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A New Approach to Fully Unstructured Three-dimensional Delaunay Mesh Generation with Improved Element Quality

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Mesh generation is known to play a critical role in semiconductor device and process simulation. We present a new approach suitable for dealing with the increasing complexity of the device boundaries and interfaces as well as moving boundaries. It is recently understood that techniques which have worked well in the past (octree methods, intersection and bisection based methods, cartesian methods) are at their limits today. It is in this spirit that we developed a fully unstructured gridding method which we believe is the only potential way to deal with the complexity of future devices and to handle moving boundary situations. Our algorithm also incorporates local improvement of element quality by non-delaunay quality measures, while still maintaining the Delaunay property.

The meshing algorithm consists of a Delaunay tetrahedrization module and a nonplanar Delaunay surface triangulation module. The tetrahedrization module uses a new modified advancing front technique. It is provided with the initial front by the nonplanar Delaunay surface triangulator.

Nonplanar Delaunay surface triangulation. The input to our mesh generator is a general polygonal boundary description of the various parts of the geometry and if it is desired a set of grid nodes to be reused. (Fig. 3) First each polygon is triangulated itself not regarding the Delaunay property. This results in a complete list of triangles representing the boundaries and interfaces of the input. A feature edge parameter defines which edges formed by two triangles must be preserved. If the angle between the two planes formed by the triangles is near 180 degrees the edge can be flipped. If the edge is formed by more than two triangles (e.g. triple lines) it must generally be preserved. The idea is to determine which edges may be flipped without changing the shape of the geometry. With such local transformations of triangles and refinement of non-flippable edges by point insertion a triangle representation of the boundaries and interfaces is generated, where each triangle fulfills the Delaunay criterion in three-dimensional space. (Refer to [3] for the more general usage of local transformations in Delaunay triangulations.) This Delaunay triangle surface (Fig. 4) can be viewed as a three-dimensional oriented front where each triangle is facing the domain which has to be meshed. Depending on the application this can be the domain inside or outside of the geometry.

Modified Advancing Front algorithm. The triangles of the initial front are put into a queue. The queue holds at any time the triangles of the current active front. A triangle is taken from the queue and a tetrahedron is attached to it. The gridding process can be viewed as a growth process of tetrahedra where the triangles of the queue are the seeds. Upon generating a new tetrahedron and attaching it to its seed triangle the front advances. The seed triangle does not belong to the front anymore and is taken out of the queue. The other three triangles which form the new tetrahedron are new seed triangles for future tetrahedra and are inserted into the queue. The queue holds once again the active front as it advances. (Fig. 1 shows snapshots of the front while it advances for the example of meshing a hand.) The modifications to known advancing front algorithms (refer to [2]) are: (i) The point set of grid nodes and boundary vertices is fixed and known at the time instance when the front advances. (ii) Only Delaunay tetrahedra are generated. Given a seed triangle the correct point among the existing points is searched for, with which a valid Delaunay tetrahedron can be formed. The key to the performance of such an algorithm is a powerful point location method. We use a point bucket octree which allows us to quickly find all points within a given region. The growth of tetrahedra is stopped when parts of the advancing front merge with each other. Since the boundaries and interfaces belong to the initial front, the growth of tetrahedra is restricted to the domain which has to be meshed. This is a big advantage to known other unstructured meshing methods, where the entire convex hull is meshed and the grid outside the domain is thrown away in a postprocessing step. The growing tetrahedra will fill the entire meshing domain no matter how complex the boundaries and interfaces are. The rigorosity in which it is dealt with arbitrary complex structures is apparent. Only if every boundary triangle and every interface has been merged with tetrahedra will the queue be empty and the meshing process be finished.

Improved Quality mesh generation. We use a special refinement method for a Delaunay grid to satisfy other quality measures (e.g. aspect ratio of a tetrahedron) and to avoid slivers ([4]) typically existent in Delaunay tetrahedrizations. If a bad Delaunay tetrahedron is encountered, we insert a grid node at the circumcenter of its circumsphere (Steiner Point, [1]). This method has not yet been exploited in three-dimensional space, however it has sucessfully been used in two dimensions. Fig. 5 shows the final grid after quality refinement. In Fig. 2 one can see clearly the difference of a regular Delaunay grid compared to the quality improved grid. The key idea of this refinement method is that we do not have to necessarily refine the bad tetrahedron itself. If the circumcenter is located outside of the bad tetrahedron, a neighboring tetrahedron will be refined instead. The refined tetrahedron is split into four subtetrahedra. We then repair the Delaunay property by local regridding (as will be described below). We can be sure that the original bad tetrahedron will not be present in the restored Delaunay tetrahedrization, because we destroyed its Delaunay property by having inserted a point at the circumcenter. Moving boundary situations and deformations are treated in the same way. They can be viewed as locally destroying the quality and/or the Delaunay property of elements where refining and regridding steps repair the grid. The key, however, lies in the regridding potential of our advancing front algorithm. Locally removing all non-delaunay tetrahedra is simply the reverse process to the growth of tetrahedra. The queue once again holds the front surrounding the gap where tetrahedra have been removed. Reprocessing the queue results in regridding the local region.

Conclusion. Our new approach deals with any complexity, allows reusing grid nodes if desired, and results in improved quality meshes compared to Delaunay-only grids. This is a significant progress to existing methods, especially not fully unstructured techniques like octree or intersection based techniques.

[1] L.P. Chew, "Guaranteed-quality triangular meshes", Tech. Rep. TR-89-983, Cornell University, 1989.

[2] T.D. Blacker and M.B. Stephenson, "Paving: A new approach to automated quadrilateral mesh generation", *Int. J. Numer. Meth. Engng.*, 32(4), pp. 811–847, 1991.

[3] B. Joe, "Three-dimensional triangulations from local transformations", *SIAM: J. Sci. Stat. Comput.*, 10(4), pp. 718–741, 1989.

[4] M. Bern and D. Eppstein, "Mesh Generation and Optimal Triangulation", Computing in Euclidean Geometry, pp. 201–204, 1992.

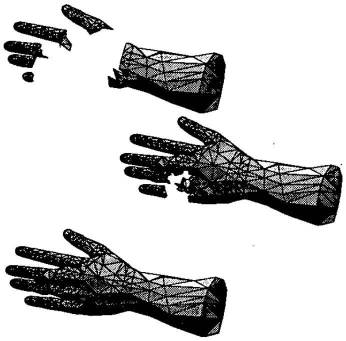

Figure 1: Snapshots of the advancing front, final meshed hand.

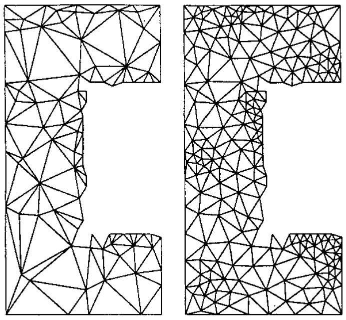

Figure 2: Ordinary Delaunay triangulation vs. Delaunay triangulation with Steiner Points.

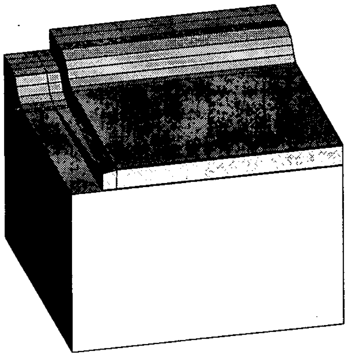

Figure 3: MOS Transistor as polygonal input.

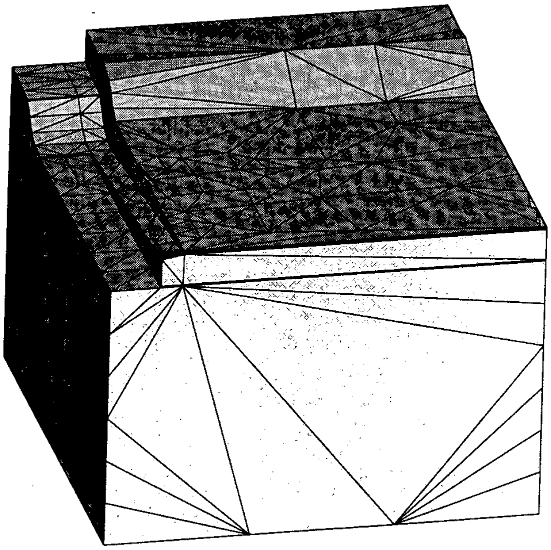

Figure 4: Delaunay surface triangulation of input transistor.

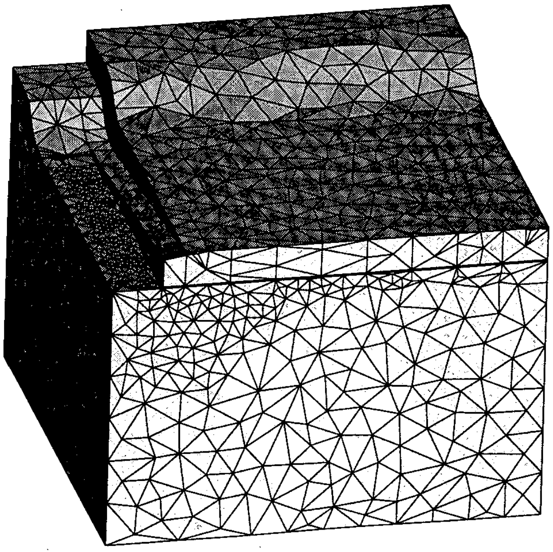

Figure 5: Final grid with 20134 elements.

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