Periodic Steady-State Analysis for Coupled Device and Circuit Simulation

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Abstract

A time-domain shooting method based coupled device and circuit simulator suitable for accurate simulation of RF circuits is presented. The simulator supports accurate numerical models for diodes, BJTs, and MOSFETs. These combined with the accelerated steady-state method allow accurate and efficient steady-state simulation of RF circuits.

1. Introduction

With the continued increasing demand for RF ICs there is a critical need for accurate and efficient simulation of circuits in the periodic steady state. Certain aspects of system performance are easier to characterize and verify in steady state. Examples of these are distortion, power, frequency, noise, and transfer characteristics such as gain and impedance.

Although several techniques have been developed for accelerated steady-state solution [1-6] not much attention has been given to the model accuracy. For RF applications distributed device effects are important and must be included in compact models [7]. In the absence of accurate compact models a coupled circuit and device simulator can be used whereby critical devices are solved using physical (numerical) models. Such an approach has been applied for the simulation of RF power amplifiers [8] using a harmonic-balance method. However, no coupled device and circuit simulator supports the time-domain steady-state method which is useful for several circuits [9]. This paper presents the first implementation of the time-domain steady-state method in the context of coupled device and circuit simulation.

The paper is organized as follows. An overview of the time-domain shooting method and the coupled device and circuit simulator CODECS [7] is provided in Section 2. Implementation details and heuristics are described in Section 3 while application examples and runtime performance are presented in Section 4. Conclusions and future work are summarized in Section 5.

2. Overview

2.1 Time-Domain Steady-State Analysis Method

There are a variety of methods that directly compute the steady-state solution more efficiently than numerically integrating the differential equations of a circuit from an arbitrary initial condition. These methods can be classified as frequency-domain and time-domain methods [1]. Harmonic balance is a frequency-domain method in which the coefficients for a truncated Fourier series expansion of the steady-state solution are determined. In the time domain, the shooting method is a popular method to find an initial condition which leads to the steady-state solution. Our focus is on the time-domain method using Newton's algorithm.

Consider the system of equations

$$f(\dot{X}, X, t) = 0 \quad (1)$$

where X and f are n vectors; f is periodic in t with period T. We assume that Eq. (1) has a periodic solution of period T. The solution of Eq. (1) can be obtained by numerical integration. The objective of the time-domain method is to determine the initial condition $X(0)$ such that the solution of Eq. (1), with an initial condition $X(0)$, over the interval $[0, T]$ results in a final state $X(T)=X(0)$. $X(T)$ is a function of $X(0)$ and is denoted as $X(T, X(0))$. This is a two-point boundary value problem in which the solution to Eq. (1) in the interval $[0, T]$ must satisfy the boundary condition:

$$X(0) - X(T, X(0)) = 0 \quad (2)$$

To solve this problem using Newton's algorithm, the following iteration equation is used [2]:

$$X^{i+1}(0) = X^{i}(0) - [I - J^{i}]^{-1}[X^{i}(0) - X^{i}(T)] \quad (3)$$

where $$J^{i} = \frac{\partial X^{i}(T, X(0))}{\partial X^{i}(0)} \quad (4)$$

is the sensitivity matrix of the final state $X^i(T)$ corresponding to period i with respect to the initial condition $X^i(0)$ for period i, I is the identity matrix, and $X^{i+1}(0)$ is the initial guess for period $i+1$. For an autonomous system, the period T is added as an unknown and one of system variables is fixed.

The sensitivity matrix J in Eq. (4) can be readily determined by use of forward/backward substitutions. When a transient analysis has been carried out for the period i, $X^i(T)$ and J^i, which are required for the Newton iteration, are computed simultaneously and both are available at the end of the period. The initial value of J^i is the identity matrix.

2.2 Coupled Device and Circuit Simulator (CODECS) [7]

CODECS is a coupled device and circuit simulator that allows accurate and detailed simulation of semiconductor circuits. The simulation environment of

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CODECS enables one to model critical devices within a circuit by physical (numerical) models based upon the solution of Poisson's equation and the current-continuity equations. Analytical models can be used for the noncritical devices. CODECS incorporates SPICE3 for the circuit-simulation capability and for analytical models of semiconductor devices.

Numerical models include one- and two-dimensional models for diodes and bipolar transistors, and a two-dimensional model for MOSFETs. The numerical models in CODECS include physical effects such as bandgap narrowing, Shockley-Hall-Read and Auger recombinations, concentration- and field-dependent mobility, concentration-dependent lifetimes, and avalanche generation.

CODECS supports dc, transient, small-signal ac, and pole/zero analysis of circuits containing one- and two-dimensional numerical models. However, periodic steady-state analysis is not available in CODECS. In this work we extend the capabilities of CODECS by implementing the time-domain periodic steady-state method.

3. Implementation Considerations

In this section, the implementation details and convergence heuristics are described.

3.1 State Elimination

Experiments have shown [5] that states due to diode and transistor parasitics need not be considered because they have only a second order effect on the periodic response. Elimination of these states makes convergence of Newton's algorithm faster and reduces the computational effort. Therefore, only the states for the capacitors and inductors in the circuit are considered.

3.2 Numerical Device Biasing

In CODECS, the numerical devices are biased up to an initial condition by stepping from the equilibrium state. This stepping is necessary for obtaining a converged solution for large bias steps. To help convergence when biasing the numerical devices for a new initial guess, we make use of the final state of the last period. At the end of the period i, we obtain the final state $X^i(T)$ and the initial guess $X^{i+1}(0)$. Usually the difference $\Delta X^i = X^{i+1}(0) - X^i(T)$ is small. To bias numerical devices to $X^{i+1}(0)$, we only need to bias them by a small step ΔX^i since at that time the numerical devices have already been biased to $X^i(T)$ by the transient simulation for period i.

3.3 Heuristics for Autonomous Systems

An example of an autonomous system is an oscillator. The oscillator is a strongly nonlinear system for which the period of oscillation is an unknown. Heuristics are necessary to ensure reliable convergence of Newton's algorithm for such a system. In our implementation the following heuristics are used.

A transient analysis is performed in the beginning for three periods without sensitivity computation such that the extremely fast transients in the start-up phase have died out. Also in this interval a pulse is applied to a voltage source to build up the oscillation. The sensitivity computation is carried out for the current period to calculate the new initial guess only when the error of the last period is less than an acceptable threshold. Otherwise, the transient analysis continues to the next period. This heuristic reduces the probability of the iteration process going astray or leading to a wrong solution. To prevent overshoot, a damped Newton iteration is used. Damping reduces the effect of the sensitivity matrix J on the iteration. Finally, the change in the period is not allowed to exceed ten percent of the current period to prevent overshoot of the Newton iteration.

4. Examples and Results

Several example circuits with a periodic steady state have been chosen to verify our implementation. The semiconductor devices in these circuits are modeled by either 1D or 2D numerical models.

4.1 Examples

The steady-state solution for the frequency multiplier circuit of Fig. B.5 of [10] is obtained after 6 periods of transient analysis using Newton's algorithm. While, the conventional transient simulation takes 1500 periods to reach the steady state. Newton's algorithm speeds up the convergence of the steady-state simulation significantly. The output voltage waveforms for several periods are plotted in Fig. 1. The Newton iteration is carried out at the end of each period to calculate the initial guesses for the next period.

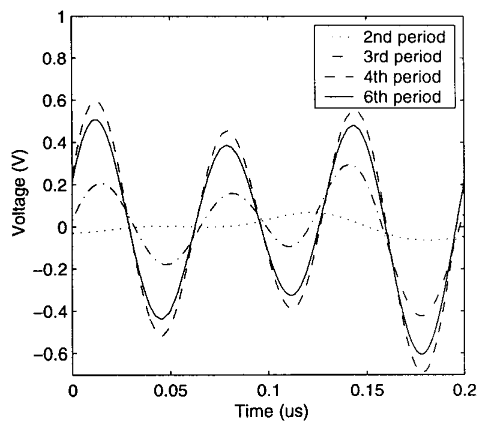

Fig. 1: Output voltage waveforms for four periods during the steady-state simulation of the frequency multiplier circuit.

The steady-state solution for the DC power supply circuit of Fig. B.1 of [10] is obtained after 6 periods of transient analysis using Newton's algorithm. The normalized harmonics of the steady-state voltage waveform at node 2 are plotted in Fig. 2. From this figure, the magnitude of the harmonics drops slowly. If the harmonic balance method is used to determine the steady state of this circuit, many harmonics have to be computed to obtain an accurate result. With the time-domain shooting method this circuit can be readily simulated. The time-domain shooting method is robust for accurate steady-state simulation of strongly nonlinear circuits [9].

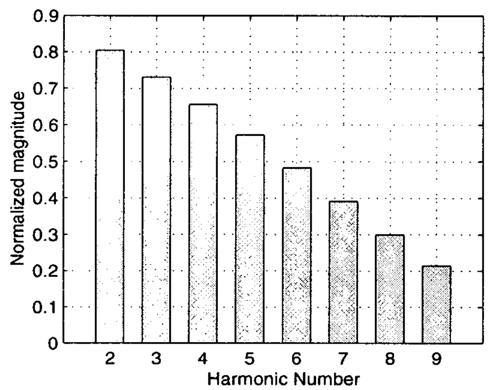

Fig. 2: Normalized harmonics of the steady-state voltage waveform at node 2 for the DC power supply circuit.

As an autonomous system, a typical BJT Colpitts oscillator is chosen from [11]. The steady-state solution of this circuit is obtained with 18 periods of transient simulation using the Newton shooting algorithm. Among them, sensitivity computation is carried out in 13 periods and Newton iteration as in Eq. (3) is performed 10 times. At the end of the 6th and 12th periods, the sensitivity matrix J has been calculated but the Newton iteration is not performed. This is because the error is larger than the acceptable threshold as discussed in the heuristics of the last section. The steady-state solution is verified with a regular transient simulation.

To demonstrate the effect of the numerical model on the steady-state solution, a high frequency Colpitts oscillator (Fig. B.8 of [10]) is used. The steady-state solutions of this circuit with analytical and numerical models are obtained by the time-domain steady-state method. The oscillation frequency is 0.8GHz for the analytical model and 0.72GHz for the numerical model. The normalized harmonics of the output waveforms for both models are shown in Fig. 3. The total harmonic distortions are 7.7% and 13.4% for the analytical and numerical models, respectively. The same simulation has been performed for the Colpitts oscillator from [11]. The oscillation frequency is 60.9 MHz for the analytical model and 61.6MHz for the numerical model. The normalized harmonics of the output waveforms are shown in Fig. 4.

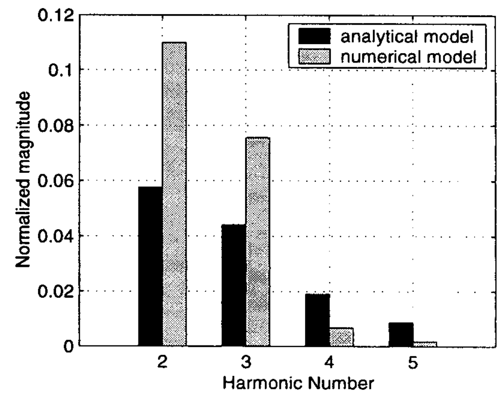

Fig. 3: Normalized harmonics of the steady-state solution for the high frequency Colpitts oscillator comparing analytical and numerical models.

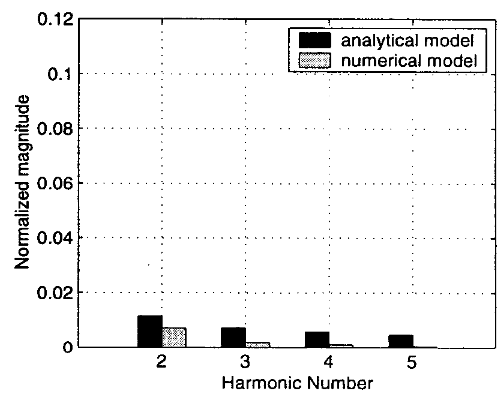

Fig. 4: Normalized harmonics of the steady-state solution for the BJT Colpitts oscillator from [11] comparing analytical and numerical models.

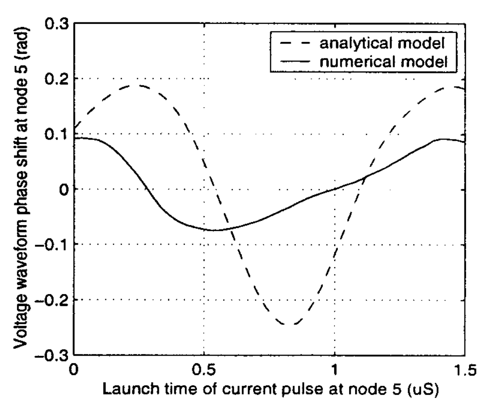

Fig. 5: Phase shift due to a current impulse at node 5 for phase noise analysis of the high frequency oscillator comparing analytical and numerical models.

The total harmonic distortions are 1.6% and 0.8% for the analytical and numerical models, respectively. The differences in oscillation frequencies and harmonic distortion are much larger for the high frequency oscillator than the low frequency oscillator.

Finally, for phase noise analysis of the high frequency oscillator [10], the impulse sensitivity functions (ISF) [11] at node 5 are simulated with both the analytical and numerical models. The phase shift is plotted in Fig. 5 and it can be seen that the difference between the two models is significant. These results show that numerical models are essential for accurate simulation of high frequency RF circuits.

4.2 Performance Results

To evaluate the performance efficiency of the time-domain steady-state method, the conventional transient simulation is used to determine the steady state for several example circuits. The number of periods required by the conventional transient simulation and the time-domain steady-state method is summarized in Table 1. It is seen that the time-domain steady-state method is much more efficient than the conventional transient simulation method.

Example circuits	Conventional transient simulation (# of periods)	Time-domain steady-state method (# of periods)
DC supply*	80	6
CB amplifier*	30	4
EC xfrmr osc*	185	25
Freq multiplier*	1500	6
LC EC osc*	22	9
SCP amplifier	182	6
H.F. Colpitts*	20	12
Colpitts [11]	84	18
Demodulator [12]	12000	4

Table 1: Performance comparison for conventional transient simulation and the time-domain steady-state method. The # of periods is directly proportional to the runtime performance. ('*'--circuits are from [10]).

In transient simulation of circuits containing numerical devices, the computationally intensive part is the numerical model evaluation. The overhead due to the sensitivity computation required by the time-domain steady-state method is negligible, especially when the state elimination in Section 3.1 is implemented. Therefore, the ratio of the number of periods required is approximately equal to the ratio of the simulation time required. The time-domain steady-state method is much more efficient with high-Q and lightly damped circuits. Furthermore, it will result in a significant reduction of simulation time when fine meshed numerical devices are used in circuits.

5. Conclusions

The first implementation of the time-domain steady-state method in the context of coupled device and circuit simulation is presented. With the implementation heuristics that were described, the time-domain shooting method is reliable and converges rapidly to the steady state. Compared with the results of a conventional transient simulation and an analytical model, this simulator is efficient and more accurate.

Future work will focus on improved heuristics for autonomous systems to improve the efficiency and reliability of steady-state simulation for oscillators.

Acknowledgement

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