# Accuracy and Convergence Properties of a One-Dimensional Numerical Non-Quasi-Static MOSFET's Model for Circuit Simulation

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#### Introduction

Accurate modeling of static currents, conductance and charge dynamics are essential for the design of digital and specially for analog circuits. In the analog domain, the shortcomings of many modeling approaches often originate from transistors biased between linear and saturation regimes where discontinuities limit the accuracy and the convergence properties. Moreover, the finite charging/discharging time of the channel may significantly degrade the performances of modern circuit architectures due to charge injection [1]. However, most MOSFET's models reveal poor prediction capabilities for high frequency operations for which quasi-static (QS) operation is often violated. In this paper we discuss the accuracy and numerical properties of a one-dimensional CAD-oriented model [2]. It is shown that the proposed model is continuous over all operating regimes and suitable for the analysis of long and short channel MOSFET's. The most interesting feature of our model, an implicit non-quasi-static (NQS) treatment of the charge redistribution, is outlined. Finally, convergence properties are discussed with a special emphasis on the mobility model and on the related non-linear resolution scheme.

### Model formulation [2]

The one-dimensional resolution of the Poisson equation is extended beyond the gradual channel approximation to account for short channel effects, velocity saturation and channel length modulation. By applying the Gauss law to a narrow vertical strip in the channel the surface potential can be related to the depletion and inversion charges and to the lateral electric field in the channel direction:

## $-\varepsilon_{si}E_{\mathbf{y}}(\mathbf{x},0)\,d\mathbf{x} + \int \varepsilon_{si}E_{\mathbf{x}}(\mathbf{x}_{n+1/2},\mathbf{y})\,d\mathbf{y} - \int \varepsilon_{si}E_{\mathbf{x}}(\mathbf{x}_{n-1/2},\mathbf{y})\,d\mathbf{y} = \mathrm{Qi}(\mathbf{x}_{n})\,d\mathbf{x} + \mathrm{Qd}(\mathbf{x}_{n})\,d\mathbf{x}$

where Qi and Qd denotes the inversion and depletion charges per unit area, respectively. Assuming that Ex(x,y) at position 'x' remains quasi constant in the bulk

direction (y-direction), the above equation is integrated over a distance corresponding to the depletion depth  $D(x) = (C_{\alpha x} \gamma \sqrt{\phi(x_n)}) / (qC_{sub})$ . Finally, a linear finite-difference scheme is used to discretize the resulting equation. This approach is less restrictive than the model proposed in [3]. A one-dimensional version of the current equation is assumed according to a charge sheet formulation. Special care must be devoted to discretize this equation to correctly account for the exponential relation between the potential and the electron concentration. The final form is given by:

$$_{n+1/2} = \frac{\mu(E)_{n+1/2} W \cdot V_t}{dx_n} \cdot \left\{ Qi_{n+1} B\left(\frac{\phi_{n+1} - \phi_n}{V_t}\right) - Qi_n B\left(\frac{\phi_n - \phi_{n+1}}{V_t}\right) \right\}$$

where B(x) is the Bernouilli function while others variables have their classical meaning. The current continuity and Poisson equations are consistently solved using a Newton method. In practice, 10 discretization points along the channel proved to be sufficient to achieve a precision comparable to 2D simulation results performed on a dense mesh. The mobility dependences are classically modeled according to [4] and [5]. No empirical parameters are introduced to improve the model accuracy. The model was coded in C and introduced in the ELDO circuit simulator [6] which retains the same functionality than SPICE.

### Static results and measurement

The present model proved to be very efficient in simulating the DC characteristics of short and long channel n-MOSFET. Fig. 1-a and Fig. 1-b report the drain current and conductance variations with the drain voltage for a 1  $\mu$ m (L<sub>eff</sub>) technology [7]. The largest discrepancy between simulated and measured results is obtained at high V<sub>GS</sub> and V<sub>DS</sub> which are not bias conditions of practical relevance in most of analog applications.

# **Dynamic results and 2D simulations**

Fast transient excitations have been applied to the gate (5V/0.5 ns) of relatively long devices (L=5  $\mu$ m) to enhance the effect of the NQS charge redistribution in the channel. The results are systematically compared to 2D numerical calculations using IMPACT3.4 [8] and to simulations performed with BSIM3 using the defaulted 0/100 channel charge partition [9]. Fig. 2 illustrates the charging of the channel through negative source and drain currents in linear operation. An excellent agreement is found between our model and 2D physical simulations. In particular, an excellent continuity is obtained when the high state is reached on the gate terminal (t=0.6 ns). Beyond that point a smooth relaxation of currents to their DC values is obtained. In contrast, BSIM3 simulations suffer from unphysical current spikes and overlooks the NQS delay.

#### Non linear treatment of the mobility expression

Accurate circuit simulations require the introduction of a field-dependent mobility model. However, the number of Newton iterations needed to converge toward an acceptable solution can increase drastically as a function of bias conditions. This is due to the fact that the mobility is a non linear function of the lateral and vertical electric field. Different refinements have been considered for the description of the Jacobian matrix:

i) A constant mobility µ0.

ii) A field dependent mobility model with an incomplete Jacobian matrix solution (legend: Inc. Jacobian).

iii) A field dependent mobility model, where the derivatives of mobility with respect to the electric field and charge are fully accounted for in the Jacobian (legend: Full Jacobian).

A fast transient simulation is considered in Fig. 3. The MOSFET operates in saturation regime ( $V_d=5V$ ), with a gate voltage rising from 0 to 5V in 0.3ns. The transient current variations are given in Fig. 3-a for the three mobility cases. As expected, current characteristics computed with the field dependent mobility model are identical. However, Fig. 3-b clearly indicates a much smaller number of iterations when a complete derivation procedure is used to fill the Jacobian matrix. In addition, Fig. 3-c shows that a complete Jacobian formulation ensures better current conservation at the end of the transient. The same comparison holds in the case of a fast turn-on in linear regime (Fig. 4-a, 4-b, 4-c). Finally Fig. 5 reports results for static  $I_{DS}$ - $V_{DS}$  characteristics for two gate bias conditions, 2.5V and 5V, respectively. Again, Fig. 5-b clearly shows that a full treatment of the Jacobian matrix leads to reduced numbers of non-linear iterations (=50%) in saturation regime. Current conservation is also found incomparably better as illustrated in Fig. 5-c.

# Conclusion

Based on a physical solution of the Poisson and current continuity equations, a continuous model suitable for fast transient analysis has been developed and implemented in ELDO for circuit simulation applications. According to our experience, this model proved to be very useful for the simulation of critical part of large analog circuits without an excessive extra computational burden (10% to 300%) particularly if we take care of the electric field dependence of the mobility when filling the Jacobian matrix in the Newton resolution scheme.

#### References

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