A Fast Algorithm for 3-D Inductance Extraction Based on Investigation of Open-Circuit Current

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Abstract - As a famous interconnect extraction software, FastHenry can compute frequency-dependent interconnect inductance with high accuracy. But its low speed prevents it from being applied to deal with industry examples with large scale. Up to now, almost all acceleration methods for inductance extraction, e.g. the K-element method, are based on omitting the current of victim conductors, which is shorted and far away from aggressor conductor with 1 volt bias voltage. In this paper, we investigate the filament currents distribution on conductors while setting victim conductor open or shorted. Based on observation of less current on open-circuit conductors, we propose a new acceleration method for inductance extraction. The numerical results show that this method is faster than FastHenry for hundred times and preserves the similar accuracy.

I. INTRODUCTION

On-chip parasitic inductance has become an important concern as VLSI design is entering the deep sub-micron (DSM) regime and the operation frequency approaches to the gigahertz range. Because of higher frequency of signal, denser geometries, and lower resistance by copper technology, it becomes more important to extract on-chip complete coupling inductance matrix. Inductance affects the signal time delay, as well as the ringing behavior and the signal integrity. Therefore, it is crucial to extract the on-chip parasitic inductance quickly when designing the circuits with high performance requirements [1, 2].

The relationship between conductor voltage and current can be described as $YV=I$, in which V and I are voltage and current vector of conductors, Y is the inverse of impedance matrix Z. The algorithm of extracting interconnect inductance includes the following steps: set the voltage on some conductor l as 1 volt, the others zero, and then solve the current vector I_l which is equal to the lth column of the Y matrix. Impedance matrix Z can be obtained through inversing Y, then inductances can be gotten from the imaginary part of Z matrix entries. FastHenry [3] is based on this algorithm, which is a famous software for 3-D inductance extraction. It can give accurate results, but consumes too much memory and time because of solving the dense matrix. Therefore, it is difficult to apply FastHenry on practical examples.

To improve the efficiency of inductance extraction, the sparsification techniques become research focus in recent years. Some of them limit the return path to a small range to reduce the conductor coupling, such as the method of shift-truncation [4], return-limited loop inductance [5], and equipotential shell [6]. Devgan et al. proposed a new K element to improve the efficiency of inductance extraction in 2000 [7, 8]. These algorithms all inherit the basic steps as that in FastHenry, i.e., setting bias voltage of some conductor 1 volt and the others zero. The main approach to accelerate the inductance extraction is to omit the currents on those conductors far away from aggressor conductor with 1 volt bias voltage.

But because most conductors are set shorted, the currents on most of them are too large to be omitted without much accuracy lost. In this paper, we analyze the current distribution of conductors set as open and shorted respectively, and find that the currents on open conductors are enough little to be omitted. Based on this assumption, we can draw a conclusion that the filament currents on open conductors is little enough to be omitted while preserving enough accuracy. Based on this investigation, we propose a fast inductance extraction algorithm with computational complexity of $O(n)$, where n is number of conductors in the structure. The numerical results show that this method can preserve similar accuracy compared with Fasthenry, and is hundreds of faster than it.

Section 2 of this paper will show that the difference of the currents on open and shorted conductors. Section 3 will give a formula to compute the interconnect inductance based on open-circuit environment. Section 4 will compare the accuracy and speed of this algorithm with Fasthenry through an example.

II. CURRENT DISTRIBUTION ON OPEN AND SHORTED CONDUCTORS

At first, we will observe the filament currents on open and shorted conductors respectively through a simple structure. Given two parallel conductors a and b with size of 2×1×100μm, we set conductor a 1 volt and b open, divide them into 5×5 filaments respectively to capture the skin and proximity effect at 100 GHz. When the space between a and b is changed from 1.5μm to 5μm and 10μm, the distribution of filament currents on conductor a and b is shown in Fig. 1. Then, we set conductor b 0 volt (make its two ends shorted) and repeat the above experiment. Corresponding distribution of filament currents on both conductors is shown in Fig. 2.

In Fig. 1 and 2, X-axis denotes 25 filaments, Y-axis denotes current value, and S denotes space between conductors a and b. From Fig. 1, we can observe that the filament currents on conductor a have little change, and filament currents on b are always much less than that on a. When the space is 5μm, the average of filament currents on b is only 2.9% of that on conductor a. As a comparison, we can observe that the filament currents on shorted conductor b increase largely and cannot be omitted easily in fig.2.

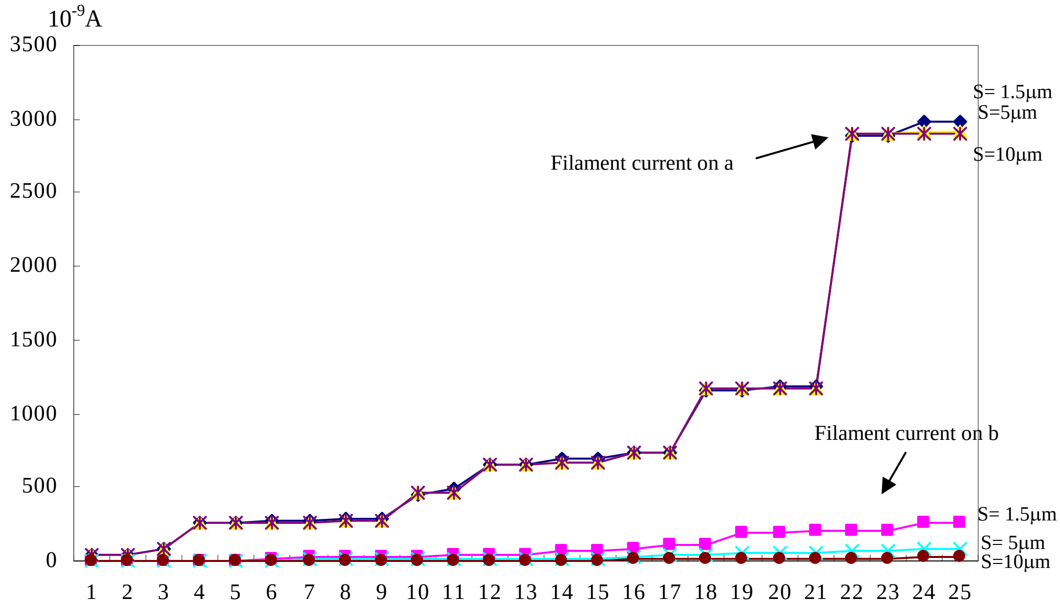

Fig. 1. The distribution of filament currents on a and b when voltage on a is 1 volt and b is open.

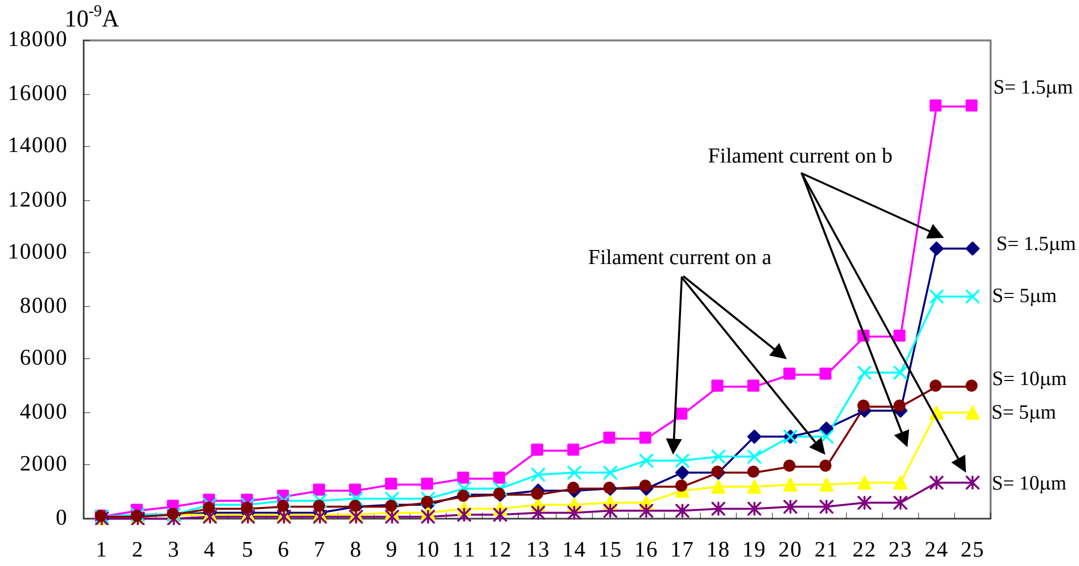

Fig. 2. The distribution of filament currents on a and b when voltage on a is 1 volt and voltage on b is 0.

The above experiments show that we cannot omit much current entries when environment conductors are set shorted in the former accelerated methods based on Fasthenry. Therefore their acceleration effect is limited in order not to lose much accuracy. In the other hand, our experiments show that the filament currents on open conductors can be omitted as zero though the open conductor is near from the aggressor conductor with 1 volt. In the following we will give a new acceleration strategy based on open-circuit assumption.

III. AN FAST EXTRACTION ALGORITHM BASED ON OPEN-CIRCUIT INVESTIGATION

To introduce our fast algorithm, we take a simple structure including only two conductors a and b as example. Conductor a is divided into filaments 1, 2, and 3, and conductor b is divided into filaments 4 and 5. The self and mutual inductances among these filaments can be calculated with analytical formulas. If conductor a is set 1 volt and conductor b is set open, the filament currents I_4 and I_5 on b can be omitted as zero based on our assumption. Then, we can get the filament currents I_1, I_2, and I_3 on conductor a by solving circuit equations about filaments on conductor a without consideration of conductor b.

Next, with the computed I_1, I_2, and I_3 on conductor a and the following equation, we can get the voltages of filaments on conductor b (V_4 and V_5):

$$\begin{pmatrix} Z_{41} & Z_{42} & Z_{43} \\ Z_{51} & Z_{52} & Z_{53} \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} V_4 \\ V_5 \end{pmatrix} \qquad (1)$$

Denoting the impedances between conductor a and filaments 4, 5 are Z_{aa}, Z_{4a}, Z_{5a}, the following equations can be obtained:

$$\begin{pmatrix} Z_{aa} & Z_{a4} & Z_{a5} \\ Z_{4a} & Z_{44} & Z_{45} \\ Z_{5a} & Z_{54} & Z_{55} \end{pmatrix} \begin{pmatrix} I_a \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ V_4 \\ V_5 \end{pmatrix} \qquad (2)$$

where $I_a=I_1+I_2+I_3$ and V_4, V_5 are obtained from (1). Then we can obtain Z_{aa}, Z_{4a}, Z_{5a} from (2).

Then we can set conductor b 1 volt, and conductor a open, the current in a can be omitted according to the same assumption. Equation (2) become the following form:

$$\begin{pmatrix} Z_{aa} & Z_{a4} & Z_{a5} \\ Z_{4a} & Z_{44} & Z_{45} \\ Z_{5a} & Z_{54} & Z_{55} \end{pmatrix} \begin{pmatrix} 0 \\ I_4 \\ I_5 \end{pmatrix} = \begin{pmatrix} V_a \\ 1 \\ 1 \end{pmatrix} \qquad (3)$$

In the other hand, the impedance equation between conductor a and b is:

$$\begin{pmatrix} Z_{aa} & Z_{ab} \\ Z_{ba} & Z_{bb} \end{pmatrix} \begin{pmatrix} 0 \\ I_b \end{pmatrix} = \begin{pmatrix} V_a \\ 1 \end{pmatrix} \qquad (4)$$

the impedance between conductor a and b can be obtained from equation (3) and (4), which has the following form:

$$\begin{pmatrix} \dfrac{1}{I_1+I_2+I_3} & \dfrac{\sum_{p=1}^{3}\sum_{q=4}^{5} I_p I_q Z_{pq}}{(I_1+I_2+I_3)(I_4+I_5)} \\ \dfrac{\sum_{p=1}^{3}\sum_{q=4}^{5} I_p I_q Z_{pq}}{(I_1+I_2+I_3)(I_4+I_5)} & \dfrac{1}{I_4+I_5} \end{pmatrix} \begin{pmatrix} I_a \\ I_b \end{pmatrix} = \begin{pmatrix} V_a \\ V_b \end{pmatrix} \qquad (5)$$

where I_4, I_5 are current on filament 4 and 5 which can be obtained by solving circuit equations on filaments without consideration of conductor a.

The formula can be extended to those structures with n conductors as the following steps:

1. For every conductor, divide it into some filaments, set it 1 volt, solve filament impedance equation and get all filament currents. In the whole procedure, we don't consider the influence of other conductors.
2. Computing the self and mutual inductance between any two conductors according to formula (5)

For a structure with *n* conductors, if every conductor is divided *m* filaments, our method only need to solve *n* equations with the dimension of *m*, whose time complexity is $O(nm^2)$. Then, the self impedances and mutual impedances are calculated with (3). On the other hand, FastHenry need to solve *n* equations whose dimension is $m \times n$, with time complexity of $O(n^3m^2)$. Therefore our method has a nearly linear complexity. To demonstrate the efficiency of our method, a numerical experiment on a six-layered interconnect P/G structure is presented. The operating frequency is 10 GHz and 100 GHz respectively. FastHenry is taken as standard for accuracy evaluation.

IV. NUMERICAL RESULT

We write a program named FEO (Fast Extraction based on Open-circuit) to implement the proposed algorithms. FEO is able to handle complex 3-D interconnect structures for high-frequency impedance extraction. In this section, it will be compared with FastHenry through a bus structure with 270 conductors, and all experiments are carried out on a SUN Ultra V880 Server with frequency 750 MHz.

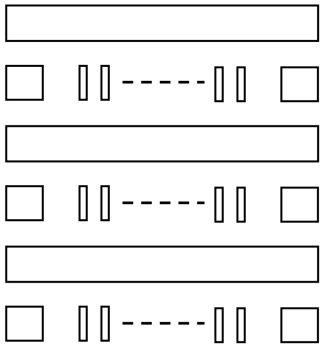

Fig. 3. A multi-layer interconnect structure.

This example is a multi-layer interconnect structure. In each layer, there are 5 P/G lines with size of 2×2×60μm and 10 signal lines with size of 0.4×2×60μm between every two adjacent P/G lines. The space between P/G and signal line is 2μm, the space between signal lines is 0.8μm, and the space between two layers is 4μm. The lines in layer 1, 3, 5 are parallel to X-axis and those in layer 2, 4, 6 are parallel to Y-axis. There are totally 270 conductors distributed in six layers (see Fig. 3 for cross section view). The working frequency is set to 10GHz and 100GHz, and all conductors are partitioned into 4×4 filaments. The numerical results are listed in Table 1 and 2. It is obvious that our method holds good accuracy and is several hundreds of times faster than FastHenry.

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Table 1 Distribution of inductance errors at different frequency.

Error	<0.3%	<0.6%	<0.9%	<1.2%	<1.5%	<1.8%	<2.1%	<2.4%
10GHz	59.68%	28.25%	9.33%	2.48%	0.26%	0.00%	0.00%	0.00%
100GHz	9.82%	17.87%	27.30%	23.96%	12.15%	6.06%	2.35%	0.49%

Table 2 Computational time at 100GHz frequency.

FastHenry	Building filament matrix	Building precondition matrix	Solving equation	Total
	12.14s	1224.75s	11467.22s	12735.7s
Our method	Building filament matrix	Fast $O(n)$ algorithm for inductance extraction		Total
	12.32s	0.46s		12.83s