# Fast Algorithms for Computing Electrostatic Geometric Sensitivities

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*Abstract*—In this paper we show that for the standard piecewise-constant collocation schemes used in programs like FASTCAP, computing the change in electrostatic forces due to geometric perturbations is equivalent to resolving for the charge distribution given a potential generated by linearly varying dipole distributions. Second, we derive closed form analytic expressions for the potential due to the linearly varying dipole distributions. Third, results are given from implementing the dipole approach in the precorrected-FFT code FFTCAP and combining it with the adjoint method so as to rapidly compute the sensitivity of capacitances to parameter variations.

## I. INTRODUCTION

Accelerated boundary-element methods based on Multipole and precorrected-FFT algorithms are now commonly used to compute accurate capacitances and electrostatic forces in complicated 3-D geometries [1], [2]. For applications such as propagating process sensitivities, fitting capacitance formulas for large scale extraction, or analyzing coupled effects in micro-electro-mechanical structures [3], [4], it is also necessary to quickly determine the geometric sensitivities of the electrostatic forces or capacitances.

In this paper we combine three contributions. First, we show that for the standard piecewise-constant collocation schemes used in programs like FASTCAP [1], computing the change in electrostatic forces due to geometric perturbations is equivalent to resolving for the charge distribution given a potential generated by linearly varying dipole distributions. Second, we have derived closed form analytic expressions for the potential due to the linearly varying dipole distributions. Third, we implemented the dipole approach in the precorrected-FFT code FFTCAP [2], and combined it with the adjoint method so as to rapidly compute the sensitivity of capacitances to parameter variations. Some computation results are presented below.
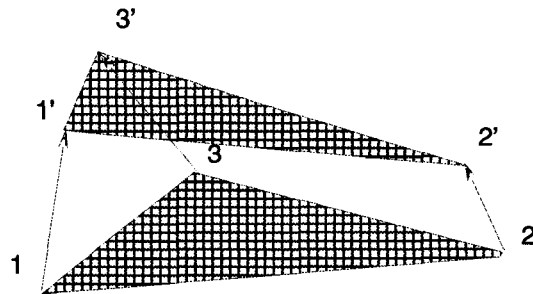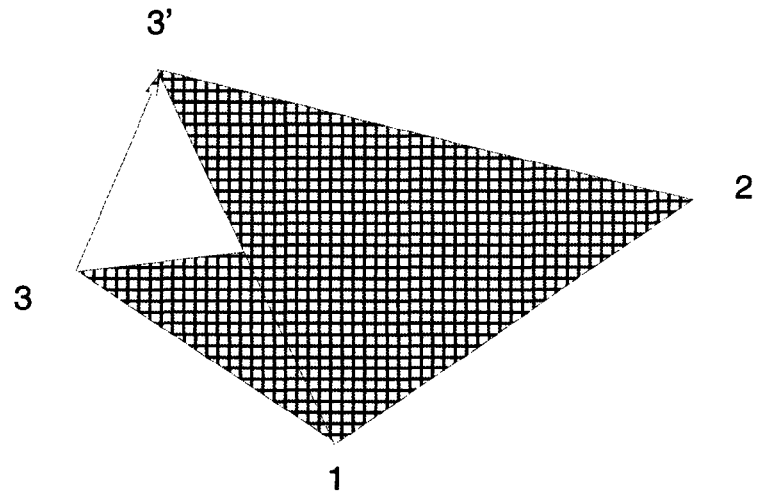
The perturbation due to the displacement of the vertices



The perturbation of the panel by one vertex

Fig. 1. Top: Perturbed panel, Bottom: One vertex perturbed.

## II. Computing Geometric Variations

To compute charge densities given conductor potentials, one can solve the integral equation $\Psi(x) = \int_S \frac{\sigma_l(x')}{\|x'-x\|}\, da'$, for $x$ on the union of conductor surfaces $S$. If the conductor surfaces are discretized into $N$ panels, and piecewise-constant collocation is used, the result is an $N$-dimensional system of the form $P(u)q = \Psi(u)$, where $q \in \Re^N$ is the vector of panel charges, $u \in \Re^3 M$ is the vector of three-dimensional panel vertex displacements, and $P_{kl}(u) = \frac{1}{a_l}\int_{panel_l(u)} \frac{1}{\|x'-x_k(u)\|}\, da'$. For a given $u$, this dense system of equations can be "sparsified" and solved efficiently using precorrected-FFT accelerated iterative methods [2].

To compute $\frac{\partial q(u)}{\partial u}$ for a given direction $\hat{u}$, one must solve $P(u)\frac{\partial q}{\partial u}\hat{u} = \frac{\partial P(u)}{\partial u}\hat{u}q(u)$. The term $\frac{\partial P(u)}{\partial u}\hat{u}q(u)$ is the change in potential due to a change in charge position along direction $\hat{u}$. This "delta" in potential can be generated by a dipole distribution on the unperturbed panels, as motivated by Figure 1top. To derive a analytic expression for this dipole potential, one can decompose the panel perturbations in to a set of three terms, one for each vertex motion, as shown in Figure 1bottom. Then, modifications of the formulas in [5] can then used to analytically evaluate the potential due to the panel dipole distributions. The formulas are valided by comparing to finite-differences and letting the perturbation shrink to zero, as in Figures 2top, bottom. Finally, the grid projection step in the precorrected-FFT algorithm can easily be modified to use these dipole potential formulas [2].

## III. Computational Results

In coupled electromechanical analysis, all that is needed is a fast approach to computing $\frac{\partial P(u)}{\partial u}\hat{u}q(u)$ for many different $\hat{u}$'s. The obvious approach is to approximate $\frac{\partial P(u)}{\partial u}\hat{u}q(u)$ by finite differences as in $\frac{1}{\epsilon}P(u + \epsilon\hat{u})q - P(u)q$, where $\epsilon$ is small. For the example of the interconnect structure below, computing $\frac{\partial P(u)}{\partial u}\hat{u}q(u)$ by the precorrected-FFT accelerated dipole approach requires 0.153 seconds, where as using precorrected-FFT accelerated finite differences requires 4.38 seconds. Not only is the new method nearly thirty times faster, it is also more robust as the finite-difference result is very sensitive to the choice of $\epsilon$.

In order to compute the sensitivity of $C_5$, the self-capacitance of conductor five in the interconnect structure below, to the seventeen process variations, one first solves the adjoint system $P(u)^T \hat{q} = \Psi$, where $\Psi_i = 1$ for panels on conductor five. Then, $\frac{\partial C_5}{\partial p_i} = \hat{q}^T \frac{\partial P(u)}{\partial u}\frac{\partial u}{\partial p}q(u)$. Using the precorrrected-FFT accelerated dipole approach, the sensitivity of $C_5$ to all seventeen parameters can be computed in under thirty seconds. For example, $\frac{\partial C_5}{\partial d_1} = -0.012$ and $\frac{\partial C_5}{\partial d_3} = 0.492$.
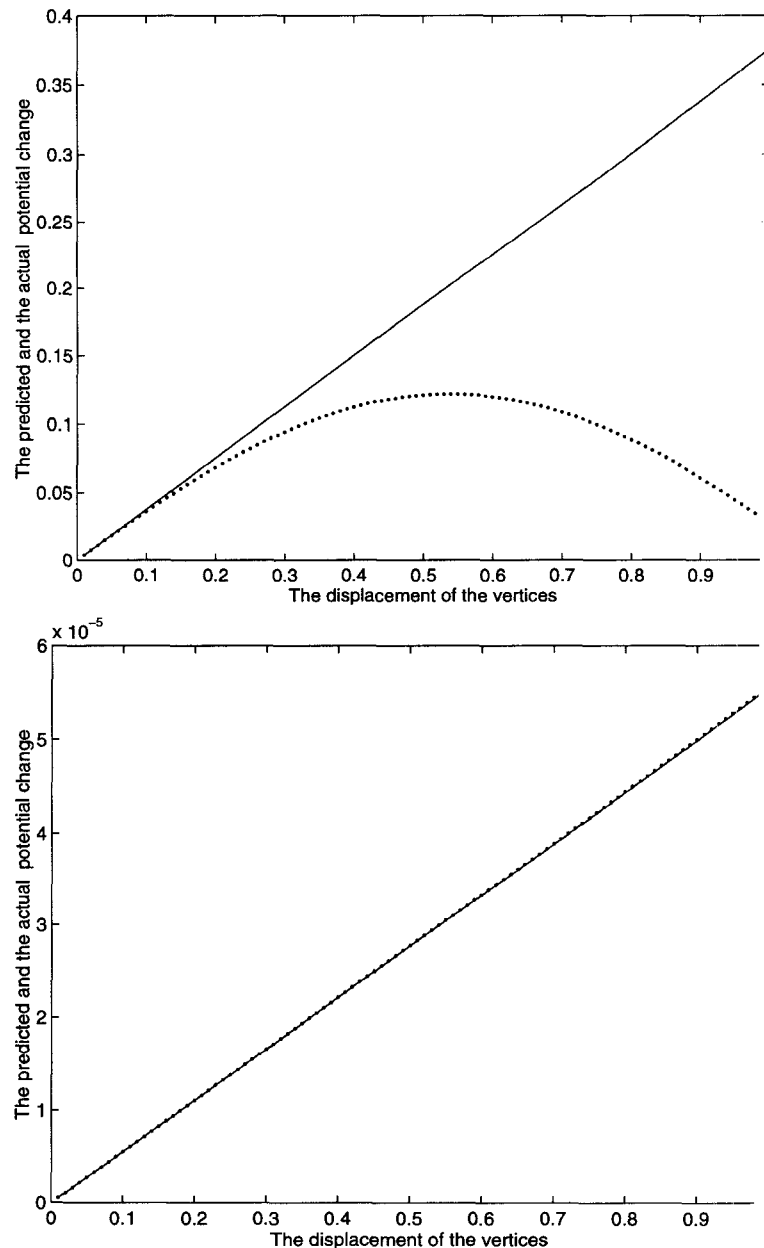


Fig. 2. (Top, bottom) Finite-difference versus analytic formula for unit triangle, with evaluation points at $x = 1/3$, $y = 1/3$, $z = 1$ and $x = 2$, $y = 5$, $z = 100$, respectively. $X$-axis is perturbation size.
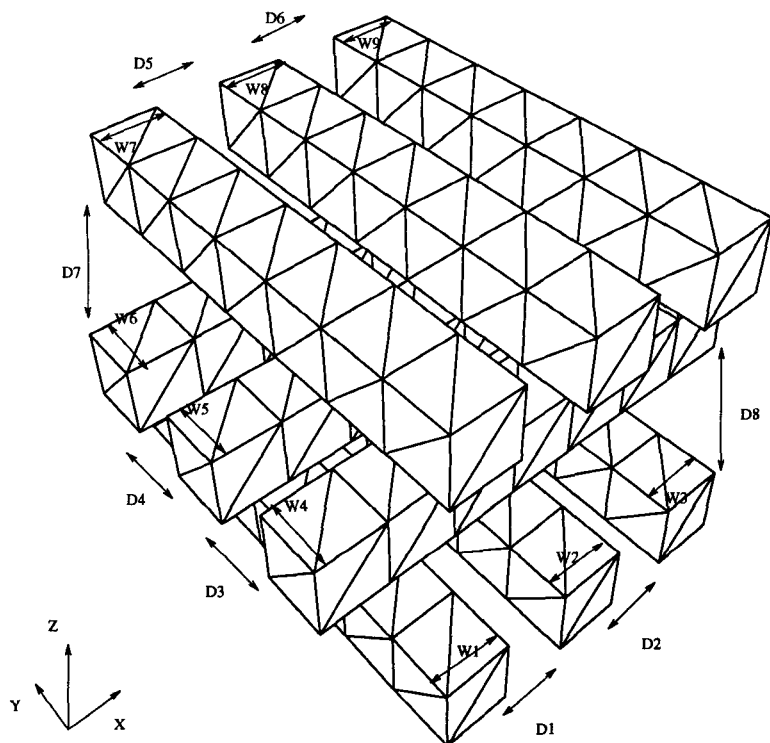
## IV. Conclusions and Acknowledgements

In this paper we show that for the standard piecewise-constant collocation schemes used in programs like FAST-CAP, computing the change in electrostatic forces due to geometric perturbations is equivalent to resolving for the charge distribution given a potential generated by linearly varying dipole distributions. Second, we derive closed form analytic expressions for the potential due to the linearly varying dipole distributions. Third, results are given from implementing the dipole approach in the precorrected-FFT code FFTCAP. On a complicated interconnect structure, the geometric sensitivity based on the dipole approach was shown to be thirty times faster than finite differences. In addition, when combined with the adjoint approach, the method can be used to quickly compute parametric sensitivities.

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The parameters of the problem