# Threshold Voltage Model of Single Gate SOI MOSFETs Derived from Asymptotic Method

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***Abstract* - A compact model for threshold voltages of SOI MOSFETs with ultra-thin top Si layer is presented. The model is based on potential distribution solution of the two-dimensional Poisson equation using an asymptotic method. The validity of the model is verified by comparison with results of two-dimensional numerical simulations.**

## I. INTRODUCTION

SOI MOSFETs are attracting a great deal of attention as low voltage, low power and high speed devices. Analytic modeling for SOI MOSFETs is useful for gaining insights into device operation and for device models in circuit simulation. Analytic models of threshold voltage for SOI MOSFETs have been proposed in [1,2,3]. In the present paper we propose a new model which is derived from a solution of 2D Poisson equation obtained analytically using an asymptotic method. The asymptotic method is a novel analysis method in the MOS device modeling field and it is useful to derive analytic models. The presented model is verified by using comparison with numerical simulation results.

## II. MODEL

A cross section of a SOI MOSFET is shown in Fig.1 which defines a top Si Layer thickness $d_s$, a buried oxide thickness $t_{box}$, a channel length $L_c$, a source layer length $L_s$ and a drain layer length $L_d$. We impose the following assumptions. The lateral sizes of the device are much smaller than $t_{box}$, that is, the channel length and others are expressed as $L_c=\mu l_c$, $L_s=\mu l_s$ and $L_d=\mu l_d$ where $\mu$ is a small parameter ($\mu$<<1) , and $l_c$ ,$l_s$ and $l_d$ have the same orders of magnitude as $t_{box}$. The top Si layer thickness $d_s$ is of the order of $\mu^2$, oxide thickness $t_{ox}$ is of the order of $\mu^{1.5}$, and the space charge term in Poisson equation for the region B is of the order of $\mu^{-3.5}$. Potentials in the region B, S, D and O can be obtained by solving Poisson equation using asymptotic method [4]. In the asymptotic method Poisson equation is transformed using scaled variables $x^*$ and $y^*$ ( $x^*=x/\mu^2$ and $y^*=y/\mu$ in B,S and D; whereas $x^*=x$ and $y^*=y/\mu$ in O) and potential $\psi$ is expanded in powers of $\mu^{0.5}$.

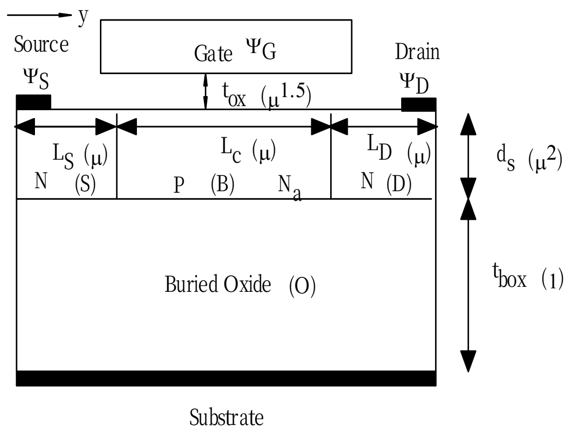


Fig.1 Cross section of SOI MOSFET. Order of sizes are expressed by ($\mu$), etc. ($\mu$ <<1).

Substituting the power series into the scaled Poisson equation and equating coefficients multiplying the same power of $\mu^{0.5}$ we obtain a series of equations. Potential terms in the expansion can be obtained successively. We also used the boundary function method for determining the asymptotic expansion [4].

The lowest order approximation for the potential solution $\psi$ in the region B is given by

$$\psi = \Psi_G - \frac{qN_A t_{ox} d_s}{\varepsilon_{ox}} + \left(\Psi_S - \Psi_G + \frac{qN_A t_{ox} d_s}{\varepsilon_{ox}}\right)\exp\left(-\frac{y-L_S}{\lambda}\right)$$

$$+\left(\Psi_D - \Psi_G + \frac{qN_A t_{ox} d_s}{\varepsilon_{ox}}\right)\exp\left(-\frac{L_S+L_D-y}{\lambda}\right),$$

where $\lambda = \sqrt{\frac{\varepsilon_s t_{ox} d_s}{\varepsilon_{ox}}}$ .

The lowest order approximations for the potential solution $\psi$ in the regions S and D are given by $\psi = \Psi_S$ , and $\psi = \Psi_D$ , respectively.

The threshold voltage $V_{th}$ is obtained from the condition that the minimum of the surface potential in B is equal to twice of the hole Fermi potential $\psi_B$. The expression for $V_{th}$ is

$$V_{th} = V_{FB} + 2\psi_B + \frac{qN_A t_{ox} d_s}{\varepsilon_{ox}}$$

$$-2\sqrt{(\Psi_S - 2\psi_B)(\Psi_D - 2\psi_B)} \exp\left(-\frac{L_c}{2\lambda}\right)$$

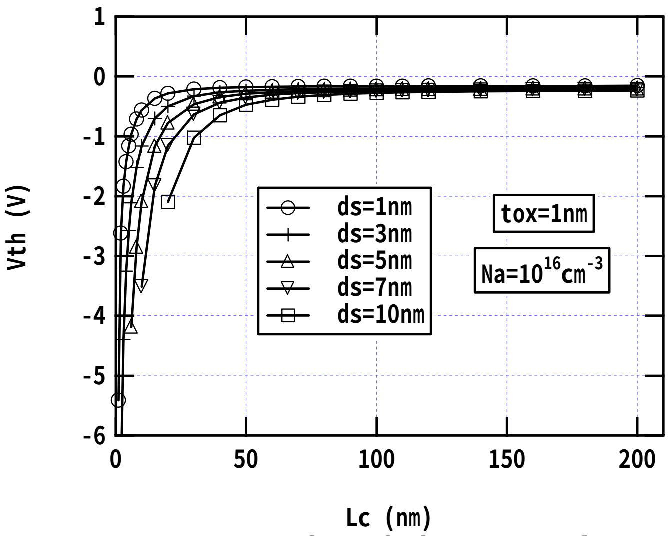


Fig.2 $V_{th}$-$L_c$ curves obtained by numerical simulation.

## III. COMPARISON WITH NUMERICAL SIMULATION

The present model is derived under the assumptions mentioned above. Now we examine how well it agrees with two-dimensional numerical results from a simulator. We calculated $V_{th}$-$L_c$ characteristics for cases with $t_{ox}$, $d_s$ =1,3,5,7,10nm and $N_A$=$10^{16}$,$10^{17}$,$10^{18}$ cm$^{-3}$ (that is, 5x5x3=75 cases). We don't stick to the assumptions used in the derivation, expecting agreement in wider range of parameters. Threshold voltage is defined as a gate voltage for which the drain current is equal to $10^{-11}W/Lc$ [A] , where $W$ is the channel width. An example of $V_{th}$-$L_c$ characteristics is shown in Fig.2.

We tried to fit these curves with an expression $V_{th} = V_{th0} - K_1 \exp(-L_c / K_2)$ with fitting parameters $V_{th0}$, $K_1$ and $K_2$. The short channel effect is determined by parameters $K_1$ and $K_2$. To extract these parameters properly, an expression

$$\log \frac{dV_{th}}{dL_c} = \log \frac{K_1}{K_2} - \frac{L_c}{K_2}$$

is used. $\log(dV_{th} / dL_c)$ vs. $L_c$ relation is linear and therefore the regression analysis to obtain $K_2$ is efficiently carried out.

Examples of fitting using regression analysis are shown in Fig.3. Set of parameters $V_{th0}$, $K_1$ and $K_2$ were obtained for all the devices. The parameter $K_2$ corresponds to twice of characteristic length $\lambda$ [1], [2].

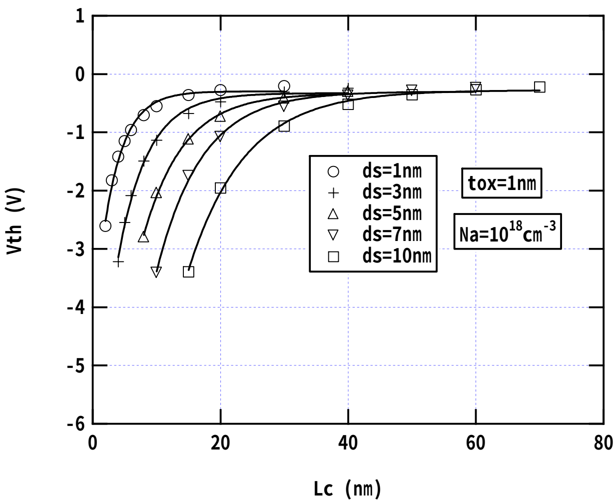


Fig.3 Regression analysis to obtain parameters $V_{th0}$, $K_1$ and $K_2$.

For $d_s$ =1nm, $K_2$ agrees very well with 2 $\lambda$ as a function of $t_{ox}$ as shown in Fig.4. This is reasonable considering that $d_s$ is assumed to be very small, of the order of $\mu^2$, in the derivation of the model. However, the values of $K_2$ for $t_{ox}$ and $d_s$ larger than 3 nm does not agree well with 2 $\lambda$. In Fig.4 , the extracted values of $K_2$ are compared with 2 λ multiplied by a fitting parameter 2/3.

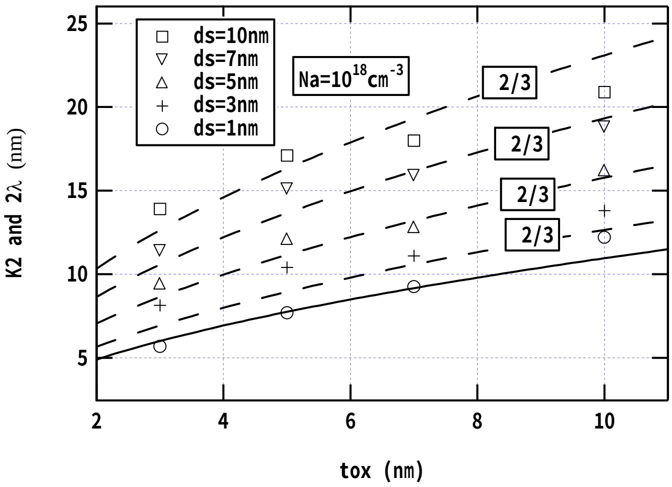


Fig.4 Comparison of $K_2$ with $2\lambda$ and 2 $\lambda$ *2/3

($\lambda$: characteristic length).

Thus, the threshold voltage for devices with $t_{ox}$ and $d_s$ larger than 3 nm is expressed by

$$V_{th} = V_{th0} - K_1 \exp(-3L_c / 4\lambda) .$$

Extracted values of $V_{th0}$ are shown in Fig.5. They align well with the lines expressing the relation obtained above

$$V_{th0} = const. + qN_A t_{ox} d_s / \varepsilon_{ox} .$$

Threshold voltages numerically obtained and the model $V_{th} = V_{th0} - K_1 \exp(-3L_c / 4\lambda)$ are compared in Fig.6. Agreement between them is satisfactory.

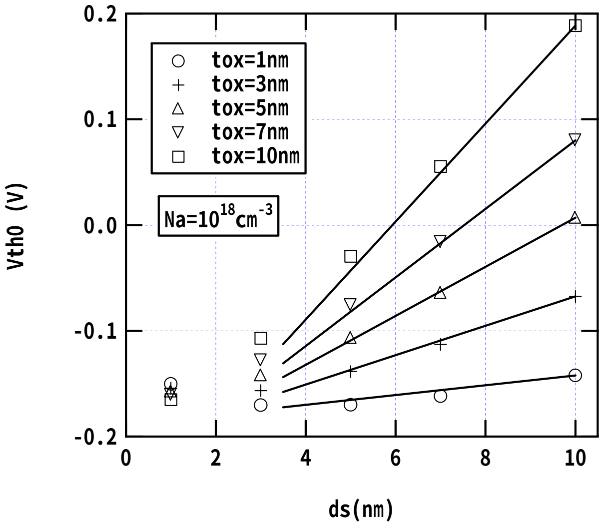


Fig.5 $V_{th0}$ showing dependence on $qN_a t_{ox} d_s / \varepsilon_{ox}$ .

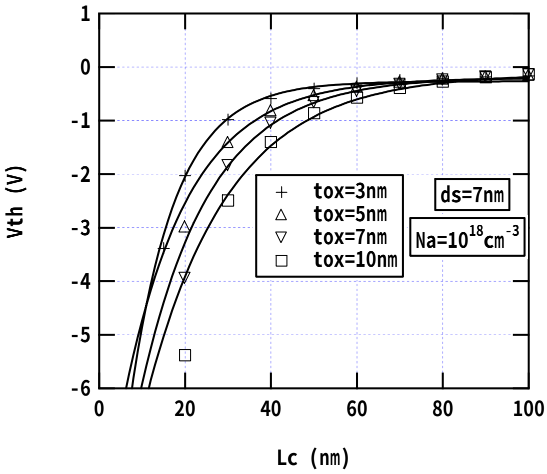


Fig.6 Comparison between the compact model and numerical results.

## IV. CONCLUSIONS

The asymptotic analysis with the boundary function method is applied to the MOS device modeling. Potential distributions in SOI MOSFETs with very thin Si top layers are obtained analytically by the asymptotic method, and the compact model of threshold voltage is derived. The model is verified through the comparison with numerical results.

## ACKNOWLEDGMENT

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