# Electrothermal Simulation of SiGe HBTs and Investigation of Experimental Extraction Methods for Junction Temperature

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Abstract—We present an electrothermal simulator for lattice temperature calculation in Silicon-Germanium (SiGe) heterojunction bipolar transistors (HBTs) by including self-heating in Boltzmann transport equations (BTEs) of electrons and holes, where the BTEs are solved by deterministic methods based on a spherical harmonics expansion. Using the I-V characteristics of the device with taking self-heating into account, junction temperature is calculated with the thermal resistance extraction method, where the extracted temperature corresponds exactly to the average temperature increase over the base-emitter junction. Finally, the influence of both electrothermal simulation with phonon temperature distributions and isothermal simulation with homogeneous junction temperature on the I-V characteristics of HBT is investigated.

#### I. INTRODUCTION

In order to precisely model self-heating in bipolar transistors, the coupled BTEs for electrons, holes and phonons have to be considered simultaneously. To make the electrothermal simulation more feasible, two energy balance equations for the optical and acoustic phonon branches are derived from the corresponding BTEs [1], [2]. For these equations, interactions between carriers and phonons have to be modeled properly. However, the conventional methods for modeling carrierphonon interactions such as joule-heating and hydrodynamic model have their own drawbacks [3]. Hot carriers do not release their energy to the lattice exactly at the same place as they receive it and they diffuse several mean free paths (10-50 nm) before relaxation. Therefore, joule-heating is not an appropriate approach for representing the spatial distribution of the energy dissipation. Despite more accurate modeling of the spatial distribution of heat generation with the hydrodynamic model, it is still based on a single averaged carrier temperature and the relaxation time approximation, and also does not account for the spectral information regarding the emitted phonons. Since lattice heating is due to in-elastic carrierphonon scattering, the model which is used for including scattering mechanisms in the BTEs for the carriers can also be the most accurate approach for the comprehensive investigation of the carrier-phonon interactions and is employed in our thermal simulations.

On the other hand, there are several experimental methods to extract thermal resistance and consequently the average temperature increase over the base-emitter junction of HBTs based on I-V characteristics measurements [4], [5]. However, using the I-V characteristics obtained from simulations without self-heating does not lead to meaningful results. Therefore, by applying self-heating via electrothermal simulation, we calculate the required parameters for thermal resistance extraction. In consequence, a comparison for thermal and electrical results of both electrothermal and isothermal simulations is performed, where the extracted junction temperature is used as the homogeneous temperature through the whole simulation domain for the latter simulation.

#### II. THEORY

# A. Thermal Simulation

Carriers with high energies mainly scatter with the optical phonons which have negligible group velocity. The optical phonons cannot participate in heat conduction and decay into long wavelength acoustic phonons, known as the dominant heat carriers in semiconductors. However, the interaction between phonons is relatively slow compared with the carrier-phonon interaction, and as a result a non-equilibrium energy bottleneck for the hot optical phonons can be produced in high field domains [6]. Energy balance equations for the optical and acoustic phonons are written, respectively, as [1], [2]

$$\frac{\partial W_{op}}{\partial t} = \left(\frac{\partial W_{c-op}}{\partial t}\right)_{col} - \left(\frac{\partial W_{op-ac}}{\partial t}\right)_{col} \tag{1}$$

$$\frac{\partial W_{ac}}{\partial t} = \nabla \cdot \left(k_{ac} \nabla T_{ac}\right) + \left(\frac{\partial W_{c-ac}}{\partial t}\right)_{col} + \left(\frac{\partial W_{op-ac}}{\partial t}\right)_{col} \tag{2}$$

where  $k_{ac}$  is the lattice thermal conductivity,  $W_{op}$  and  $W_{ac}$  are the optical and acoustic phonon energy densities, respectively, which are related to their corresponding temperatures with optical phonon heat capacity  $C_{op}$ , and acoustic phonon heat capacity  $C_{ac}$ , as

$$dW_{op} = C_{op}dT_{op}, \quad dW_{ac} = C_{ac}dT_{ac} \tag{3}$$

The phonon-phonon interaction term, which describes the decay of optical to acoustic phonons, can be written in the relaxation time approximation as

$$\left(\frac{\partial W_{op-ac}}{\partial t}\right)_{col} = C_{op}\left(\frac{T_{op} - T_{ac}}{\tau_{op-ac}}\right) \tag{4}$$

where  $\tau_{op-ac}$  is the relaxation time for the optical phonons. To model heat generation accurately, carrier-phonon interactions

can be written as the energy loss rate due to in-elastic phonon scattering separately for the optical and acoustic modes as

$$\left(\frac{\partial W_{c-op}}{\partial t}\right)_{col} = \frac{2}{(2\pi)^3} \sum_{v} \sum_{\eta:op} \int \hbar \omega_{\eta} f^v(r, \vec{k}) \\ \left[S^v_{\eta,em}(r, \vec{k}, T_{op}) - S^v_{\eta,ab}(r, \vec{k}, T_{op})\right] d^3k$$
(5)
$$\left(\frac{\partial W_{c-ac}}{\partial t}\right)_{col} = \frac{2}{(2\pi)^3} \sum_{v} \sum_{\eta:ac} \int \hbar \omega_{\eta} f^v(r, \vec{k}) \\ \left[S^v_{\eta,em}(r, \vec{k}, T_{ac}) - S^v_{\eta,ab}(r, \vec{k}, T_{ac})\right] d^3k$$
(6)

where  $\omega_{\eta}$  is the fixed phonon frequency,  $f^v(r, \vec{k})$  is the carrier distribution function, and v denotes number of valleys for electrons and number of bands for holes.  $S_{\eta,em}^v(r, \vec{k}, T)$  stands for phonon emission by carriers (phonon generation) and  $S_{\eta,ab}^v(r, \vec{k}, T)$  for the phonon absorption by carriers (phonon annihilation). The scattering rate for intervalley carrier-phonon scattering in silicon which includes all in-elastic phonon scatterings is given by [7]

$$S_{\eta}^{v}(r,\vec{k},T) = \frac{\pi K_{\eta}^{2}}{(2\pi)^{3}\rho\omega_{\eta}} \sum_{v'} \int \left(\frac{1}{\exp\left(\frac{\hbar\omega_{\eta}}{T}\right) - 1} + \frac{1}{2} \pm \frac{1}{2}\right)$$
$$\delta\left[\varepsilon(k) - \varepsilon(k') \mp \hbar\omega_{\eta}\right] r(\eta,v,v') d^{3}k' \tag{7}$$

where  $K_{\eta}$  is the coupling constant,  $\rho$  is the mass density, and  $r(\eta, v, v')$  is the selection rule of phonons which distinguishes between g- and f- processes. The upper signs stand for phonon emission and the lower signs for absorption. The temperature dependence of the Bose-Einstein distribution function for phonons in the scattering rate causes two non-linear coupled energy balance equations for phonons, which have to be solved using the distribution function of carriers. The acoustic and optical phonon temperatures are also fed back into the BTEs for the carriers through all temperature-dependent parameters to constitute a self-consistent electrothermal simulator.

### B. Thermal Resistance Extraction Method

For a bipolar transistor operating in forward active mode, if the collector-base voltage  $V_{CB}$  and the collector current  $I_C$  are limited to values sufficiently low to disregard impact-ionization and high injection effects, the lattice temperature over the baseemitter junction can be approximated using some parameters derived from I-V characteristics of the device. The extraction procedure is based on the calculation of the temperature coefficient of the base-emitter voltage ( $\phi = -\frac{\partial V_{BE}}{\partial T_B}|_{I_C}$ ), for which the base-emitter voltage has to be calculated for different ambient temperatures at constant collector current, and also the the slope of the  $V_{BE}$ - $V_{CB}$  characteristic at constant collector current ( $\gamma = \frac{\partial V_{BE}}{\partial V_{CB}}|_{I_C}$ ) has to be evaluated. Using these parameters, the thermal resistance can be written as [5]

$$R_{TH} = -\frac{\gamma}{I_E \phi} \tag{8}$$

where  $I_E$  is the emitter current. This thermal resistance can be attributed to the base-emitter junction of the bipolar transistor. Therefore, using the thermal equivalent of the Ohm's law, the



Fig. 1. Doping and Ge content profiles of the investigated SiGe HBT.



Fig. 2. Spatial distribution of power density calculated from joule-heating and energy loss due to in-elastic phonon scattering approaches. The portion of the energy-loss rate for the optical and acoustic phonons are shown separately.

average temperature increase above ambient over the baseemitter junction of the HBT is calculated as

$$\Delta T_i = R_{TH} \times P_D \tag{9}$$

where  $P_D$  is the total dissipated power through the device.

# III. RESULTS

The doping and Ge profiles of the investigated THz SiGe HBT are derived from Ref. [8] and shown in Fig. 1. The BTEs for electrons and holes are solved based on the spherical harmonics expansion method [9] including full-band effects [10]. In addition to intervalley phonon scattering, intravalley phonon scattering [11], impurity scattering [11], and alloy scattering [12] are included in the BTEs. The required constant parameters for thermal simulation are considered as  $\tau_{op-ac}$ =10ps, and  $C_{op}$ =0.65J/(K.cm<sup>3</sup>). Fig. 2 depicts the spatial distribution of joule-heating, which is electric field



Fig. 3. Optical and acoustic phonon temperature profiles through the device, and constant junction temperature calculated by the extraction method for the bias conditions of  $V_{BE}$ =0.9V, and  $V_{CB}$ =0.6V, and assuming a constant thermal conductivity  $K_{ac}$ =10WK<sup>-1</sup>m<sup>-1</sup>.

multiplied by current density, and the energy loss rate due to inelastic phonon scattering. This figure shows the positional shift that carriers experience before releasing their energy to the lattice and clarifies the importance of using the latter approach for modeling heat generation in nano-scale devices.

Fig. 3 shows the optical and acoustic phonon temperature profiles calculated self-consistently from the coupled system of BTEs and energy balance equations. The boundary conditions are fixed for the acoustic phonon temperature at the contacts to the ambient temperature ( $T_B$ =300K). Using this coupled system which has already included self-heating in I-V characteristics, the parameters required for thermal resistance extraction are calculated from the curves in Fig. 4. In Fig. 4(a), the  $I_C - V_{BE}$  characteristic of the HBT for different homogeneous temperatures which is used for the temperature coefficient derivation is shown. Fig. 4(b) shows the  $V_{BE}$ - $V_{CB}$  characteristic for different constant thermal conductivities. While thermal conductivity changes the calculated temperatures from thermal simulation, self-heating tunes the thermal resistance by changing the slope of the  $V_{BE}$ - $V_{CB}$  characteristic. Therefore, regardless of the value of thermal conductivity, the extraction method yields the correct thermal resistance. The junction temperature which is calculated from these characteristics  $(\Delta T_j = 22.5 \text{K})$  is in very good agreement with the average lattice temperature (acoustic phonon temperature) around the base-emitter junction (Fig. 3).

The impact of self-heating using both electrothermal and isothermal simulations on the I-V characteristics of the SiGe HBT is shown in Fig. 5. These results show that isothermal simulations based on the junction temperature as a constant temperature throughout the simulation domain have approximately the same effect on the currents as the electrothermal simulations with temperature distributions. This approach is also extensible to the higher dimensional case. For instance, the lattice temperature distribution calculated from electrothermal simulation for a 2D SiGe HBT is shown in Fig. 6.



Fig. 4. (a)  $I_C$ - $V_{BE}$  characteristic at  $V_{CB}$ =0.6V for different homogeneous temperatures. (b)  $V_{BE}$ - $V_{CB}$  characteristic at constant collector current for different constant values of thermal conductivity.

# IV. CONCLUSION

We have presented a deterministic solver for the BTEs of the carriers and phonon equations to provide an electrothermal simulator. It is shown that the junction temperature extracted by the experimental thermal resistance extraction method is consistent with the average temperature increase over the baseemitter junction of the investigated HBT. Furthermore, the results showed that for bipolar junction transistors, isothermal simulations with constant junction temperature throughout the device can yield similar currents as the self-heating due to microscopic simulations.

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Fig. 5. (a)  $I_C$ - $V_{BE}$  characteristic at  $V_{CB}$ =0.6V, and (b)  $I_C$ - $V_{CB}$  characteristic at  $V_{BE}$ =0.9V, for the investigated SiGe HBT with electrothermal and isothermal simulations.

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Fig. 6. Lattice temperature distribution through the 2D SiGe HBT using electrothermal simulation for  $V_{BE}$ =0.9V,  $V_{CB}$ =0.6V, and  $K_{ac}$ =10WK<sup>-1</sup>m<sup>-1</sup>.

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