Plasma Instability and Non-linear Wave Propagation in Gate-controlled Semiconductor and Graphene Conduction Channels.

Sergey Rudin and Greg Rupper.

U.S. Army Research Laboratory,
Adelphi, Maryland 20783.
sergey.i.rudin.civ@mail.mil

Abstract— **The plasma wave in the conduction channel of a semiconductor heterostructure high electron mobility transistor can be excited at frequencies significantly higher than the cut-off frequency in a short channel device. The hydrodynamic model predicts a resonance response to applied harmonic signal at the plasma oscillation frequency. When either the ac voltage induced in the channel by the signal at the gate or the current applied at the drain or source contact are not very small, the plasma waves in the semiconductor channel will propagate as a shock wave. The device can be used either as a detector or a tunable source of terahertz range radiation. We show that in both configurations the charge flow develops shock waves due to hydrodynamic nonlinearities. In the graphene based channel there is an additional mechanism of the shock wave formation, related to nearly linear dispersion of the conduction band.**

Keyword-plasma resonator; shock waves; hydrodynamic model

I. INTRODUCTION.

The plasma wave in the conduction channel of a semiconductor heterostructure high electron mobility transistor (HEMT) is an electron density excitation, possible at frequencies significantly higher than the cut-off frequency in a short channel device. The hydrodynamic model predicts a resonance response to electromagnetic radiation at the plasma oscillation frequency, which can be used for detection, mixing, and frequency multiplication in the terahertz range. [1] In particular, the hydrodynamic nonlinearities produce a constant source-to-drain voltage when gate-to-channel voltage has a time-harmonic component. In the Dyakonov-Shur detector a short channel HEMT is used for the resonant tunable detection of terahertz radiation. The non-linear plasma response has been observed in InGaAs [2] and GaN [3] HEMTs, in the frequency range from 0.2 to 2.5 THz. Recently, a broad band plasma mediated response to 0.3 THz electromagnetic signal was shown in a graphene field effect transistor at room temperature. [4]

The plasma waves in the gated two-dimensional channels have linear dispersion law $\omega(q) = sq$ where s is the plasma wave velocity, q is the in-plane wave vector. For example, if gate voltage U_0 is 1 V above the threshold voltage for the formation of the conduction channel, U_{th}, the plasma wave velocity in GaAs and GaN channels is s ~ 10^8cm/s, usually much higher than the electron drift velocity. [1] In a graphene channel the wave velocity s exceeds the Fermi velocity, $v_F = 10^8$ cm/s, by a factor of 5 to 9. [5] In a channel with length L with asymmetric boundary conditions the eigen-frequencies of the plasma standing waves are odd multiples of the fundamental plasma frequency, given by $\omega_0 = \pi s/2L$. When the mean free path of electron-electron collisions is much smaller than the channel length L, the plasma transport in the channel can be studied within the hydrodynamic model of the electron density waves. [6] If the momentum relaxation time τ due to electron-phonon and electron-impurity collisions is such that $\omega_0\tau >> 1$, the HEMT can operate as a resonant detector tunable by the gate voltage. If the ac voltage induced in the channel by the signal at the gate is not very small compared to the dc gate voltage, we show that the plasma waves in the channel will propagate as a shock wave. With no applied ac voltage at the gate but with large enough dc current at the drain contact and constant voltage at the source, the charge flow becomes unstable because of plasma wave amplification at the boundaries. [6] The device then can be used as a tunable source of terahertz range radiation. We show that in such configuration the charge flow also develops shock waves due to hydrodynamic nonlinearities. In the graphene channel if the drift velocity is not very small compared to the Fermi velocity, there is an additional mechanism of the shock wave formation, related to the strongly non-parabolic dispersion of the conduction band.

II. HYDRODYNAMIC MODEL FOR SEMICONDUCTOR CHANNELS.

The hydrodynamic model here was derived as the balance equations from the quasi-classical Boltzmann equation, starting with a drifted Fermi-Dirac distribution as a zero order term in the expansion of the distribution function in orders of the Knudsen number, the ratio of the mean free path of electron-electron collisions to the characteristic

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length of electron density variations. Using the electron–electron scattering rate $1/\tau_{ee}$ in the relaxation time approximation for the first order correction to collision integrals, we obtain the pressure tensor and the heat flux vector, and the temperature dependence of the hydrodynamic transport coefficients, i.e. viscosity ν and the heat conductivity κ. [7] In the long wave length limit, kd <<1 where d is the distance from the channel to the gate contact, we obtain the linear dispersion $\omega_{pl}(k) = sk$ with $s = (eU_0/m)^{1/2}$, where $U_0 = U_g - U_{th}$ and m is the effective electron mass..

We obtain the equations of the viscous hydrodynamic model for the electron fluid in the gated semiconductor channel as the two-dimensional density balance equation, the Navier-Stokes equation, and the heat equation. The first two equations are:

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{u}) = 0$$

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} + \frac{e}{m}\nabla U + \frac{1}{n}\nabla P + \frac{\mathbf{u}}{\tau} - \nu\nabla^2\mathbf{u} = 0$$

where $n(\mathbf{r},t)$ and $\mathbf{u}(\mathbf{r},t)$ are the density and drift velocity respectively. $P(\mathbf{r},t)$ is the scalar pressure and $U(\mathbf{r},t)$ is the electrical potential in the channel determined here in the gradual channel approximation [6,7]. For a GaN channel with gate to channel distance d = 35 nm, electron effective mass $m/m_0 = 0.2$ and $\varepsilon_s = 8.9$ we obtain the gate to channel capacitance per unit area $C = 2.25\times10^{-3}$ F/m^2, and $n = UC/e$. To the order in which the hydrodynamic model is derived from the Boltzmann quasiclassical equation, the pressure and internal energy per electron ε at temperature T are given by

$$P = \frac{n\varepsilon}{m} = \frac{nk_B T F_1(\xi)}{F_0(\xi)}$$

where ξ is chemical potential in units of k_BT: $\xi = \ln[\exp(E_F/k_BT) - 1]$ and Fermi energy $E_F = \pi\hbar^2 n/m$. $F_n(\xi)$ are the Fermi integrals, defined as

$$F_m(y) = \int_0^\infty dx \frac{x^m}{1+\exp(x-y)}$$

The heat equation and the effects of the temperature gradient on plasma waves were discussed elsewhere [7,8] and will not be considered in this work. The temperature dependence of the viscosity for a semiconductor channel was also derived in references 7 and 8. The relaxation time τ in the friction term in Eq. (22) can be related to the electron mobility η in the channel, $\eta = e\tau/m$, and its temperature dependence can be found from electron-phonon scattering rates [9].

The system of the hydrodynamic equations is solved numerically. In the one-dimensional transport [1,6] the boundary conditions where $U(x=0,t) = U_0 + U_a\cos(\omega t)$ at the source side of the channel and fixed current density $j_0 = en(x=L,t)u_x(x=L,t)$ at the drain side. Because of viscosity terms we need an additional boundary condition for spatial derivatives of velocity and we set these derivatives to zero at the boundaries.

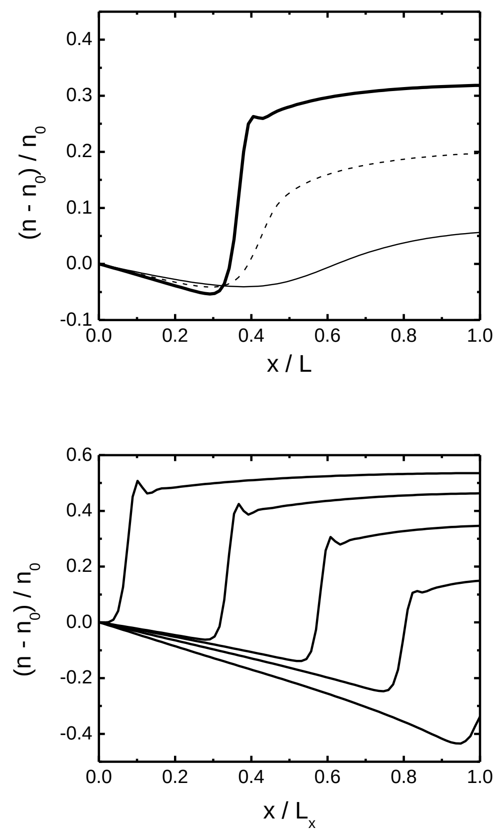

Figure 1. Formation and propagation of shock waves in a semiconductor channel. **Top panel**: electron density variation as function of position at three different values of dimensionless time ts/L, 162.4 (thin solid line), 177 (broken line), and 179.6 (thick solid line), for GaN channel at 60 K. **Bottom panel**: propagation of a shock wave from drain to source is seen in the density profiles shown at dimensionless times ts/L separated by 0.4, starting at ts/L = 244 for the rightmost curve

In Fig. 1, we model a GaN channel of length L=0.5μm used as a terahertz source (no external ac signal, $U_a = 0$, and the dimensionless value of the applied dc current j_0 at the drain given by $j_0/esn_0 = 0.12$), where n is the surface density of the electrons with n_0 denoting its dc value. We find that initially the channel acts as a quarter wave plasma resonator with a harmonic plasma wave. As the plasma wave grows shock waves develop. The final steady state plasma wave in the channel has a very significant shock wave component. We note that this shock wave may cause the terahertz source to emit higher order harmonics as well as the desired fundamental frequency.

We can obtain an analytical expression for the instability threshold using the perturbation treatment of the plasma response to the small applied ac source-to-gate voltage $U_a\cos(\omega t)$ [1, 10]. The induced dc component of the source-to-drain voltage has a resonant dependence on the signal frequency ω. In the case of a non-zero drain current the width of the resonant response is reduced to $\Delta\omega = 1/\tau + \pi^2\nu/4L^2 - 2u_0/L$ where $u_0 = u(x=L)$. Thus in perturbation treatment the width vanishes at $u_0 = L/2\tau + \pi^2\nu/4L$. This also gives the threshold condition for plasma wave generation by drain current [10]. For GaN channel of length L=0.5μm at 60 K this gives $j_0/sn_0 > 0.069$ as condition of instability, close to the value obtained from a numerical solution [9].

III. HYDRODYNAMIC MODEL FOR GRAPHENE CHANNELS.

The carrier densities and hence the plasma resonance frequencies in a graphene conduction channel can be tuned by the gate voltage, e.g. in the back-gated device configuration proposed in reference 5. The plasma wave velocity in graphene was shown there to always exceed the electron velocity, i.e. the Fermi velocity, and as a result the intraband Landau damping of the plasma waves in gated graphene conduction channel is reduced. The quality factor of the plasma resonator will be mobility limited and in graphene it will only weakly depend on temperature. For conduction channel with length L = 1 μm, the gate to channel distance 0.3μm and U = 1 to 10 V, the frequency $f_0 = \omega_0/2\pi$ is in the range from 1 to 5 THz. The wave velocity s exceeds the Fermi velocity, $v_F = 10^8$ cm/s, by a factor of 5 to 9.

When a positive potential is applied to the gate at temperatures significantly below the Fermi temperature the density of holes is much smaller than the density of electrons. [7] As an example consider graphene on a silicon oxide substrate, with d = 0.3 μm, U = 1 V. With $\varepsilon_s = 6$, we obtain gate to channel capacitance per unit area $C = 1.77\times10^{-4}$ F/m^2, and electron density $n_e = 1.1\times10^{11}$ 1/cm^2. With the Fermi velocity $v_F = 10^8$ cm/s, we obtain $k_F = 0.59\times10^6$ 1/cm and $E_F = 39$ meV. If the gate voltage increases to 10 V the Fermi energy increases to 123 meV. The plasma wave speed as a function of the gate voltage U in the gradual channel approximation is given by

$$s = \left(\frac{g_s g_v e^3 v_F{}^2}{4\pi\hbar^2 C}\right)^{1/4} U^{1/4}$$

where $g_s = 2$ and $g_v = 2$ are spin and valley degeneracy factors. In the presence of both electrons and holes with $n_e > n_h$ the Fermi energy is given by $E_F = \hbar v_F k_F = \hbar v_F[4\pi(n_e - n_h)/g_s g_v]^{1/2}$.

The hydrodynamic variables are electron and hole densities $n_\alpha(\mathbf{r},t)$, drift velocity $\mathbf{v}(\mathbf{r},t)$, and drift momentum $\mathbf{p}(\mathbf{r},t)$. They are obtained by an ensemble averaging from the corresponding microscopic variables. As in the semiconductor channels the hydrodynamic equations for graphene are obtained from the Boltzmann transport equation by expansion of the non-equilibrium distribution functions in orders of the Knudsen number, λ_{cc}/L. The flow is described by the Navier-Stokes equation, a non-linear relation between hydrodynamic momentum and velocity, and the diffusion between electrons and holes. We also define the total carrier density $n = n_e + n_h$. In this work we ignore the temperature gradients and use the energy balance equation to the zero order, and obtain the following hydrodynamic equations describing 2D flow of electrons and holes [7]:

$$\frac{\partial n}{\partial t} + \nabla\cdot(n\mathrm{v}) = 0$$

$$\frac{\partial n_\alpha}{\partial t} + \nabla\cdot(n_\alpha \mathrm{v}) + \nabla\cdot j_\alpha = 0$$

$$\frac{\partial \mathbf{p}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{p} + e\nabla U + \frac{\mathbf{p}}{\tau} + \frac{1}{n}\nabla P - \nu(p_F/v_F)\nabla^2\mathbf{v} = 0$$

where the relaxation time τ is due to the carrier- phonon and carrier- impurity scatterings and is related to the mobility $\mu = e\tau v_F{}^2/E_F$. The hydrodynamic momentum and velocity are related through the following relation:

$$\mathbf{p} = \frac{3}{2}\mathbf{v}(1 - v^2/v_F{}^2)^{-1}\frac{k_B T}{v_F{}^2}\left[\frac{n_e F_2(\xi)}{nF_1(\xi)} + \frac{n_h F_2(-\xi)}{nF_1(-\xi)}\right].$$

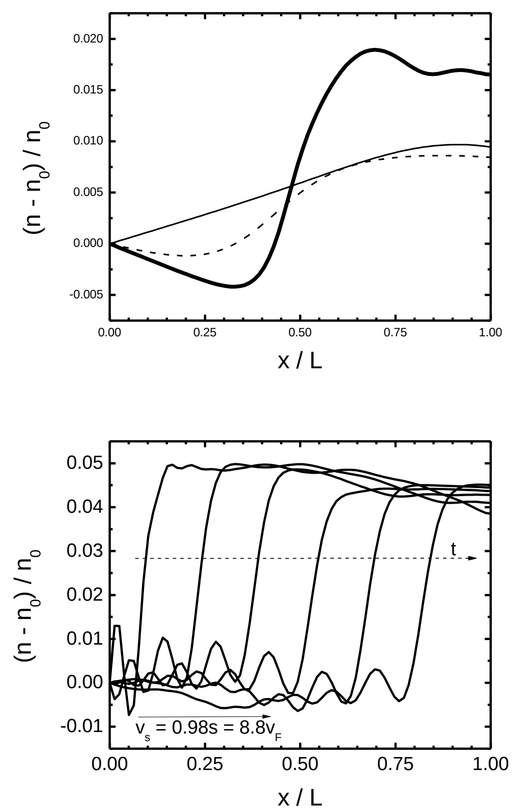

Figure 2. Formation and propagation of shock waves in a graphene channel. **Top panel**: electron density variation as function of position at three different values of dimensionless time ts/L, 454.5(thin solid line), 504.5 (broken line), and 515.5 (thick solid line). **Bottom panel**: propagation of a shock wave from drain to source is seen in the density profiles shown at dimensionless times ts/L separated by 0.15, starting at ts/L = 624.5. The shock propagation speed is indicated by v_s.

The pressure P is given by

$$P = \frac{k_B T}{2}\left[\frac{n_e F_2(\xi)}{F_1(\xi)} + \frac{n_h F_2(-\xi)}{F_1(-\xi)}\right].$$

The temperature dependences of the viscosity coefficient and the diffusion currents j_e and j_h were discussed in reference 7. Here ξ is the chemical potential in units of k_BT. It is found by solving the following equation

$$2\left(\frac{k_B T}{E_F}\right)^2 (1 - v^2/v_F{}^2)^{-3/2}\left[F_1(\xi) - F_1(-\xi)\right] = 1.$$

The development of shock waves in the charge flow in a graphene channel with L= 0.25μm and U = 10 V is shown in Fig. 2. In the case shown in this figure there was no external ac signal and the dimensionless value of the applied dc current j_0 at the drain is given by $j_0/sn_0 = 0.06$. With the gate voltage of 10 V above the threshold the Fermi energy is 123 meV, much larger than the room temperature. Then the hole density is much smaller than the electron density and the diffusion currents can be ignored in the numerical solution.

The nonlinear relation between hydrodynamic momentum and velocity is responsible for the onset of shock waves in graphene channel at the lower values of drift velocities than in semiconductor channels.

IV. Oblique Wave Propagation on the Two-Dimensional Semiconductor Channels.

We also studied the case of oblique wave propagation in a two dimensional channel. We define the longitudinal direction x along the channel from source to drain and the transverse direction y. The channel has width W and length L. The wave vector of a linearized plasma wave $\exp(-i\omega t + ikx + iqy)$ has a longitudinal and transverse components k and q, respectively. In order to introduce oblique waves, the source boundary condition is $U(0,y,t) = U_0 + U_a\cos(\omega t - qy)$. Oblique propagation will occur for $q \neq 0$. The drain boundary condition is $en(L,y,t)u_x(L,y,t) = j_0$ and the conditions of zero transverse velocity can be used, $u_y(y=0) = u_y(y=W) = 0$. For the additional boundary condition for spatial derivatives of velocity we choose to be $\partial u_x/\partial x(0,y,t) = \partial u_x/\partial x(L,y,t) = 0$. For the initial conditions we chose the density and velocity profiles obtained analytically from the perturbation approximation in the small signal analysis [1]. The oblique waves were shown to result in a widening of detection resonant characteristics and lowering the quality factor [2].

In addition to the instability discussed in section II an additional instability at nonzero values of transverse wave-vector q was predicted in reference 11, using a linear approximation for a flow in an infinitely wide channel. The corresponding threshold condition was given by $qu_0 > 1/\tau$ [11]. However, our numerical investigation showed that for all values of the transverse wave-vector q for the large enough longitudinal boundary velocity the transport was dominated by the one-dimensional instability, leading to the propagation of shock waves in the longitudinal direction. In our numerical implementation we added randomly generated components to the initial density and velocity fields so that the initial values are $n(\mathbf{r},t=0) + \Delta n(\mathbf{r})$ and $\mathbf{u}(\mathbf{r},t) + \Delta\mathbf{u}(\mathbf{r})$, with Δn and $\Delta\mathbf{u}$ given by the pseudorandom (white noise) generator. They have Fourier components with arbitrary large values of q. For any value of the longitudinal boundary velocity u_0 and zero transverse boundary velocity the transverse velocity anywhere in the channel eventually decays, $\max\{|u_y(r,t)|\} \to 0$ at large t. We conclude that the only current driven instability is of the type predicted in reference 6, which was shown here to lead to the formation of shock waves in the longitudinal direction in the channel.

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