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Automatic Adaptive Meshing for Efficient Electrostatic Boundary Element Simulations

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The boundary element method (BEM) is well suited for the electrostatic analysis of large, geometrically complex structures. When highly accurate solutions are required, the discretization (meshing) of the geometry becomes increasingly important. A scheme is required to construct optimal meshes, in the sense that maximum accuracy of the solution can be achieved with minimal computational effort. Automatic adaptive meshing allows to automatically generate a good mesh, by iteratively refining the elements that contribute strongly to the overall error. An error indicator for BEM simulations is presented and an adaptive meshing scheme, involving both p- and h-type refinement. The generated discretizations lead to significantly higher accuracy for a given simulation size.

The BEM in Electrostatic Analysis The BEM is widely applied to the solution of electrostatic simulations [1]. The method divides the domain surfaces and interfaces into separate elements, each containing a number of boundary nodes. The electric potential and the flux in the outside normal direction on each element are computed from the node values using the element's interpolation functions, using Eqns. (b) in Table 1. For each node, a boundary integral equation (BIE) is formulated, relating its electric potential and flux to the values at the other nodes, using Eqn. (a). Boundary conditions impose additional restrictions on the node values, leading to a system of equations which determines potential and flux for the entire domain surface.

The accuracy of an electrostatic simulation is often estimated from the accuracy of an integral quantity, such as the computed cross capacitance matrix. However, in MEMS actuator simulations, the primary interest lies in computing electrostatic pressures. This requires highly accurate flux values at all boundary positions. The mesh is a crucial factor in obtaining such local accuracy.

Automatic Adaptivity and Error Indicator Automatic adaptivity is an iterative method to construct good meshes, in the sense that maximum accuracy is achieved for a given simulation size. A coarse initial mesh is improved by solving the simulation and then refining those elements where a large error is recognized. This involves evaluating an *error indicator*, an estimation of the error in each element. The proposed error indicator is based on the boundary integral equation Eqn. (a). For the exact solution, the equation is satisfied at every location on the boundary. Due to the discretization process, the BIE is only satisfied at node positions. Between the node positions, the potential and flux are computed using interpolation functions, see Eqns. (b). After having solved a simulation using a given initial mesh, the error at a boundary position is estimated from the mismatch between the interpolated potential value and the potential value computed by solving the BIE using the current boundary values. The error indicator of an entire element can be computed by integrating the mismatch over element's area, using numerical quadrature. The computational cost of calculating the error indicator can be greatly reduced using multipole acceleration [2]. This allows to evaluate each element's error indicator in constant time, regardless of the overall simulation size.

Mesh Refinement Once an element has been marked for refinement, different strategies are possible to increase its accuracy: The element can be split into several, smaller elements (h-type refinement), or the number of nodes on the element can be increased, along with the order of the element's interpolation functions (p-type refinement). The proposed scheme uses a combination of the two approaches, by applying p-type refinement to elements of less than cubic order and using h-type for any further refinement.

Results Fig. 1 shows the layout of an electrostatically actuated MEMS gyroscope. As initial mesh for the electrostatic analysis we chose a uniform discretization using constant boundary elements of 3 μm length. Four iterations of automatic adaptive refinement were applied to the initial mesh, leading to the solution and distribution of boundary nodes shown in Fig. 2. To estimate the accuracy of the computed results, a reference solution with uniformly sized, cubic order elements and 12,354 boundary nodes was computed. The error over an element with respect to the reference solution is computed using the first of Eqns. (c). The correlation between the error indicator and the actual error can be considered a measure for the indicator's quality. As the solution improves, the indicator quality will increase but even on the coarse, initial mesh bad elements are correctly identified (Fig. 3). The dependence of the overall error on the simulation size (number of boundary nodes) for different refinement strategies is shown in Fig 4. The automatic adaptive method allows to obtain high solution accuracy with significantly less boundary nodes than when using uniform mesh refinement.

[1] C.A. Brebbia, J.C.F. Telles, L.C. Wrobel, "Boundary Element Techniques, Theory and Applications in Engineering", *Springer Verlag Berlin*, 1994

[2] M. Bächtold, J.G. Korvink, H. Baltes, "Enhanced Multipole Acceleration Technique for the Solution of Large Poisson Computations", *submitted to IEEE Transactions on Computer Aided Design*, 1996

Table 1: Boundary integral equation, interpolation functions and error measures.

Boundary integral equation	$u(\xi) = \frac{1}{c(\xi)}\int_{\Gamma} q(x) \cdot u^*(\xi, x) \cdot d\Gamma(x) - \frac{1}{c(\xi)}\int_{\Gamma} u(x) \cdot q^*(\xi, x) \cdot d\Gamma(x)$	(a)
Symbols: Ω, Γ, ξ	: Simulation domain, simulation domain boundary, position on the boundary.	
$u(x)$, $q(x)$	: Electric potential and flux at the boundary location x.	
$c(\xi)$	: Boundary integration singularity factor at position ξ.	
$u^*(\xi, x)$	: Fundamental solution: The potential generated by a unit-sized charge at ξ.	
$q^*(\xi, x)$	: The flux through the boundary position x generated by a unit-sized charge at ξ.	
Interpolation on element Γ_k	$u(\eta) = \sum_i N_i(\eta) \cdot u_i$ and $q(\eta) = \sum_i N_i(\eta) \cdot q_i$	(b)
Symbols: $N_i(\eta)$	: Element interpolation functions at the position $\eta \in \Gamma_k$.	
u_i, q_i	: Potential and flux values at the nodes of element Γ_k.	
Error measures	$E_{\Gamma_k} = \int_{\Gamma_k} (q_{ref}(x) - q(x))^2 d\Gamma(x)$ and $E_{\Gamma} = \sum_k E_{\Gamma_k}$	(c)
Symbols: E_{Γ_k}, E_{Γ}	: Error measure for element Γ_k, overall error measure.	
$q_{ref}(x)$	: Reference solution for the flux distribution over the boundary.	
$q(x)$	: Approximate solution for the flux distribution over the boundary.	

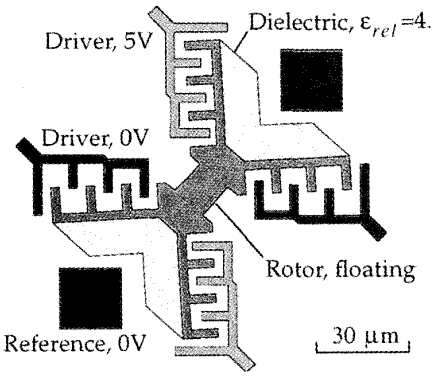

Fig. 1. Layout and boundary conditions of an electrostatically actuated MEMS gyroscope.

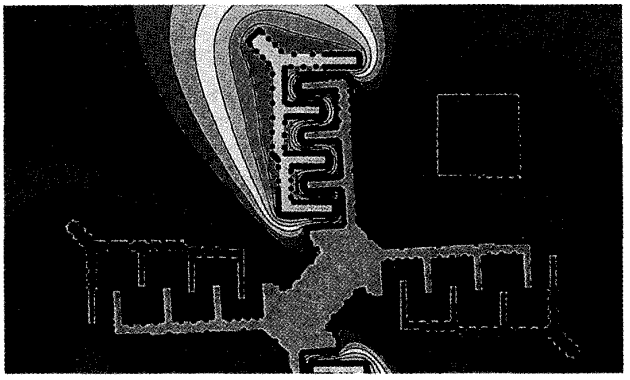

Fig. 2. Potential distribution and automatically adapted mesh. The marked positions at the interfaces indicate boundary nodes.

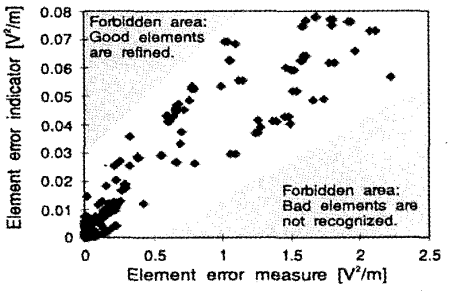

Fig. 3. Indicator value versus error measure for each element (constant elements, 3µm length).

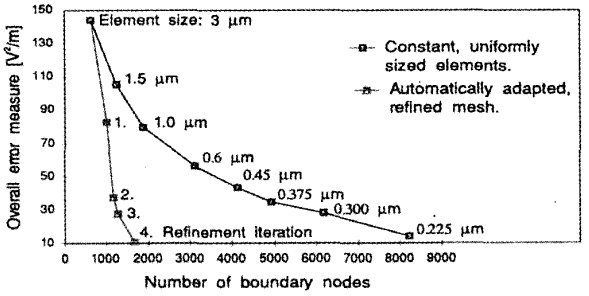

Fig. 4. Overall error measure vs. simulation size for different mesh refinement strategies.