RESONANCE QUANTUM SWITCH : SEARCH OF WORKING PARAMETERS

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Abstract **Design of a three-terminal Quantum Switch is suggested in form of a network consisting of a circular quantum well and a four quantum wires attached to it. The conditions of functioning are defined in dependence of the desired working temperature, Fermi level and effective mass of an electron. The speed of switching is estimated.**

1 Introduction

The interference of wave functions is intensely used for manipulation of quantum current since eighties, see for instance [2, 3, 4]. The Resonance Quantum Switch in [5, 9, 6, 11] was suggested as a quantum network Ω ("fattening graph", see [7, 8]) constructed on the surface of a semiconductor of a circular quantum well Ω_0 (vertex domain)radius R and quantum wires Ω_1, Ω_2, Ω_3, Ω_4 width δ attached to it centering at the points a_s, $s = 1, 2, 3, 4$ on the boundary of the well $\partial\Omega_0$. The wire Ω_1 is selected for the input, others are terminals. The quantum wires are connected to the well directly or via tunnelling under potential barriers situated on the contact zone $(-l, 0)$ of the wires $(-l, \infty)$ (split-gates). The width l of the barrier may vary from zero to few nano-meters. The height of the barrier over the Fermi-level and the depth of the channel may vary from zero to few eV, see [10].The effective potential $V_\infty + \frac{\hbar^2\pi^2}{2m^\perp\delta^2}$ in the wires at infinity is choosen such that the resonance energy level $E_0 = E_f$ is in the middle of the first spectral band.

We neglect scattering on impurities both in the quantum well and on the wires assuming that the dynamics of electrons is ballistic and single-mode on large intervals of the wires.The quantum dynamics of a single electron is described by the Schrödinger equation with the linear potential $V(x)$ and the average effective mass m inside the well. The potential on the wires is piecewise constant or constant,but the tensor of effective mass is non-trivial : the effective mass across the wire $m^\perp$ and along the wire $m^\parallel$ are different [1]. When assuming that the radius of the vertex domain is *small for given temperature* , we have in mind that the spacing $\rho_0^R(E_f) = \min_{s\neq 0}|E_s - E_0|$ of energy levels in the vertex domain Ω_0 radius R at the resonance energy $E_0 = E_f$ (with wires disjoint from the domain) is large compared with temperature:

$$\kappa T < \frac{1}{2}\inf_{E_s\neq E_f}|E_f - E_s| = \rho(E_f). \qquad (1)$$

We assume that the inverse "re-normalized" spacing is estimated by the distance of the Fermi-level from the second threshold :

$$\frac{1}{\rho^1} := \frac{\hbar^2}{2m_0R^2\rho^R(E_f)} < \frac{R}{\delta}\ \pi\sqrt{\frac{3}{2}\ \frac{m^\parallel}{m^\perp}}. \qquad (2)$$

The non-dimensional spacing on the resonance level for the circular quantum well radius 1 is 2.3 so that the above condition (2) is fulfilled if the width δ of the wires and the radius R of the well are connected by the condition $\delta\ R^{-1} < 8.5\sqrt{m^\parallel\ (m^\perp)^{-1}}$, for instance if $\delta \leq \frac{R}{2}$.

2 Scattering and resonances

We consider a single act of an electron transmission from the incoming wire Ω_1 to one of terminals Ω_s, $s = 1, \ldots$ as a scattering process in the network Ω. The Schrödinger equation on the wires admits separation of variables $u = \sum_{l=1}^\infty u_l(x)\mathbf{e}_l(y)$ when using the expansion over eigenfunctions of the cross-sections $\mathbf{e}_l(y) = \sqrt{\frac{2}{\delta}}\sin\frac{\pi l y}{\delta}$, $0 < y < \delta$, . Denoting by $V_1(x)$ the potential on the wire

$$V_1(x) = \begin{cases} V_{barrier} & \text{if} \quad (-l < x < 0) \\ V_\infty & \text{if} \quad (0 < x < \infty), \end{cases}$$

we may present the equations for the amplitudes u_l, $l = 1, 2, \ldots$ as

$$-\frac{d^2u_l}{dx^2} + \frac{2m^\parallel}{\hbar^2}\left[V_1(x) - V_\infty\right]u_l =$$

$$\left[\frac{m^\parallel}{m_0}\lambda - \frac{m^\parallel}{m^\perp}\left(\frac{\pi^2l^2}{\delta^2} - \frac{\pi^2}{\delta^2}\right)\right]u_l,\ l = 1, 2, \ldots. \qquad (3)$$

[1]On the lower valleys on (100) plane in Si $m^\perp = 0.190$, $m^\parallel = 0.916$

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with the positive spectral parameter $\lambda = p^2 = \frac{2m_0}{\hbar^2}\left[E - V_\infty - \frac{\hbar^2}{2m^\perp}\frac{\pi^2}{\delta^2}\right]$. Assuming that the effective mass of electron inside the well coincides with conventional electron mass $m = m_0$ we present the spectral problem on the whole network by combination of the equation (3) and

$$-\triangle u_0 + \frac{2m_0}{\hbar^2}\left[\mathcal{E}e\langle x,\nu\rangle + V_0 - V_\infty - \frac{\hbar^2}{2m^\perp}\frac{\pi^2}{\delta^2}\right]u_0 =$$

$$\frac{2m_0}{\hbar^2}\left[E - V_\infty - \frac{\hbar^2}{2m^\perp}\frac{\pi^2}{\delta^2}\right]u_0 = \lambda u_0, \qquad (4)$$

with the matching boundary conditions:

$$[u-u_0]\Big|_{\Gamma_s} = 0,\ \left[\frac{1}{m^\parallel}\frac{\partial u}{\partial n} - \frac{1}{m_0}\frac{\partial u_0}{\partial n}\right]\Big|_{\Gamma_s} = 0. \qquad (5)$$

The *intermediate* perturbed operator l^r is defined by the above Schrödinger equations on the whole network with partial Dirichlet boundary condition *chopping the first channel off*:

$$P_+ u = 0, \qquad (6)$$

and partial matching conditions in all closed channels. Removing the trivial component of the intermediate operator in the first channel we consider the orthogonal component l^r. The continuous spectrum of l^r begins from the second threshold $\frac{\hbar^2}{2m^\perp}\frac{4\pi^2}{\delta^2}$. The eigenvalues $E^r_s = \frac{\hbar^2}{2m_0}\lambda^r_s$ of it below the second threshold play a role of resonances .

The 4-dim branch of scattered waves of the original spectral problem corresponds to the components combined of Jost solutions of the equation (3) in the first channel with compactly-supported potential $[V_1(x) - V_\infty] = 0, x > 0$ and $[V_1(x) - V_\infty] = H\frac{\hbar^2}{2m_0},\ -l < x < 0$:

$$-\frac{d^2u_1}{dx^2} + \frac{2m^\parallel}{\hbar^2}\left(V_1(x) - V_\infty - \frac{\hbar^2}{2m^\perp}\ \frac{\pi^2}{\delta^2}\right)u_1 = \frac{m^\parallel}{m_0}p^2u_1,$$

$$u_1(x) = e^{-i\sqrt{\frac{m^\parallel}{m_0}}px}\nu + e^{i\sqrt{\frac{m^\parallel}{m_0}}px}S_1\nu,\ \ x > 0,\ \nu \in E_+, \qquad (7)$$

and is accomplished with exponentially decreasing components in upper channels $l \geq 2$:

$$u_l(x) = S_l\ \nu\ e^{-\sqrt{\left(\frac{\pi^2 l^2}{\delta^2} - \frac{\pi^2}{\delta^2}\right) - p^2}\ x},\ x > 0,\ l > 1.$$

The Scattering Matrix S_1 and the amplitudes S_l in upper channels are defined from the matching condition of u_1 to the solutions of the corresponding homogeneous equation inside the well. The two-dimensional matching problem may be solved with use of the Dirichlet-to-Neumann map (DN-map) Λ^r of the operator l^r, see Appendix below and [17, 20]. Then , introducing the notation

$$\frac{m_0}{m^\parallel}\frac{-i\sqrt{\frac{m^\parallel}{m_0}}p - \sqrt{H^2 - \frac{m^\parallel}{m_0}p^2}\tanh\sqrt{H^2 - \frac{m^\parallel}{m_0}p^2}l}{i\sqrt{\frac{m^\parallel}{m_0}}p\frac{\tanh\sqrt{H^2 - \frac{m^\parallel}{m_0}p^2}l}{\sqrt{H^2 - \frac{m^\parallel}{m_0}p^2}}\ + 1} = \mathcal{D}$$

we obtain :

$$S_1(p) = -\frac{P_+\Lambda^r P_+ - P_+\ \mathcal{D}}{P_+\Lambda^r P_+ - P_+\ \overline{\mathcal{D}}}\times$$

$$\frac{i\sqrt{\frac{m^\parallel}{m_0}}\ p\ \tanh\sqrt{H^2 - \frac{m^\parallel}{m_0}p^2}\ l\ +\ \sqrt{H^2 - \frac{m^\parallel}{m_0}p^2}}{-i\sqrt{\frac{m^\parallel}{m_0}}\ p\ \tanh\sqrt{H^2 - \frac{m^\parallel}{m_0}p^2}\ l\ +\ \sqrt{H^2 - \frac{m^\parallel}{m_0}p^2}} \qquad (8)$$

We estimate the speed of transition processes by the width of the corresponding resonance, see [19] $\lambda_s = p_s^2$ for the quantum well radius $250A$ in two cases:

1. In case when equivalent split-gates are constructed on the initial part of each wire $-l < x < 1$, we assume that the barrier is $1eV$ over the Fermi level E_f which is 1 eV over the effective bottom $V_\infty + \frac{\hbar^2}{2m^\perp}$ in the wire. The width δ of the wire and the width l of the barrier both are 2 nm [10] Neglecting the contribution to DN-map from the neighboring (non-resonance) eigenvalues compared with the momentum we obtain an approximate position of the solution $\hat{\lambda}^r$ of the previous equation near to the resonance eigenvalue $\hat{\lambda}^r_0$:

$$\hat{\lambda} \approx \hat{\lambda}^r_0 - 0.11 + \ 0.14i\,\varepsilon$$

with $\varepsilon = 3.45\ 10^{-8}$. Then the life-time is calculated as

$$\tau_0 = 4.08\, m_0\, (\hbar)^{-1}\, R^2\, 10^8 = 7.6\ 10^{-3} sec.$$

2. In case when the split-gates at the entrances to the wires are absent, $l = 0$, we also assume that the contribution to DN-map from the neighboring non-resonance eigenvalues is dominated by momentum. Then the non-dimensional resonance may be calculated as

$$\hat{\lambda} = \hat{\lambda}^r_0 - 5.5\ 10^{-3} + i\ 0.11$$

and the life-time of the resonance is found as $\tau_0 = 10\pi s$.

In both cases we neglected the contribution to DN-map from the non-resonance eigenvalues, using actually the *one-pole approximation* for expressions staying in the numerator and denominator of the Scattering matrix. In particular, in the second case we have the following "one-pole approximation " for the Scattering matrix on the first spectral band

$$S(\lambda) \approx -\frac{\frac{P_+\frac{\partial\varphi^r_0}{\partial n}\rangle\langle P_+\frac{\partial\varphi^r_0}{\partial n}}{\lambda - \lambda_0} + i\sqrt{\frac{m^\parallel}{m_0}}pI}{\frac{P_+\frac{\partial\varphi^r_0}{\partial n}\rangle\langle P_+\frac{\partial\varphi^r_0}{\partial n}}{\lambda - \lambda_0} - i\sqrt{\frac{m^\parallel}{m_0}}pI}, \quad \lambda = p^2 \qquad (9)$$

which actually is a Scattering matrix for the corresponding "zero-range potential with inner structure" similar to one discussed in [13].

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3 Switching phenomenon and working point

We see from (9) the transmission of the electron's waves across the vertex domain is implemented via excitation of the resonance mode inside the vertex domain Ω_0. The transmission coefficients are defined by the portions $\phi_0^r = \frac{\partial \varphi_0^r}{\partial n}$ of the resonance eigenfunction on the bottom sections Γ_s of the wires.

We use the non-dimensional Schrödinger equation obtained from the original equation by scaling $x = R\xi$ in the geometrical form (4) of the Schrödinger equation for the values of energy on the first spectral band :

$$-\triangle_\xi u + \frac{2m_0 e\mathcal{E}R^3}{\hbar^2}\langle\vec{\xi},\,\vec{\nu}\rangle u + \frac{2m_0R^2}{\hbar^2}\left[V_0 - V_\infty - \frac{\hbar^2}{2m^\perp}\left(\frac{\pi}{\delta}\right)^2\right]u = \hat{p}^2 = \hat{\lambda}u. \quad (10)$$

Choosing $\varepsilon = \frac{2m_0\,|e|\mathcal{E}R^3}{\hbar^2} = 18.86$ one may see that the eigenfunction corresponding to the second lowest eigenvalue $\hat{\lambda}_0 = 14.62$ of the even series of eigenfunctions of the Dirichlet problem the zeroes of the normal derivative on the boundary divide the boundary in ratio 1 : 2. The portion $\phi_0^r = P_+\frac{\partial\varphi_0^r}{\partial n}$ of the eigenfunction of the intermediate operator coincides, up to small error, with the corresponding portion of the eigenfunction of the Dirichlet problem. The relative shift of the corresponding non-dimensional eigenvalues is estimated (by Doctor V. Oleinik, with variational method) as $\hat{\Delta}_0 = \hat{\lambda}_0^r - \hat{\lambda}_0 = 14.55 - 14.62 = -0.07$.In this section we do not distinguish these eigenvalues.The resonance portion was computed (by Mr. Kieran Robert) for the potential defined by the vector ν directed to the point a_1 as :

$$\hat{\phi}_{0+} = (1,\,0.1,\,3,\,0.1), \quad (11)$$

hence $\hat{\alpha}^2 = |\hat{\phi}_0|^2 \approx 10$. Then the transmission coefficients may be calculated from (9) as : $|T_{12}| = |T_{14}| = 0.02$, $|T_{13}| = 0.6$ so that the ratio of the amplitudes of the signal in closed $\Omega_{2,4}$ and open $\Omega_{1,4}$ wires is $\approx 1:30$ and the conductance from the input wire to the open channel Ω_3 may be obtained from the Landauer formula, $\sigma_{13} \approx e^2\,h^{-1}0.36$,see [1], for spin-polarized electrons. For non-polarized electrons the result should be doubled. Due to the symmetry of the domain the resonance eigenfunction is rotated together with vector ν, blocking different pairs of entrances.

The working regime of the switch is stable if the bound states in the well corresponding to the neighboring eigenvalues are not excited at the temperature T as:

$$R^2 \le \frac{2.3}{2\kappa T}\frac{\hbar^2}{2m}, \quad \delta < \frac{R}{2} \quad (12)$$

Then the shift potential V_0 on the well Ω_0 may be defined from the condition $2m\,R^2[E_f - V_0 + V_\infty + \frac{\hbar^2}{2m^\perp}\,\frac{\pi^2}{\delta^2}] = \hbar^2\,\hat{\lambda}_0$, and the electric field $\mathcal{E}$ may be found from the condition $\varepsilon\hbar^2 = 18.86\hbar^2 = e\mathcal{E}\,2m\,R^3$, where e is the absolute value of the electron charge. The magnitude of the governing electric field for the quantum well radius $R = 1000A$ is $38\ 10^{-6}$ v/mkm which is not yet greater than admissible level 10^{-2}v/mkm of an accelerating electric field in Si. For the quantum well radius $R = 100A$ the governing electric field is already $38\ 10^2$ v/mkm $> 10^{-2}$ v/mkm.

Calculation of the radius of the quantum well (in Angstroms) based on the previous estimation (12) of the radius for different materials gives the following results,[16, 14, 15]:

Material	R_{300K}	R_{77K}	$R_{4.2K}$
$Cd_{0.15}Hg_{0.85}Te$	160	310	1350
InSb	110	230	930
InAs	90	170	760
GaAs	50	100	420
Si	10	25	85

For Si the local values of the mass may essentially deviate from the average value 0.8, depending on local positions of valleys. For quantum wells with $R > 100A$ one may describe the electron's dynamics on the well by the Schrödinger operator with the average mass, since the period of the Si lattice is $5.43A$.

4 Appendix

For the Schrödinger equation in geometric form the DN-map is a map of the boundary values u_Γ of the solution

$$-\triangle u + Vu = \lambda u,\ u|_{\partial\Omega_0} = u_\Gamma$$

on the border $\partial\Omega_0 = \Gamma$ of the domain into the boundary values of it's normal derivative:

$$\Lambda : u_\Gamma \to \frac{\partial u}{\partial n}|_{\partial\Omega_0}.$$

For the boundary condition $u\big|_\Gamma = u_\Gamma$ we obtain the solution of $Lu = \lambda u$ as a re-normalized double-layer potential : $u(x) = \int_\Gamma \mathcal{P}_\lambda(x,\,s,\,\lambda)u_\Gamma(s)d\Gamma$. The DN-map is represented for regular points λ of L_D as a generalized integral operator with a singular kernel:

$$[\Lambda(\lambda)u_\Gamma\,](x_\Gamma) = \frac{\partial}{\partial n}|_{x=x_\Gamma}\int_{\partial\Omega}\mathcal{P}_\lambda(x,\,s,\,\lambda)u_\Gamma d\Gamma. \quad (13)$$

The DN-map for the intermediate operator is defined for given boundary data $u_\gamma \in E_+$ as a projection $\Lambda^r = P_+\frac{\partial u}{\partial n}$, of the normal derivative of the solution $u : lu - \hat{\lambda}u = 0$ on the whole network which is properly decreasing at infinity in all upper channels

Theorem 4.1 *The Scattering Matrix on the whole network Ω may be presented in terms of the DN map Λ of the vertex domain Ω_0 as*

$$S^1(\lambda) = -\frac{\Lambda^r - \bar{K}^+}{\Lambda^r - K^+}. \quad (14)$$

A special combination of matrix elements of the matrix representation of Λ *with respect to the orthogonal decomposition* $I = P_+ \oplus P_-$*:*

$$\Lambda^r = \Lambda_{++} - \Lambda_{+-} \left[K^{-} + \Lambda_{--} \right]^{-1} \Lambda_{-+} \qquad (15)$$

coincides with the DN-map of the intermediate operator l^r *with the "chop-off" boundary condition.*

The eigenfunctions of the operator l^r may be found by variational method.

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