A Highly Efficient Statistical Compact Model Parameter Extraction Scheme

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ABSTRACT

A new method of determining statistical compact model parameters is proposed. The variations of measured device characteristics can be efficiently translated into the variations of a set of transistor model parameters. Since the target of fitting is not the *I-V* curves of individual samples, but the statistically analyzed results of the *I-V* data, the extraction is fulfilled in only one optimization step. The method is applicable to any compact model platforms. Therefore, high accuracy and efficiency can be achieved at the same time.

I. INTRODUCTION

As the feature sizes of LSIs are scaled down, the impact of variations is becoming serious for both analog and digital circuit design. Therefore, statistical compact modeling, where some of the compact model parameters are treated as statistical variables to simulate the device variations, is becoming increasingly important. The statistics of the compact model parameters, required for the modeling, can be determined by performing parameter extraction procedure for many individual sample devices, then statistically analyzing the resulting sets of the parameters obtained [1-6]. However, this method tends to be time-consuming, due to the repeated extraction steps required. Though this drawback can be alleviated by simplifying the compact models or extraction procedure, the accuracy will be degraded depending on the degree of the simplification.

In this presentation, a totally different approach is proposed, which realizes highly efficient statistical compact model parameter extraction.

II. EXTRACTION METHOD

The concept of the proposed "direct fitting" method is shown in Fig.1. While the conventional straightforward method requires *N* times parameter extraction procedure (*N* is the number of samples, as large as 100 or 1000 or more), only one fitting step is necessary for the new method. Therefore, it is quite fast and efficient. Statistical analysis is done *before*, not after, the fitting. That is, measured data (a set of measurable quantities, such as current vs voltage sweep data, I_{ON} vs V_{TH} data, etc) are subject to principal component analysis (PCA). PCA decomposes the variations into mutually uncorrelated "principal components (PCs)". An example of PCA for a two-dimensional case is shown in Fig.2. The target of the fitting is not the raw measured data, but the major PCs (a group of PCs with the largest variance).

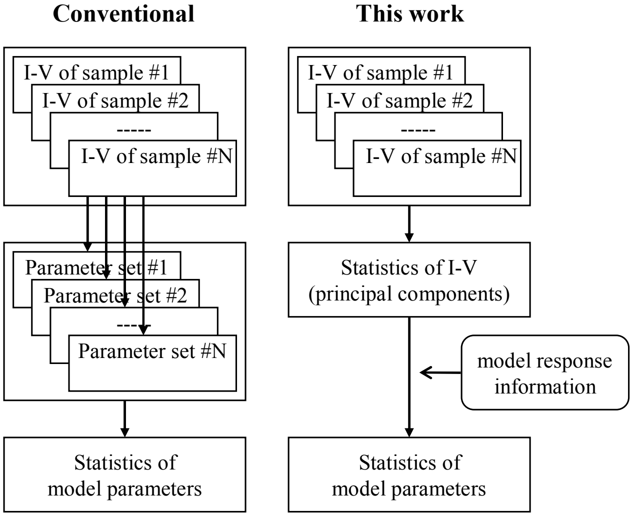

Fig.1 Statistical parameter extraction flows. Proposed method can determine the statistics of model parameters by one-step fitting. Statistical analysis is done before the fitting.

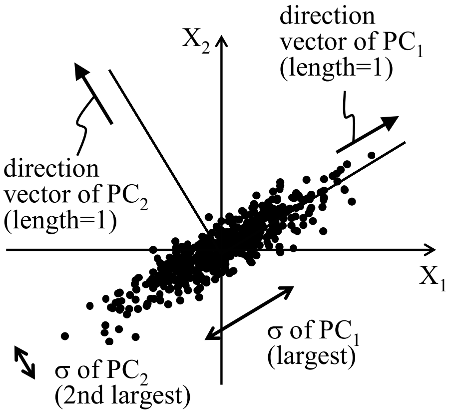

Fig.2 Explanation of principal component analysis. Statistical variables X_1 and X_2 are linearly transformed into PC_1 and PC_2, which are mutually uncorrelated. PCs are sorted and numbered in the order of σ. Higher PCs are similarly defined for higher dimensions.

The fitting is performed by using "model response information", which is given by a response matrix

$$R \equiv \begin{pmatrix} \dfrac{\partial x}{\partial p_1} & \dfrac{\partial x}{\partial p_2} & \cdots & \dfrac{\partial x}{\partial p_m} \end{pmatrix}, \quad x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}, \tag{1}$$

where x_i is the simulated data at the i-th bias point, and p_j the j-th model parameter. The simulated data and bias points should correspond to the measured data. The differentiation is done around the center (typical) model prepared in advance. Once the response matrix is obtained, the fitting can be achieved as follows:

(Step 1) Assume standard deviation values σ_j for all p_j.

(Step 2) Calculate trial matrix

$$A = R \begin{pmatrix} \sigma_1 & 0 & \cdots & 0 \\ 0 & \sigma_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \sigma_m \end{pmatrix}. \tag{2}$$

(Step 3) Apply singular value decomposition to A as

$$\begin{aligned} A &= USV^T \\ &= \begin{pmatrix} u_{11} & u_{12} & \cdots & u_{1m} \\ u_{21} & u_{22} & \cdots & u_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ u_{n1} & u_{n2} & \cdots & u_{nm} \end{pmatrix} \\ &\times \begin{pmatrix} s_1 & 0 & \cdots & 0 \\ 0 & s_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & s_m \end{pmatrix} \begin{pmatrix} v_{11} & v_{21} & \cdots & v_{m1} \\ v_{12} & v_{22} & \cdots & v_{m2} \\ \vdots & \vdots & \ddots & \vdots \\ v_{1m} & v_{m2} & \cdots & v_{mm} \end{pmatrix}. \end{aligned} \tag{3}$$

(Step 4) Compare the k-th column vector of U with direction vector of PC$_k$, s_k with standard deviation σ of PC$_k$, for a desired range of k (e.g. 1 to 3).

(Step 5) Repeat (Step 1) to (Step 4) until sufficient match is achieved.

The validity of the above procedure can be understood by considering that *Var(x)*, the variance matrix of x, can be expressed as

$$\begin{aligned} Var(x) &= \overline{(x-\bar{x})(x-\bar{x})^T} \\ &\cong AA^T = USV^T VSU^T = US^2U^T \end{aligned} \tag{4}$$

and hence,

$$Var(x)U = US^2. \tag{5}$$

Eq.5 shows that the eigenvalue decomposition of *Var(x)* (principal component analysis of x) directly corresponds to singular value decomposition of A. In the above equations, it is assumed that the model parameters p_j are uncorrelated with each other. However, this restriction can be easily eliminated, by assuming that p_j are functions of independent statistical variables, instead of assuming p_j themselves are independent variables. The details of this extension will be reported elsewhere.

In addition to the fast single step fitting, there are other advantages with the proposed method. As described above, there is no need to know the detailed internal structures of the compact models. Therefore, the method can be combined with any compact model platforms (BSIM, HiSIM, etc). Existing center models can be used without any modification. PCA tells which component of the variations is the most important, second important, and so on. Since the PCA results are known before the fitting, one can control the accuracy (or complexity) of the fitting, by simply choosing the number of PCs to evaluate in Step 4. Due to the high efficiency, trial and error approach is not so costly. Therefore, it is possible to choose an appropriate set of compact model parameters out of a wide range of possible combinations.

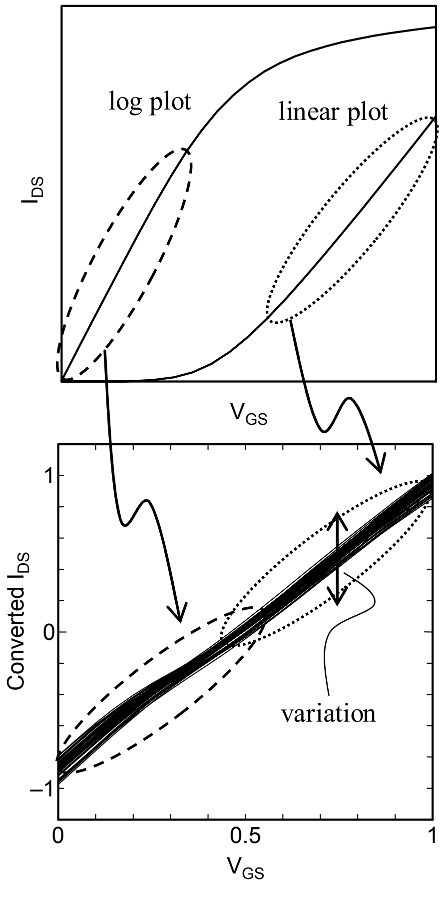

Fig.3 Conversion of I_{DS} prior to PCA by Eq.6.

III. EXTRACTION RESULTS

The method was applied to the variations of MOSFET current vs voltage (*I-V*) characteristics. Drain current I_{DS} of several n-channel 90nm node FETs was measured across a wafer, sweeping gate voltage V_{GS} and also varying drain and substrate voltages V_{DS} and V_{BS}. I_{DS} at each of 150 bias points is considered a statistical variable here. Before PCA, the acquired data should be appropriately weighted. Here, I_{DS} was converted using the following equation.

$$I_{DS,CONVERTED} = \frac{I_{DS}}{a} + \frac{\log(I_{DS}/a)}{\log(a/b)}, \qquad (6)$$

where a and b are constants. Eq.6 transforms the *I-V* curves into a near linear form, from sub-threshold to strong inversion, and the magnitude of the variation becomes nearly constant over the bias range, as shown in Fig.3. This was intended to weight both ON and OFF bias ranges equally. Without this weighting, the variations in the subthreshold region will be ignored, since the off-state I_{DS} is orders of magnitude smaller than that in the on-state. By selecting a weighting function, the policy of the fitting can be changed as desired.

The PCA results for the measured and converted I_{DS} are shown in Figs.4 and 5 (labeled "measured"). In Fig.4, direction vector elements for the first and second PCs (PC_1 and PC_2) are plotted (marks). The dimension of the vectors is 150, which is equal to the number of the bias points. The near periodic appearance of the plots is due to arranging V_{GS} sweep data for different combinations of V_{DS} and V_{BS} in a single row. Note that the curves in Fig.4 directly correspond to the shapes of the *I-V* shift caused by each PC. This is explained in Fig.6, where the magnified view of the shaded region in Fig.4, and its relation ship with the *I-V* characteristics is shown. PC_1 is the variation component that shifts both ON and OFF current in the same direction, whereas PC_2 opposite direction. In Fig.5, the length of the vectors (i.e. standard deviation σ of PCs) is shown for PC_1 to PC_7 (filled bars). It can be seen that the third and higher PCs are relatively small. It is possible to simplify the statistical modeling (i.e. reduce the number of statistical variables) by ignoring high order PCs, while minimizing the loss of information.

Statistical parameter extraction for the measured data was performed by the proposed direct fitting method. Rather surprisingly, by using only two BSIM parameters (L and V_{OFF}) as variables, PC_1 and PC_2 could be reasonably fitted, as shown in Figs.4 (solid lines) and 5 (gray bars). Since only two statistical variables are involved here, PC_3 and higher are automatically ignored, but its impact on the accuracy is expected to be small. Since the plots in Fig.4 directly correspond to the *I-V* shift (Fig.6), the good match in Figs.4 and 5 guarantees that the statistical compact model simulation using the fitted results will reproduce the detailed shapes of the measured scatter of *I-V* curves for all the bias range under consideration.

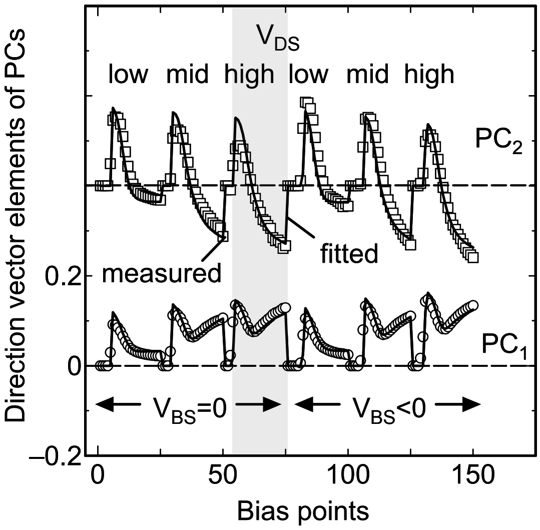

Fig.4 Direction vector elements of first and second principal components (PC_1 and PC_2). Measured plots (marks) are obtained by PCA of measured data. Fitted plots (curves) are obtained by singular value decomposition of trial matrix A. PC_2 is shifted upward for clarity.

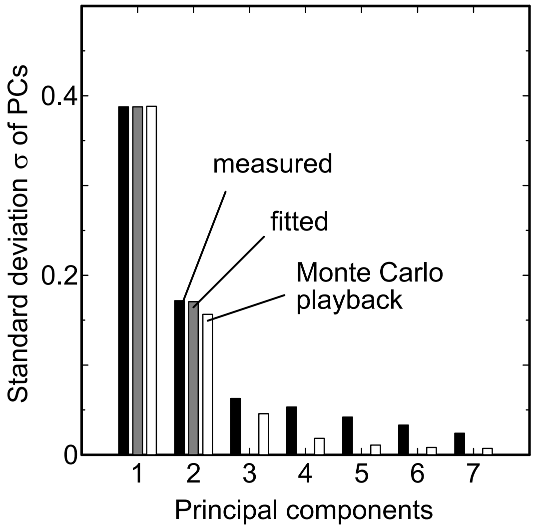

Fig.5 Standard deviation σ from 1st to 7th principal components (PC_1 to PC_7). Measured plots (filled bars) are obtained by PCA of measured *I-V* data. Fitted plots (gray bars) are obtained by singular value decomposition of trial matrix A. Monte Carlo playback plots (open bars) are obtained by PCA of Monte Carlo generated *I-V* data. The first two correspond to Fig.4, and the last to Fig.7.

To further confirm the validity of the parameter extraction scheme, verification by Monte Carlo playback was performed. That is, 100 sets of *I-V* data (the same bias conditions as the measurement) were generated by circuit simulation, randomly varying L and V_{OFF} as determined by the fitting, and then PCA was applied to the obtained simulated *I-V*. The results are shown in Figs.7

(marks) and 5 (open bars). It was confirmed that the Monte Carlo simulation actually reproduce the measured variations. Fig.8 compares the measured and Monte Carlo playback I_{ON} vs I_{OFF} characteristics. Good agreement is confirmed again.

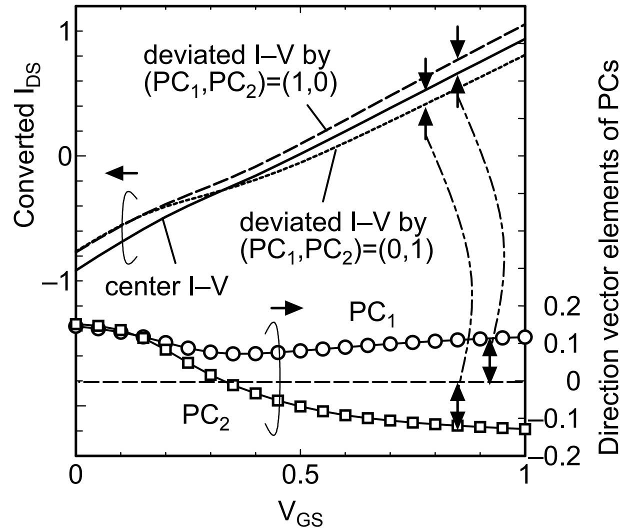

Fig.6 Relationship between PC vectors and *I-V* shift. Fitted PC_1 and PC_2 elements in the shaded region of Fig.4 is plotted here, together with the *I-V* curves for the same bias conditions. Bias number (horizontal axis of Fig.4) is translated into V_{GS}. PC direction vectors directly correspond to the *I-V* shift by each principal components.

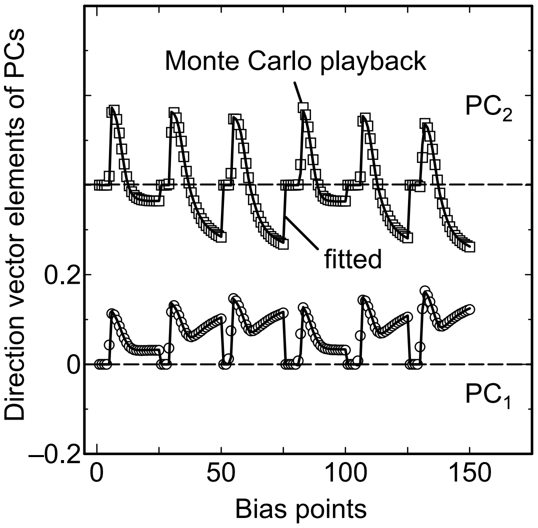

Fig.7 Comparison of direction vector elements for verification. Fitted plots (curves) are obtained by singular value decomposition of trial matrix A (same as Fig.4). Monte Carlo playback plots (marks) are obtained by PCA of Monte Carlo generated *I-V* data. Corresponding results for σ are shown in Fig.5.

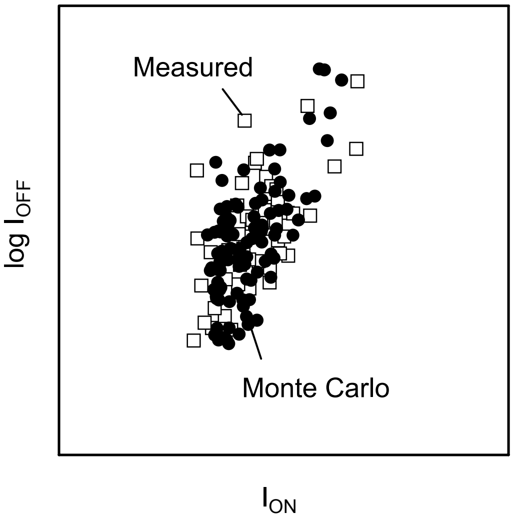

Fig.8 Monte Carlo playback of I_{ON} vs I_{OFF}, as compared with the measured data.

IV. CONCLUSION

A highly efficient method for statistical compact model parameter extraction has been proposed. The measured *I-V* variations were successfully reproduced, with minimal extraction effort. Due to the efficiency and flexibility, the method will enhance the usefulness of statistical compact modeling, and help realizing highly optimized circuit design against variations.

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