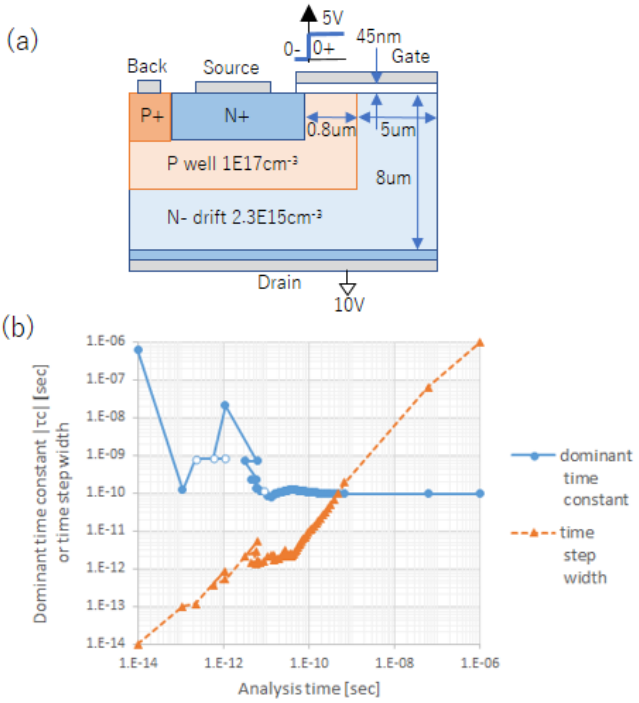

Fig. 7. (a) 2-D power DMOSFET structure. (b) Time variation of the dominant time constant and time step width.

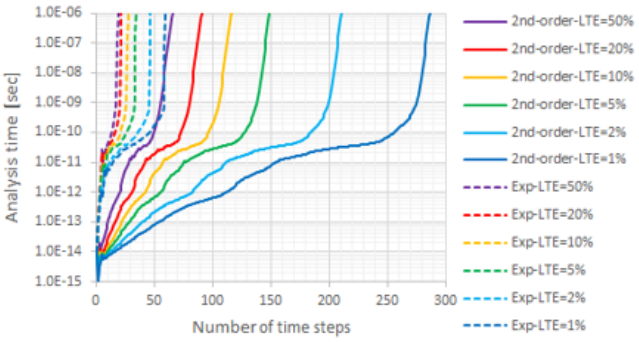

Fig. 8. Analysis time progress with respect to the number of time steps.

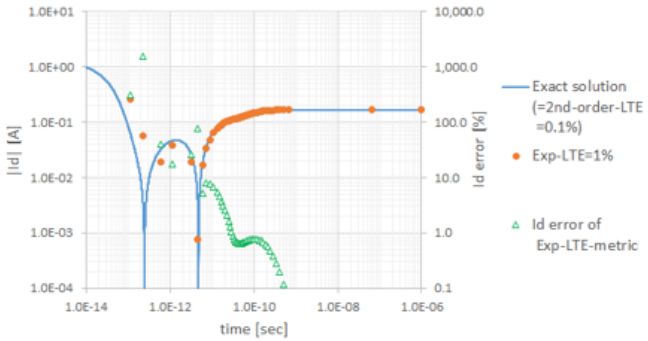

Fig. 9. Ids comparison between the exact solution and Exp-LTE=1%.

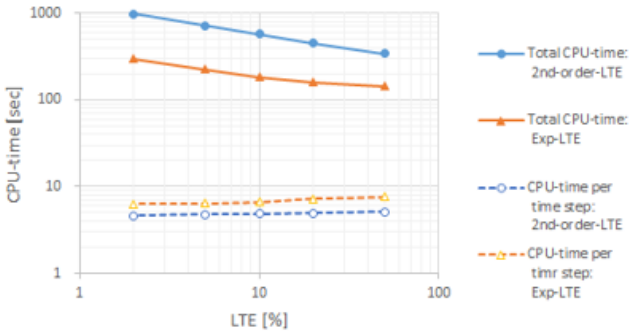

Fig. 10. CPU-time comparison between 2nd-order-LTE and Exp-LTE for 2 – 50% LTE criteria.

V. CONCLUSION

A new accurate Exp-LTE-metric for time step control for transient device simulation is derived by utilizing dominant time constant information of the whole device structure. The dominant time constant is extracted as the negative inverse of the smallest eigen value of the matrix which appears in the matrix exponential term in the formal solution of the linearized device equations. By using the proposed Exp-LTE-metric, CPU-time of the 2-D power DMOSFET transient simulation successfully decreases down to 30% of the conventional 2nd-order-LTE-metric with assuring the current accuracy of the dominant transient response.

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