Two-qbit gates based on coupled quantum wires

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Abstract– **A solid-state implementation of a set of one- and two-qbit gates for quantum computing is proposed. The qbit is defined as the state of an electron running along two quantum wires, suitably coupled through a potential barrier with variable height and/or width. Single-qbit gates are implemented using the coupling between the two wires. The two-qbit gates have been designed using a Coulomb coupler to induce a mutual phase modulation of the two qbits. A number of runs have been performed using a time-dependent 2D Schrödinger solver.**

I. Introduction

Over the last decade, considerable interest has developed in the application of quantum theory to few-particle systems. Moreover, it is now possible to fabricate very sophisticated semiconductor devices in which quantum effects play a dominant role. Even if the nanodevices behave accordingly to quantum mechanics, the algorithms that they run are classical. But quantum mechanics might be used also in a new kind of algorithms, more powerful than the classical computation: to this purpose a fundamental research, called quantum computation, has recently been developed [1].

From a physical point of view, a bit is a physical system which can be prepared in one of two different states, 0 or 1. The quantum bits, qbits, are two-state quantum systems and represent the elementary unit of quantum information: the qbit can be prepared in a coherent superposition of the two classical input states. In some sense, the quantum logic gate can process the input states in parallel due to the superposition.

The choice of the individual quantum system is crucial: since many qbits are necessary to realize a quantum computer, the physical system must be reliable and easily reproducible with high quality standards. In this work, a proposal for semiconductor qbits is presented, based on the coherent propagation of electrons in quantum wires. The fabrication of the proposed devices is within the reach of today's advanced technological processes. The main difficulty is the onset of interactions between the system and the environment, which produce decoherence and consequent loss of information.

II. The quantum system

Two identical semiconductor quantum wires separated by a high potential barrier constitute the device for the two-state quantum system. The qbit states $|0\rangle$ and $|1\rangle$ represent the localization of the electron in one of the two wires. They are linear combinations of the even and odd states (ψ_e and ψ_o) corresponding to the lowest energy levels of the quantized direction, i.e., the transversal one:

$$|0\rangle = \frac{1}{\sqrt{2}}(|\psi_e\rangle + |\psi_o\rangle),$$
$$|1\rangle = \frac{1}{\sqrt{2}}(|\psi_e\rangle - |\psi_o\rangle). \quad (1)$$

The electron is assumed to coherently propagate along the longitudinal direction. By introducing a coupling window with lower potential barrier between the wires, the wave function begins to oscillate between them with a period $\tau = h/(\epsilon_o - \epsilon_e)$, where ϵ_o and ϵ_e are the energies of ψ_o and ψ_e, respectively [2]. In our case, the barrier height and/or width between the wells changes along the wires: the heterostructure can be designed in such a way as to produce the desired transfer process of the wave function while the electron crosses the coupling regions. In this way, a set of single-qbit logic gates has been designed, that is formally described by the following evolution matrix [3]:

$$\mathbf{S}(\theta) = \begin{pmatrix} \cos(\theta/2) & i\sin(\theta/2) \\ i\sin(\theta/2) & \cos(\theta/2) \end{pmatrix}. \quad (2)$$

The length L_W of the coupling potential window is chosen in such a way as to obtain the desired value for θ. As a first example of one-qbit gate, in Fig. 1 a quantum NOT gate is shown, realized by means of two "beam splitters": the single gate $\mathbf{S}(\pi/2)$ is applied at 150 nm and at 700 nm with a 14nm-long coupling window, performing the computation $\mathbf{S}(\pi/2)\mathbf{S}(\pi/2) = \mathbf{S}(\pi)$ that is a NOT evolution.

Another useful one-qbit gate is given by the following transformation:

$$\mathbf{R}(\phi) = \begin{pmatrix} e^{i\frac{\phi}{2}} & 0 \\ 0 & e^{-i\frac{\phi}{2}} \end{pmatrix}. \quad (3)$$

Any $\mathbf{R}(\phi)$ can be realized by introducing a suitable potential barrier in one of the wires: the wave packet experiences a delay that shifts the phase between the two states $|0\rangle$ and $|1\rangle$. In order to test such an evolution, the IDENTITY gate has been realized by $\mathbf{S}(\pi/2)\mathbf{R}(\pi)\mathbf{S}(\pi/2)$, as shown in Fig. 2: the light grey barrier experienced by the wave packet along the left wire gives the phase shift $\phi = \pi$. As shown in [4], the matrices (2) and (3) are fundamental to construct any universal rotation $\mathbf{U}(\delta, \alpha, \beta, \theta)$, i. e.

$$\mathbf{U} = e^{i\delta} \begin{pmatrix} e^{i(\frac{\alpha}{2}+\frac{\beta}{2})}\cos(\frac{\theta}{2}) & e^{i(\frac{\alpha}{2}-\frac{\beta}{2})}\sin(\frac{\theta}{2}) \\ -e^{i(-\frac{\alpha}{2}+\frac{\beta}{2})}\sin(\frac{\theta}{2}) & e^{-i(\frac{\alpha}{2}+\frac{\beta}{2})}\cos(\frac{\theta}{2}) \end{pmatrix}. \quad (4)$$

In fact, by neglecting the global phase-shift factor δ, the universal transformation $\mathbf{U}$ can be decomposed in three transformations:

$$\mathbf{U}(\alpha, \beta, \theta) = \mathbf{R}(\alpha - \pi/2)\mathbf{S}(\theta)\mathbf{R}(\beta + \pi/2). \quad (5)$$

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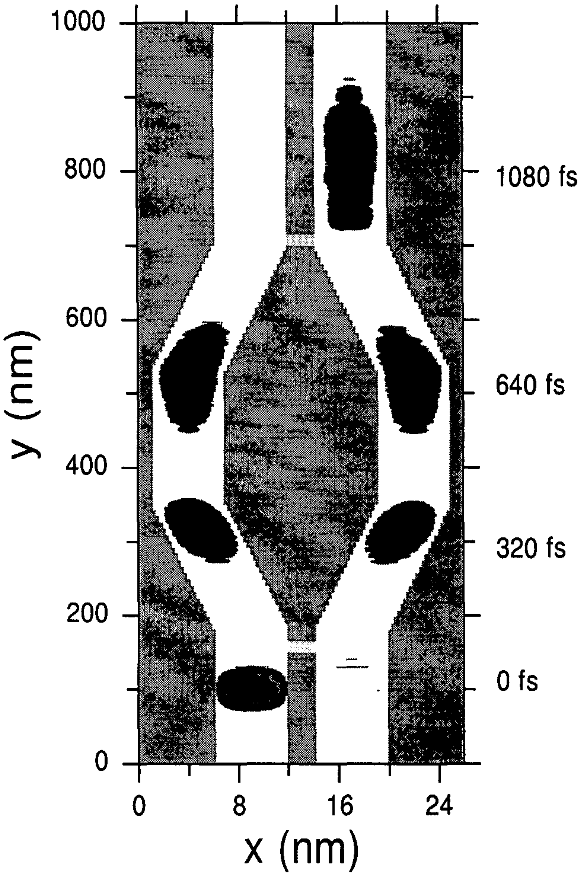

Fig. 1. *Electron density at different times in the proposed device: two coupling windows between the wires are applied, which realize the "beam splitter" gate. A quantum NOT is obtained.*

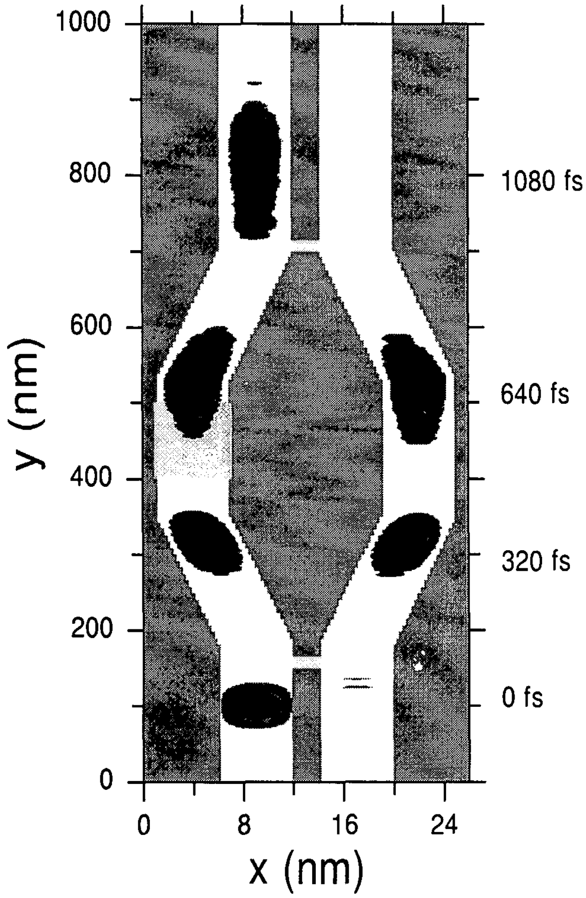

Fig. 2. *Electron density at different times in the proposed device: two "beam splitter" gates are applied with a delaying potential between them in one of the wires (light grey region). A quantum IDENTITY is obtained.*

Hence, all one-qbit logic gates can be realized by means of a suitable decomposition in terms of $\mathbf{S}$ and $\mathbf{R}$ transformations.

III. Two-qbit gates

In the previous section a set of gates has been described, which consists of all one-qbit rotations. The next step was to design a two-qbit gate. In particular, the attention has been focused on Control-U gates. The latter have two inputs and outputs; bit one is the *control* and bit two is the *target*. The Control-U evolution applies the unitary matrix U to the *target* if and only if the *control* is $|1\rangle$. As shown in [4] and [5], almost any Control-U gate is universal, with the meaning that any logic gate can be decomposed into a sequence of the Control-U gate and any arbitrary single-qbit rotation. In particular, the Control-NOT (C-NOT) is universal and gives the simplest example of a two-qbit gate.

In this work, the design of a Control-U gate starts from the purpose of using the Coulomb interaction to establish a quantum-mechanical correlation between the two qbits. The gate has been implemented as shown in figure 3, where CC is a Coulomb coupler of length L_C, able to induce a mutual phase modulation on the two qbits [6]. As the Coulomb interaction is strongly dependent on the distance between the charges, the interaction acts only on the evolution of the wave functions along the closer wires. As a consequence, the action of the gate corresponds to the following matrix:

$$\mathbf{T}(\phi) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & e^{i\phi} \end{pmatrix}, \tag{6}$$

where ϕ is a function of the coupling interaction and of the propagation time. Hence, the length of the gate, L_C, implements the desired value of ϕ. By using both (2) and (6), the C-NOT can easily be realized as shown in figure 4, where the network that simulates the C-NOT gate is reported. It is worth noting that even if the *control* qbit is not directly affected by any state evolution, whereas the *target* qbit experiences the unitary evolutions in the succession reported in the figure, the resulting final state is an "entanglement" of the single-particle states, and, as a consequence, the information is contained in the system of the two qbits. In order to show the mutual phase modulation of (6), a suitable network of logic gates has been realized and tested. In particular, the following transformation has been analyzed:

$$\begin{pmatrix} \mathbf{S}(\pi/2) & \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} \\ \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} & \mathbf{S}(\pi/2) \end{pmatrix} \mathbf{T}(\pi/2) \begin{pmatrix} \mathbf{S}(\pi/2) & \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} \\ \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} & \mathbf{S}(\pi/2) \end{pmatrix} \tag{7}$$

which represents the application of a beam splitter to the *target* qbit, followed by a Coulomb coupler applied to both qbits, and again a beam splitter applied to the *target*: the electron density along the wires depends on the initial state of the *control* qbit, as shown in Figs. 5 and 6. Actually, the *target* qbit has been taken as $|0\rangle$ at $t = 0$, then the splitting $\mathbf{S}(\theta = \pi/2)$ has been applied before and after the $\mathbf{T}(\phi = \pi/2)$ gate. If the *control* qbit is $|0\rangle$, the evolution applied to the *target* is $\mathbf{S}(\pi/2)\mathbf{S}(\pi/2) = \mathbf{S}(\pi)$, that is a NOT gate (Fig. 5). If the *control* qbit is $|1\rangle$, the mutual phase modulation of $\mathbf{T}(\pi/2)$ alters the final electron density of the *target* giving a split wave function (Fig. 6).

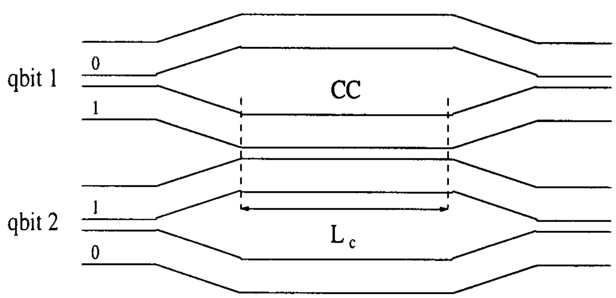

Fig. 3. *Architecture for the $\mathbf{T}(\phi)$ gate based on the Coulomb coupler CC: the phase ϕ is proportional to the coupler length L_C.*

IV. Numerical Results

The proposed system has been simulated using a time-dependent 2D Schrödinger solver. The solver has been implemented in the frame of a Crank-Nicholson method [7] by means of a simple finite-difference approximation. A uniform two-dimensional grid has been used for the discretization, with intervals of 0.25 nm along the x direction and 1 nm in the y direction. The iterative time evolution has been performed with steps of 1.6 fs. The electron-electron interaction is taken into account by means of a self-consistent Coulomb potential generated by each electron: the interaction potential is calculated at each time step and added to the structure potential. The evolution matrix is solved by means of the Conjugate Gradient Stabilized (CGS) method.

The wires have been realized as GaAs/AlGaAs heterostructures. The lateral extension of the wires is 6 nm. The high-level potential (dark regions in the figures) is assumed to be infinite. The initial condition along the x axis has been taken as the ground state of an infinite square well with the same lateral extension of the single wire. Along the y direction a minimum uncertainty

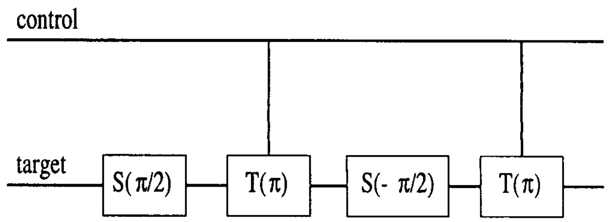

Fig. 4. *A network consisting of two $\mathbf{T}(\phi)$ gates and two $\mathbf{S}(\theta)$ gates can simulate the C-NOT.*

wave packet has been assumed:

$$\psi(x,y) = \sqrt{\frac{2}{L}}\cos[\pi(x-x_0)]\,\frac{1}{\sqrt{\sigma\sqrt{2\pi}}}e^{-(\frac{y-y_0}{2\sigma})^2}e^{ik_0y} \tag{8}$$

In Figs. 1 and 2, it is assumed that at $t = 0$ the electron is confined in the left wire with a wave function given by (8) with $\sigma = 10$ nm and a kinetic energy of 0.1 eV. The wave packet is localized at the center of the wire with coordinates $x_0 = 9$ nm and $y_0 = 100$ nm. The total energy must not exceed the energy of the first excited transversal state in the wire in order to ensure that only the ground state is occupied.

As far as the design of the one-qbit gates is concerned, the coupling potential window used to realize the $\mathbf{S}(\pi/2)$ operation has been designed with $L_W = 14$ nm and a barrier height of 0.1 eV (Figs. 1 and 2), whereas the delay corresponding to $\mathbf{R}(\pi)$ has been designed with a potential barrier 110 nm long and 10 meV high (Fig. 2). Very good results have been obtained for the one-qbit gates, while the broadening of the wave functions shown in the two-qbit gate in Figs. 5 and 6 can become a critical problem in longer gates. In particular, the $\mathbf{T}(\pi/2)$ gate has been realized with a Coulomb coupler 180 nm long, and by drawing the two wires to a distance of 2 nm. The result reported in Fig. 6 has been checked by means of an integration of the electron densities located in the wires at the final time ($t = 1120$ fs).

The search of optimal parameters for the geometry of the system and for the appropriate initial energy of the injected electrons is being performed to realize the $\mathbf{T}(\pi)$ gate.

V. Conclusions

A solid-state implementation of new quantum devices for quantum-computing purposes has been presented, based on the coherent electron transport in quantum wires. By adopting a 2D time-dependent Schrödinger solver, a number of runs have been performed to design quantum logic gates. The results obtained so far demonstrate the feasibility of a number of logic quantum gates within the proposed technology. In particular, the evolution of one and two wave packets along coupled wires has been shown. Following [4], a set of elementary gates has been addressed, which is shown to be universal.

Acknowledgments

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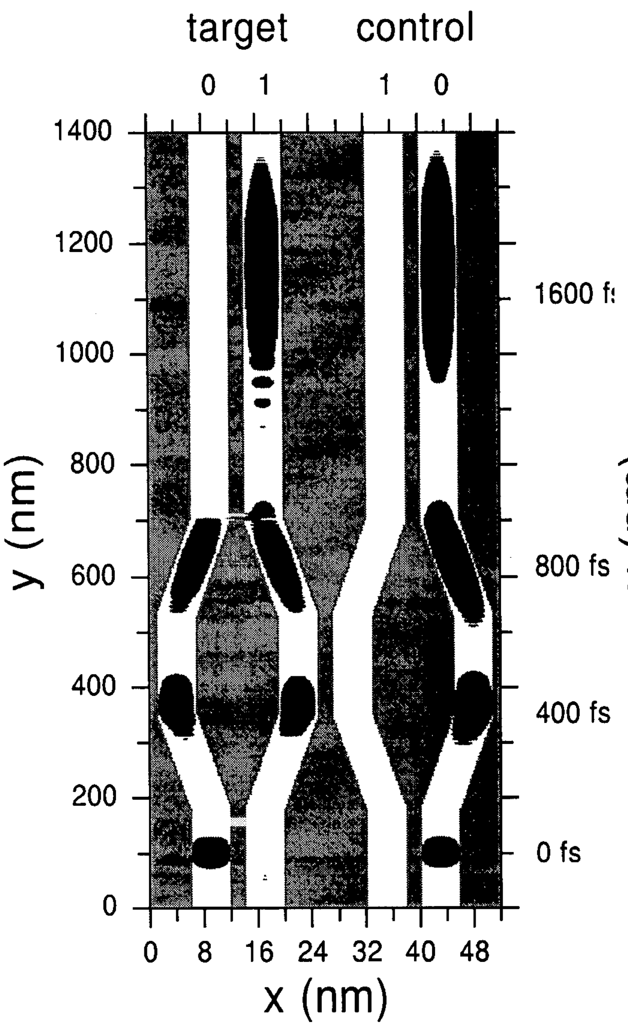

Fig. 5. *Electron density at different times in the device corresponding to Eq. (7). As the control qbit is* $|0\rangle$*, the Coulomb interaction between the electrons is not effective due to the large distance. The initial state* $|00\rangle$ *evolves to the final state* $|01\rangle$*, due to the action of the two beam splitters applied to the target qbit.*

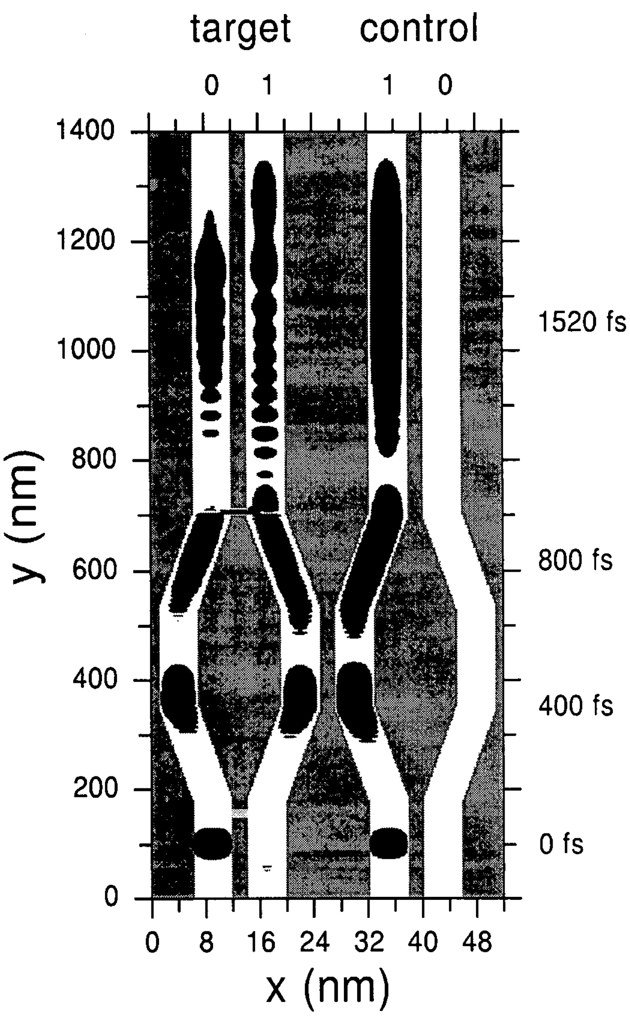

Fig. 6. *Electron density at different times in the device corresponding to Eq. (7). The control qbit is* $|1\rangle$*, hence the Coulomb interaction between the electrons delays the wave functions propagating in the inner wires. The initial state* $|10\rangle$ *is evolved to the final state* $(1-i)(|10\rangle - |11\rangle)/2$.

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