Numerical Simulation of RF Noise in Si Devices

(Invited Paper)

Christoph Jungemann
Institute for Electronics (EIT4)
Bundeswehr University
D-85577 Neubiberg, Germany
Email: Jungemann@ieee.org

Bernd Meinerzhagen
Institute for Electron Devices and Circuits
Postfach 33 29, TU Braunschweig
D-38023 Braunschweig, Germany
Email: b.meinerzhagen@tu-bs.de

***Abstract*— The workhorse of today's TCAD is the drift-diffusion (DD) model, which in the special formulation of the impedance field method has been used for a long time for noise calculation, but there has been much debate over its noise source. The derivation of the DD noise model from the Langevin-type Boltzmann equation (LBE) is discussed in detail and it is shown that the DD noise source should be local in the real space, white and given by the power spectral density (PSD) of the velocity fluctuations at zero frequency calculated under homogeneous bulk conditions in analogy to the mobility. The white noise source and frequency independent mobility of the DD model reflect the neglect of certain acceleration terms in the LBE. By comparison of solutions of the LBE with and without these terms it is found that the DD model works well up to frequencies of about 100GHz in silicon devices. Comparison of solutions of the LBE and DD model for different definitions of the noise source shows that the best device results are obtained with the PSD of the velocity fluctuations calculated under bulk conditions. Use of the Einstein relation to calculate the noise source for nonequilibrium, as is often done, leads to an underestimation of noise. While the DD model delivers good results in sub-micron devices, it fails in very small devices resulting in spurious super shot noise. Based on the LBE it is found that excess noise in devices is mostly due to scattering of cold or warm electrons, whereas hot electrons contribute little in the absence of electron-hole pair generation.**

I. Introduction

Electronic noise due to scattering, trapping or generation of charge carriers within a device leads to fluctuations of the terminal currents or voltages and cannot be avoided, because it is fundamentally linked with charge transport [1]–[6]. These random events degrade the performace of analogue circuits and limit, for example, the sensitivity of RF receivers [7]. Therefore, electronic noise has to be taken into account during the design process of an RF circuit [8], [9]. To this end, compact models have been developed, which allow to describe the electronic noise produced by bipolar and MOS devices (e.g. [10]–[13]). A more detailed approach to noise simulation is based on device simulation, where either the drift-diffusion (DD) or hydrodynamic (HD) model is solved (e.g. [14]–[22]). The well-known impedance field method can be derived from the DD model [14]. The most detailed noise models in the framework of semiclassical transport theory are based on the Boltzmann equation (BE), which is often solved by the Monte Carlo (MC) method (e.g. [23]–[27]). An alternative method is based on a spherical harmonics expansion (SHE) of the BE [28], [29], and it allows to calculate solutions directly in the frequency domain avoiding many of the problems encountered by MC simulation [30].

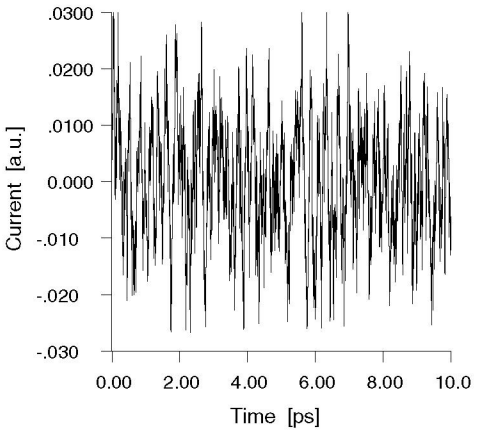

Fig. 1. Terminal current at zero bias for an N^+NN^+ structure.

Before going into the details of numerical noise simulation, noise is defined more precisely. A fluctuation is the deviation of a quantity from its expected value in the time domain [3], [31]. In Fig. 1 an example is shown for a 1D N^+NN^+ silicon structure (100nm of a donor concentration of $5 \cdot 10^{17}\mathrm{cm}^{-3}$ followed by 400nm of $2 \cdot 10^{15}\mathrm{cm}^{-3}$ and again 100nm of $5 \cdot 10^{17}\mathrm{cm}^{-3}$) biased at zero volts. The system is stationary and the expectation of the terminal current is zero. Nevertheless, the terminal current is not zero and fluctuates around zero due to particle scattering within the device. Since a particle carries a charge, it induces a displacement current, which appears at the terminals and is proportional to the particle charge and velocity. Thus, a change in the particle's velocity due to scattering leads to a change in the terminal current. The terminal current due to the moving electrons within the N^+NN^+ structure can be evaluated by the Ramo-Shockley theorem [32], [33]. In the 1D case it reads for a material with a position-independent dielectric constant and stationary bias conditions

$$I(t) = -\frac{qA}{L}\int_0^L j\,\mathrm{d}x = -\frac{q}{L}\sum_{i=1}^{N_{\mathrm{par}}} v_i \;, \qquad (1)$$

where q is the positive electron charge, L the length of the

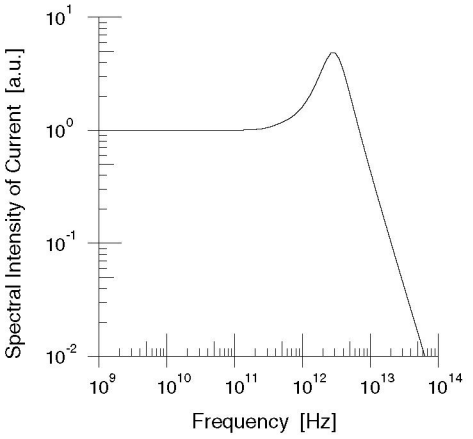

Fig. 2. PSD of the current fluctuations at zero bias for an N^+NN^+ structure.

device, A its area, j the particle current density, N_{par} the number of particles within the device and v_i the velocity of the ith electron. A generalization of this theorem to the multi dimensional case can be found in Refs. [34], [35].

A current fluctuation is defined as $\delta I(t) = I(t) - \mathrm{E}\{I\}$, where $\mathrm{E}\{\}$ indicates an expectation. The fluctuations are characterized in the stationary case by their autocorrelation function (ACF)

$$\varphi_{II}(\tau) = \mathrm{E}\{\delta I(t+\tau)\delta I(t)\} \ . \tag{2}$$

In engineering it is often more convenient to work in the frequency domain and the frequency dependence of noise is given by the one-sided power spectral density (PSD), which is the Fourier transform of the ACF

$$S_{II}(\omega) = 2\int_{-\infty}^{\infty} \varphi_{II}(\tau)\exp(-\mathrm{i}\omega\tau)\mathrm{d}\tau \ , \tag{3}$$

where the angular frequency $\omega = 2\pi f$ is assumed to be positive. $S_{II}(2\pi f)\mathrm{d}f$ is the noise power in the interval $\mathrm{d}f$ at frequency f. The PSD of the short-circuited N^+NN^+ structure is shown in Fig. 2, and the noise is white below about 100GHz neglecting $1/f$ noise. At about 3THz a plasma resonance occurs [26], which is due to the capacitance of the lowly doped region and the inductive effect of the acceleration of the electrons. This is reflected in the time dependence of the terminal current (Fig. 1), of which the strongest fluctuations occur in the THz range. The microscopic source of the noise is particle scattering by phonons and impurities, which is called diffusion noise. Since the system is in an equilibrium state, the noise can be calculated by a fluctuation-dissipation theorem [3] (here Nyquist theorem) $S_{II} = 4k_{\mathrm{B}}T\Re\{Y\}$, where $k_{\mathrm{B}}T$ is the thermal energy and Y the small-signal admittance of the N^+NN^+ structure. This device related noise is called thermal noise. It is a macroscopic manifestation of the microscopic diffusion noise at equilibrium and it is a good approximation for noise in resistors even if a current flows. Microscopic diffusion noise can also generate macroscopic shot noise ($S_{II} = 2qI$) under non equilibrium conditions. Microscopic and macroscopic noise sources should therefore not be confused. Other microscopic noise sources are particle trapping and generation of electron-hole pairs. Microscopic noise sources can be fully accounted for on the level of the BE, whereas macroscopic models like thermal or shot noise are often only approximations.

In the next part the semiclassical noise theory is outlined for the stationary case. Extension to the cyclostationary or quasiperiodic case is possible by the means presented in Refs. [36]–[38] and has been demonstrated in Ref. [30]. Many particle effects such as the short range electron-electron interaction or the Pauli exclusion principle are not included in this investigation (see for example: [3], [4], [39]). From semiclassical noise theory the DD noise model is derived and its noise source is discussed in detail. In the third section the DD approximation is investigated under homogeneous bulk conditions and in the fourth part for devices.

II. Theory

The theory of semiclassical noise was developed in the sixties [1], [3], [4]. An elegant way to formulate the theory is the Langevin-type BE (LBE), where a stochastic force $\xi(\boldsymbol{r},\boldsymbol{k},t)$ is added to the BE (only the case for electrons in unstrained silicon under stationary bias conditions is shown)

$$\frac{\partial f}{\partial t} - q\boldsymbol{E}^{\mathrm{T}}\nabla_{\boldsymbol{k}} f + \boldsymbol{v}^{\mathrm{T}}\nabla_{\boldsymbol{r}} f - \tilde{S}\{f\} = \xi \ , \tag{4}$$

where $f_\nu(\boldsymbol{r},\boldsymbol{k},t)$ is the distribution function, $\boldsymbol{E}(\boldsymbol{r},t)$ the electric field, t the time variable, $\boldsymbol{r}$ a location in the real space, $\boldsymbol{k}$ the wave vector, $\boldsymbol{v}_\nu(\boldsymbol{k})$ the group velocity, ν the valley index, and $\tilde{S}$ the scattering integral for a nondegenerate electron gas of non interacting particles [3], [25], [40]

$$\begin{aligned}\tilde{S}\{f\} \ = \ & \frac{\Omega_s}{(2\pi)^3}\sum_{\nu'=1}^{6}\int S_{\nu,\nu'}(\boldsymbol{r},\boldsymbol{k},\boldsymbol{k}')f_{\nu'}(\boldsymbol{r},\boldsymbol{k}',t) \\ & - S_{\nu',\nu}(\boldsymbol{r},\boldsymbol{k}',\boldsymbol{k})f_\nu(\boldsymbol{r},\boldsymbol{k},t)\mathrm{d}^3k' \ .\end{aligned} \tag{5}$$

The transport model is based on the analytical electron band structure and phonon and impurity scattering models of the Modena group [25]. An empirical correction of the impurity scattering model is used to account for high doping effects [41]. The transition rate $S_{\nu,\nu'}(\boldsymbol{r},\boldsymbol{k},\boldsymbol{k}')$ for scattering from the initial state $(\nu',\boldsymbol{k}')$ into the final state $(\nu,\boldsymbol{k})$ is assumed to be independent of the distribution function, where impurity scattering is screened by a fixed particle density equal to the doping concentration under equilibrium conditions for the sake of simplicity. For the same reason generation/recombination processes (and therewith $1/f$ noise [3]) are neglected.

The transition rate also enters the ACF of the Langevin force

$$\begin{aligned}&\varphi_{\xi\xi\ \nu,\nu'}(\boldsymbol{r},\boldsymbol{r}',\boldsymbol{k},\boldsymbol{k}',t,t') \\ &\qquad = \Omega_s\Bigg[\Bigg(\sum_{\nu''=1}^{6}\int S_{\nu'',\nu}(\boldsymbol{r},\boldsymbol{k}'',\boldsymbol{k})f_\nu(\boldsymbol{r},\boldsymbol{k})\end{aligned}$$

$$+ S_{\nu,\nu''}(\boldsymbol{r},\boldsymbol{k},\boldsymbol{k}'')f_{\nu''}(\boldsymbol{r},\boldsymbol{k}'')\mathrm{d}^3k''\Big)\delta(\boldsymbol{k}-\boldsymbol{k}')\delta_{\nu,\nu'}$$
$$- S_{\nu',\nu}(\boldsymbol{r},\boldsymbol{k}',\boldsymbol{k})f_\nu(\boldsymbol{r},\boldsymbol{k})$$
$$- S_{\nu,\nu'}(\boldsymbol{r},\boldsymbol{k},\boldsymbol{k}')f_{\nu'}(\boldsymbol{r},\boldsymbol{k}')\Big]\delta(\boldsymbol{r}-\boldsymbol{r}')\delta(t-t')$$
$$= S_{\xi\xi\,\nu,\nu'}(\boldsymbol{r},\boldsymbol{k},\boldsymbol{k}')\delta(\boldsymbol{r}-\boldsymbol{r}')\delta(t-t') \ , \quad (6)$$

where $f_\nu(\boldsymbol{r},\boldsymbol{k})$ is the noiseless stationary solution of the BE and $S_{\xi\xi}$ the PSD of the Langevin force, which is white and local in the real space due to the assumption of instantaneous scattering [3]. The PSD of the Langevin force is fully determined by the transition rate and noiseless distribution function. Therefore, the LBE does not contain any information beyond the BE, and the Langevin approach is just a well understood mathematical apparatus to calculate noise.

The LBE is solved self-consistently with the Poisson equation (PE) for the quasistatic potential. In order to calculate noise it is assumed that the Langevin force is so small that the fluctuations of the distribution function and potential can be calculated by a first order expansion of the LBE and PE around the steady state [15], [16]. The LBE is solved by a deterministic numerical approach based on spherical harmonics, finite differences, box integration and the maximum entropy dissipation principle [30], [42], [43]. The CPU efficient adjoint approach [44] is used to calculate the corresponding Green's functions similar to the case of the DD model [18].

Langevin-type DD and HD models are obtained from the LBE by derivation of balance equations for the respective moments of the distribution function [19], [20], [41], [45]–[48], which are given by

$$x(\boldsymbol{r},t) = \frac{2}{(2\pi)^3}\sum_{\nu=1}^{6}\int X_\nu(\boldsymbol{r},\boldsymbol{k})f_\nu(\boldsymbol{r},\boldsymbol{k},t)\mathrm{d}^3k = n\langle X\rangle_{\boldsymbol{k}} \ , \quad (7)$$

where n is the particle density, which is obtained for $X=1$ with $\langle 1\rangle_{\boldsymbol{k}}=1$. The corresponding balance equations are obtained by multiplying the LBE with the microscopic quantity $X_\nu(\boldsymbol{r},\boldsymbol{k})$ and integration over the $\boldsymbol{k}$-space

$$\frac{2}{(2\pi)^3}\sum_{\nu=1}^{6}\int X_\nu(\boldsymbol{r},\boldsymbol{k})\{\mathrm{LBE}\}\mathrm{d}^3k \ , \quad (8)$$

which leads without any approximations to

$$\frac{\partial x}{\partial t} + n\frac{q}{\hbar}\left\langle\nabla_{\boldsymbol{k}}^{\mathrm{T}}X\right\rangle_{\boldsymbol{k}}\boldsymbol{E} + \nabla_{\boldsymbol{r}}^{\mathrm{T}}(n\left\langle X\boldsymbol{v}\right\rangle_{\boldsymbol{k}}) - n\left\langle\boldsymbol{v}^{\mathrm{T}}\nabla_{\boldsymbol{r}}X\right\rangle_{\boldsymbol{k}}$$
$$+ n\left\langle\tilde{W}\{X\}\right\rangle_{\boldsymbol{k}} = \xi_{\dot{x}} \ . \quad (9)$$

The scattering integral for balance equations is defined as

$$\tilde{W}\{X\} = \frac{\Omega_s}{(2\pi)^3}$$
$$\times\sum_{\nu'=1}^{6}\int S_{\nu',\nu}(\boldsymbol{r},\boldsymbol{k}',\boldsymbol{k})\left[X_\nu(\boldsymbol{r},\boldsymbol{k}) - X_{\boldsymbol{r},\nu'}(\boldsymbol{k}')\right]\mathrm{d}^3k' \ , \quad (10)$$

and the moments of the Langevin force read

$$\xi_{\dot{x}}(\boldsymbol{r},t) = \frac{2}{(2\pi)^3}\sum_{\nu=1}^{6}\int X_\nu(\boldsymbol{r},\boldsymbol{k})\xi_\nu(\boldsymbol{r},\boldsymbol{k},t)\mathrm{d}^3k \ . \quad (11)$$

The DD model consists of two balance equations [49]. The first one, the continuity equation is derived with $X_\nu(\boldsymbol{r},\boldsymbol{k})=1$

$$\frac{\partial n}{\partial t} + \nabla_{\boldsymbol{r}}^{\mathrm{T}}\boldsymbol{j} = 0 \ , \quad (12)$$

where the particle current density is given by

$$\boldsymbol{j} = n\langle\boldsymbol{v}\rangle_{\boldsymbol{k}} \ , \quad (13)$$

and no Langevin force occurs, because for the sake of brevity only particle number conserving scattering processes are considered.

The constitutive equation for the particle current density is based on the vector mean free path $\boldsymbol{\lambda}_\nu(\boldsymbol{r},\boldsymbol{k})$ [50], which is defined in such a way that the scattering integral of the balance equations yields exactly the group velocity [51], [52]

$$\boldsymbol{v} = \tilde{W}\{\boldsymbol{\lambda}\} \ . \quad (14)$$

With $\boldsymbol{X}=\boldsymbol{\lambda}$ the balance equation for the particle current density is obtained without any approximations [41], [47], [48], [53]

$$n\underbrace{\frac{q}{\hbar}\left\langle\nabla_{\boldsymbol{k}}\boldsymbol{\lambda}^{\mathrm{T}}\right\rangle_{\boldsymbol{k}}}_{=\hat{\mu}}\boldsymbol{E} + \left[\nabla_{\boldsymbol{r}}^{\mathrm{T}}\Big(n\underbrace{\left\langle\boldsymbol{v}\boldsymbol{\lambda}^{\mathrm{T}}\right\rangle_{\boldsymbol{k}}}_{=\hat{D}}\Big) - n\left\langle\boldsymbol{v}^{\mathrm{T}}\nabla_{\boldsymbol{r}}\boldsymbol{\lambda}^{\mathrm{T}}\right\rangle_{\boldsymbol{k}}\right]^{\mathrm{T}}$$
$$+ \boldsymbol{j} = \boldsymbol{\xi}_j \ , \quad (15)$$

where $\hat{\mu}$, $\hat{D}$ are the mobility and diffusion tensors, which in the absence of a magnetic field are usually symmetric, and the time derivative has been neglected for the sake of brevity[1]. Due to the definition of the vector mean free path, the scattering integral of the LBE is solved exactly and the macroscopic relaxation time approximation usually invoked to derive balance equations [46] is avoided. The Langevin force of the particle current density reads

$$\boldsymbol{\xi}_j(\boldsymbol{r},t) = \frac{2}{(2\pi)^3}\sum_{\nu=1}^{6}\int\boldsymbol{\lambda}_\nu(\boldsymbol{r},\boldsymbol{k})\xi_\nu(\boldsymbol{r},\boldsymbol{k},t)\mathrm{d}^3k \ . \quad (16)$$

Thus, the Langevin force is directly evaluated for the balance equation of the current density (velocity) and this scheme is called velocity fluctuation scheme. In the acceleration fluctuation scheme on the other hand the Langevin force is evaluated for the time derivative of the current density ($\boldsymbol{X}=\boldsymbol{v}$) [19]. Since the latter scheme requires the macroscopic relaxation time approximation, it violates basic fluctuation dissipation theorems in contrast to the velocity fluctuation scheme [41], [54]. Furthermore, current fluctuations $\delta\boldsymbol{j}$ and the corresponding Langevin force $\boldsymbol{\xi}_j$ are two distinct quantities, which are related by (15) and which are in general not equal.

[1]The time derivative can be eliminated exactly by a more general definition of the vector mean free path (cf. Refs. [41], [47], [48]).

With the above definitions the cross correlation function for the Langevin forces of two microscopic quantities X and Y reads

$$\begin{aligned}\varphi_{\xi_{\dot{x}}\xi_{\dot{y}}}(\boldsymbol{r},\boldsymbol{r}',t,t') &= 2n(\boldsymbol{r})\Big(\langle X\tilde{W}\{Y\}\rangle_{\boldsymbol{k}}+\langle Y\tilde{W}\{X\}\rangle_{\boldsymbol{k}} \\ &\quad -\langle\tilde{W}\{XY\}\rangle_{\boldsymbol{k}}\Big)\delta(\boldsymbol{r}-\boldsymbol{r}')\delta(t-t') \\ &= S_{\xi_{\dot{x}}\xi_{\dot{y}}}(\boldsymbol{r})\delta(\boldsymbol{r}-\boldsymbol{r}')\delta(t-t')\ , \qquad (17)\end{aligned}$$

where all averages are evaluated with the noiseless distribution function. The ACF for the Langevin force of the particle current density evaluates with (14), (17) to

$$\begin{aligned}\hat{\varphi}_{\boldsymbol{\xi}_j\boldsymbol{\xi}_j}(\boldsymbol{r},\boldsymbol{r}',t,t') &= 2n(\boldsymbol{r})\left(2\hat{D}-\langle\tilde{W}\{\boldsymbol{\lambda}\boldsymbol{\lambda}^{\mathrm{T}}\}\rangle_{\boldsymbol{k}}\right) \\ &\quad\times\delta(\boldsymbol{r}-\boldsymbol{r}')\delta(t-t') \\ &= \hat{S}_{\boldsymbol{\xi}_j\boldsymbol{\xi}_j}(\boldsymbol{r})\delta(\boldsymbol{r}-\boldsymbol{r}')\delta(t-t')\ . \qquad (18)\end{aligned}$$

Since the PSD of the Langevin force of the LBE (6) is local and white and the procedure to derive the balance equations is local in real space and time, the PSD of the Langevin force of the current does have these properties as well. Furthermore, the transport coefficient (mobility) is local in real space and time (see 15), because it is derived by the same procedure[2]. Since $\langle\tilde{W}\{X\}\rangle_{\boldsymbol{k}}^{\mathrm{eq}}=0$ for equilibrium due to the principle of detailed balance [51], the PSD of the current Langevin force is under this condition reduced to [41]

$$\hat{S}^{\mathrm{eq}}_{\boldsymbol{\xi}_j\boldsymbol{\xi}_j}=4n\hat{D}=4n\hat{\mu}\mathrm{U_T}\ , \qquad (19)$$

and the microscopic Nyquist theorem is recovered, where the Einstein relation holds exactly under equilibrium conditions as can be shown by integration by parts. Since the noise source is proportional to the diffusion constant, this is called diffusion noise, which is due to particle scattering within the conduction bands. Since the equilibrium distribution function is given by the Maxwell-Boltzmann distribution, the noise source and the mobility are only determined by local quantities (doping, lattice temperature etc) and can be determined under equivalent homogeneous bulk conditions.

In order to derive a tractable set of equations, the balance equation for the particle current density has to be simplified. The mobility tensor is assumed to be isotropic and is replaced by a scalar. The complete diffusion term [all terms in the square brackets of (15)] is approximated by $\mu\mathrm{U_T}\nabla_{\boldsymbol{r}}n$, where the Einstein relation is assumed to hold under nonequilibrium conditions. This yields

$$\boldsymbol{j}+\mu(n\boldsymbol{E}+\mathrm{U_T}\nabla_{\boldsymbol{r}}n)=\boldsymbol{j}-n\mu\nabla_{\boldsymbol{r}}\phi=\boldsymbol{\xi}_j\ , \qquad (20)$$

where μ is the longitudinal electron mobility, $\mathrm{U_T}$ the thermal voltage evaluated at the lattice temperature, and ϕ the quasi Fermi potential or imref [55]. The dependence of the mobility on the distribution function is evaluated under equivalent homogeneous bulk conditions, where the system can be parameterized by a single driving force (in addition to the local doping concentration, lattice temperature etc), which is given in a device by the gradient of the imref.

While many experimental results are available for the mobility, only few are known for the noise source (e.g. [23]) and simulation has to be used to fill this gap. The noise coefficient of the DD model is determined under homogeneous bulk conditions (i.e., all dependences on the position in real space are neglected in the LBE) for a constant electric field $\boldsymbol{E}_0$ consistent with the procedure for the evaluation of the mobility. In this case the particle current density can be replaced by the drift velocity ($\boldsymbol{v}_{\mathrm{d}}=\langle\boldsymbol{v}\rangle_{\boldsymbol{k}}=\boldsymbol{j}/n$) and the balance equation for the velocity fluctuations, which corresponds to (15), reads for a single electron

$$\delta\boldsymbol{v}_{\mathrm{d}}+\delta\hat{\mu}\boldsymbol{E}_0=\boldsymbol{\xi}_v \qquad (21)$$

and

$$\hat{S}_{\boldsymbol{v}_{\mathrm{d}}+\hat{\mu}\boldsymbol{E}_0\ \boldsymbol{v}_{\mathrm{d}}+\hat{\mu}\boldsymbol{E}_0}=\hat{S}_{\boldsymbol{\xi}_v\boldsymbol{\xi}_v}\ , \qquad (22)$$

where $\hat{S}_{\boldsymbol{\xi}_v\boldsymbol{\xi}_v}=\hat{S}_{\boldsymbol{\xi}_j\boldsymbol{\xi}_j}/n$ is the single particle PSD for the Langevin force of the velocity. In principle it is possible to evaluate $\hat{S}_{\boldsymbol{\xi}_v\boldsymbol{\xi}_v}$ with (18) by solving the LBE. Although this noise source is exact, the noise calculated by the DD model would contain a large error [21]. This is due to the DD approximation, which suppresses mobility fluctuations under bulk conditions. In the DD model it is assumed that the mobility depends on the modulus of the gradient of the imref $\mu=\mu(|\nabla_{\boldsymbol{r}}\phi|)$, which is a constant under bulk conditions and equal to the (negative) external electric field. Therefore, the DD mobility does not fluctuate under bulk conditions in contrast to the exact one, and the PSD of the velocity fluctuations erroneously equals the PSD of the Langevin force in the framework of the DD approximation

$$\hat{S}^{\mathrm{DD}}_{\boldsymbol{v}_{\mathrm{d}}\boldsymbol{v}_{\mathrm{d}}}=\hat{S}_{\boldsymbol{\xi}_v\boldsymbol{\xi}_v}\ . \qquad (23)$$

Thus, if the exact PSD of the Langevin force were used, the current noise would be grossly overestimated. In order to avoid this, the exact PSD of the Langevin force is replaced by a modified one, which is equal to the PSD of the velocity fluctuations. Thus, also in the case of nonequilibrium the noise source of the DD model is given by the PSD of the velocity fluctuations evaluated under equivalent bulk conditions [5], and it remains local in the real space

$$\hat{S}^{\mathrm{DD}}_{\boldsymbol{\xi}_j\boldsymbol{\xi}_j}=n\hat{S}_{\boldsymbol{v}_{\mathrm{d}}\boldsymbol{v}_{\mathrm{d}}}\approx nS_{v_lv_l}\hat{I}\ . \qquad (24)$$

$\hat{I}$ is the identity matrix and v_l the longitudinal component of the velocity. The approximation is consistent with the approximation of the mobility tensor by a scalar in the DD model. This scheme, which can be generalized to the case of the HD model [20], reduces to the exact one in the case of equilibrium, because in this case the driving force is zero and therewith the impact of the mobility fluctuations vanishes. The mobility and modified noise source of the DD model are generated consistently with the LBE under homogeneous bulk

[2]Noise sources, which are nonlocal in real space and time, are obtained by using nonlocal moments $X_\nu(\boldsymbol{r},\boldsymbol{r}',\boldsymbol{k},t,t')$ in (9) and additional integration over $\boldsymbol{r}'$ and t' (convolution). But this leads immediately to nonlocal transport coefficients [3]. Use of nonlocal noise sources with local transport coefficients is inconsistent and results in violation of fundamental relations (e.g. Nyquist theorem).

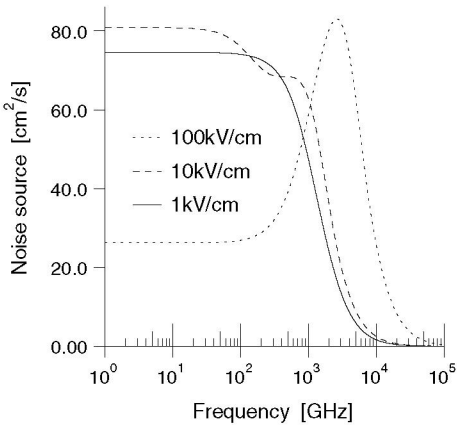

Fig. 3. PSD of the longitudinal velocity fluctuations $S_{v_l v_l}$ at room temperature for three electric fields in $\langle 100 \rangle$ direction and a donor concentration of $10^{17}/\mathrm{cm}^3$.

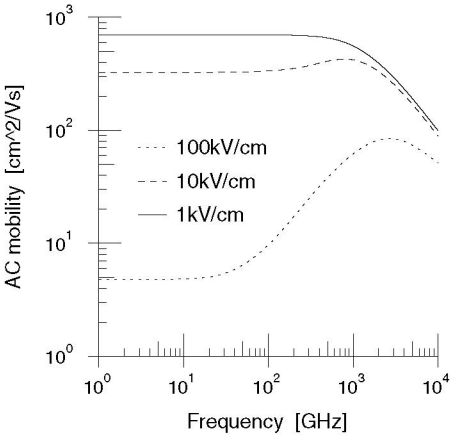

Fig. 4. Absolute value of the longitudinal AC mobility at room temperature for three electric fields in $\langle 100 \rangle$ direction and a donor concentration of $10^{17}/\mathrm{cm}^3$ (reprint from Ref. [58]).

conditions and stored in lookup tables for later use by the DD device simulator [56].

The PSD of the velocity fluctuations calculated under homogeneous bulk conditions should not be confused with the PSD of the velocity fluctuations in a device. The latter quantity is nonlocal in the real space and contains already part of the kinetics in the real space, which are also accounted for by the DD model leading to double counting of these effects if the device PSD of the velocity fluctuations is used as a noise source as proposed in Ref. [57]. Moreover, the nonlocal PSD does not reduce to the exact noise source (18) in the case of equilibrium, which is a prerequisite for the Nyquist theorem to hold for the terminal current noise of a device. In addition, the comment in footnote 2 applies.

In the balance equation for the particle current density (20) the time derivative has been neglected, as this is usually done to avoid plasma oscillations, which cause numerical problems [55]. Due to this approximation, the PSD of the velocity fluctuations of the DD model $S^{\mathrm{DD}}_{\boldsymbol{v}_\mathrm{d}\boldsymbol{v}_\mathrm{d}}$ is white in contrast to the PSD obtained from the LBE. The noise source of the DD model is therefore given by the low-frequency limit of the LBE PSD [58]. The validity of this approximation is discussed in the next section.

III. Bulk Results

The PSD of the velocity fluctuations, which is the noise source of the DD model, is shown in Fig. 3 as a function of frequency for electrons in a homogeneous bulk system. At frequencies above about 100GHz the PSD is no longer white in silicon and depends on the frequency. This frequency dependence is ignored in the DD model and could in principle be included, but it would require at the same time the inclusion of the frequency dependence of the mobility (Fig. 4). The frequency dependence of both quantities is rather complex and not captured by a single relaxation time, which is sometimes used to model these effects within the DD approximation. As mentioned above, the frequency dependence of the mobility leads to numerical problems, and the frequency dependences of the noise source and mobility are therefore consistently neglected.

The PSDs of the velocity fluctuations and of the corresponding Langevin force are shown in Fig. 5 together with the result of the Einstein relation (19) as a function of the electric field. At low fields, where mobility fluctuations do not matter and the particle gas is close to equilibrium, all three PSDs are equal. At high electric fields, on the other hand, the PSD of the Langevin force is for all important cases much larger than the PSD of the velocity fluctuations. Use of the PSD of the Langevin force as noise source in the DD model would therefore lead to strong overestimation of the device noise. The opposite result is obtained, if the Einstein relation is used to calculate the noise source of the DD model as this is often done. In addition, it can be seen that the noise source of the DD model ($S_{v_l v_l}$) decreases at high electric fields. Only at very high doping concentrations and intermediate fields does an increases of the noise source occur. In most cases, hot electrons therefore produce less noise than cold or warm ones [59].

IV. Device Results

The time derivative neglected in the DD model [cf. (20)] corresponds to an acceleration term. The impact of this omission on the noise of the N^+NN^+ structure of Fig. 1 is shown in Fig. 6 based on the LBE, where the simulations have been performed with and without the corresponding acceleration terms. Neglect of the acceleration terms modifies the results only at frequencies above 100GHz similarly to the bulk case (Fig. 3).

Results of the DD model based on the different noise sources (Fig. 5) are shown in Fig. 7 for the terminal current noise of the N^+NN^+ structure together with solutions of the LBE. As expected, the exact PSD of the Langevin force leads to an overestimation of the noise for the nonequilibrium case, whereas the Einstein relation underestimates it. Good agreement of the LBE and DD model is obtained in the case

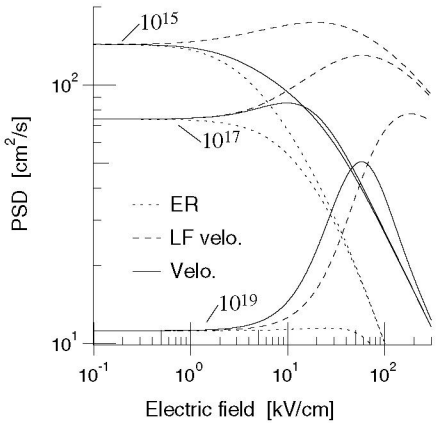

Fig. 5. PSDs of the velocity fluctuations $S_{v_l v_l}$ (solid lines), the corresponding Langevin force $S_{\xi_v \xi_v}$ (dashed lines) and obtained by the Einstein relation (dotted lines) at room temperature and zero frequency for three donor concentrations in units of cm^{-3} and an electric field in $\langle 100 \rangle$ direction.

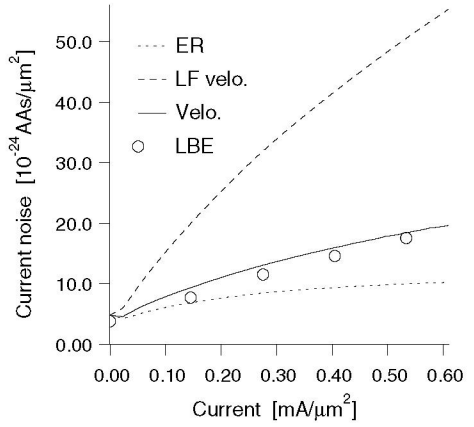

Fig. 7. PSDs of the terminal current fluctuations for the 400nm N^+NN^+ structure of Fig. 1 at room temperature for a bias in the range of 0.0 to 5.0V and current flow in $\langle 100 \rangle$ direction obtained by the LBE and DD based on the PSD of the velocity fluctuations $S_{v_l v_l}$ (solid line), the corresponding Langevin force $S_{\xi_v \xi_v}$ (dashed line) and the Einstein relation (dotted line).

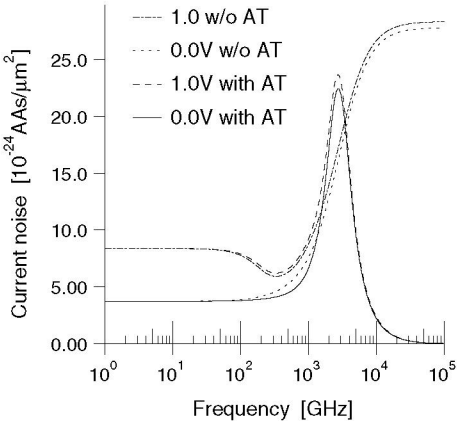

Fig. 6. PSDs of the terminal current fluctuations for the 400nm N^+NN^+ structure of Fig. 1 at room temperature for a bias of 0.0 and 1.0V and current flow in $\langle 100 \rangle$ direction based on the LBE with and without the acceleration terms (AT).

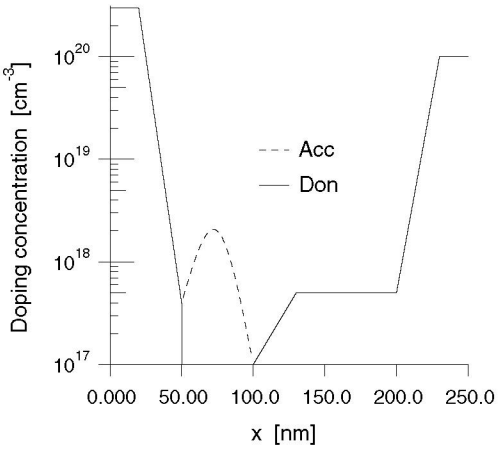

Fig. 8. Doping profile of the Si NPN BJT with a 50nm thick base.

of the PSD of the velocity fluctuations. Therefore, the PSD of the velocity fluctuations is used in the remaining part of this paper as noise source for the DD model and in a modified version for the HD. In the case of the N^+NN^+ structure (Fig. 7) the results of the HD model are almost equal to the DD data and not shown [21]. Similar good results are obtained for holes and not repeated here [21].

Since the DD and HD models are approximations of the LBE, it is doubtful, whether they will yield reliable results in small devices or not. To test this, a Si NPN BJT with a 50nm thick base has been simulated (Fig. 8). The collector current noise due to electrons should be full shot noise ($S^n_{I_C I_C} = 2\alpha_{\mathrm{Fano}} q I_C$ with $\alpha_{\mathrm{Fano}} = 1$) below high injection. The relation between the current noise and full shot noise α_{Fano} is called Fano factor. In the case of the LBE the Fano factor is one below high injection (Fig. 9), whereas the HD model yields a spuriously enhanced Fano factor of 1.33 and the DD of about 3. This error vanishes with increasing base thickness. A similar error occurs in the case of MOSFETs with very short channels in the subthreshold region and reveals the limitations of the macroscopic models.

In Fig. 10 the local contribution to the electron collector noise is shown for low injection. The noise originates from the left-hand side of the potential barrier (base/emitter region, where the minimum of the conduction band edge is at 72nm) consistent with full shot noise, which is due to the random injection of independent carriers over a potential barrier [10]. The subsequent heating of the carriers in the base/collector space charge region and hence strong particle scattering has no impact on the terminal current noise, because electrons scattered in this region cannot return to the emitter. The electron temperature at the peak of the current noise generation (68nm) is even below the lattice temperature (277K). This result again confirms that excess noise is in most cases not due to hot electrons (but nevertheless due to noneqilibrium effects). This is consistent with results for N^+NN^+ structures [59] and MOSFETs [60]. The decrease in the Fano factor at high

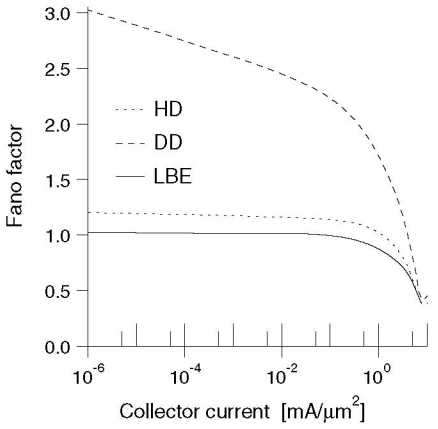

Fig. 9. Fano factor of the electron collector current noise for the Si NPN BJT biased at $V_{CE} = 0.5$V for room temperature and zero frequency.

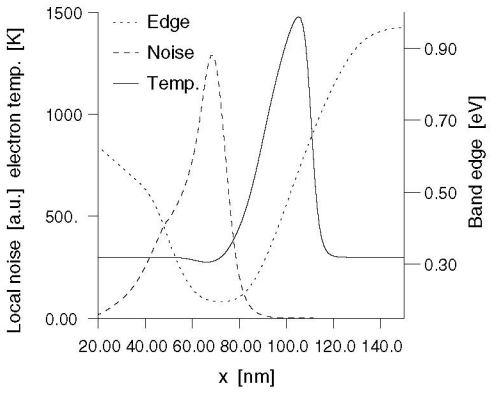

Fig. 10. Local electron collector current noise contribution, electron gas temperature and conduction band edge for the Si NPN BJT biased at $V_{CE} = 0.5$V and $V_{BE} = 0.65$V for room temperature and zero frequency based on the LBE.

injection is caused by electrostatic interaction of electrons being injected into the base/collector region with electrons already in the base/collector region. This leads to correlation of the injected electrons and reduction of the Fano factor.

A comparison of DD and HD results with experimental data for RF NMOSFETs can be found in Ref. [60] and good agreement of measurement and simulation is obtained for deep sub-micron devices.

V. Conclusion

Based on the velocity fluctuation scheme the DD noise model has been derived from the LBE. The resultant DD noise source is local in the real space, white and given by the PSD of the velocity fluctuations calculated under equivalent bulk conditions. This noise source has been shown to yield better results than exact Langevin forces or noise sources based on the Einstein relation. The DD model yields good results for devices in comparison with the LBE for frequencies below about 100GHz. Only in very small devices the DD model fails and an overestimation of shot noise occurs. The Fano factor, which is in this case larger than one, is a good indicator for the failure of the DD model. In addition, it has been shown that excess noise is mostly produced by scattering of cold and warm electrons and not by the frequent scattering of hot electrons.

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