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# Fundamental relation between local and effective transversefielddependent mobility for electrons in inversion channels

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**Abstract**

This paper describes a new modeling approach that relates the local mobility to the experimentally determined macroscopic or effective mobility as a function of the vertical electric field. Using the technique of integral representations [1] it is possible to find a mathematical condition which any local mobility model has to fulfill to be in accordance with experiment. A local mobility model used in a 2D device simulator has to meet this requirement in order to achieve quantitative agreement between experimental and simulated data.

**Introduction - derivation of the fundamental relation**

Numerical modeling of transport in semiconductor devices is playing an increasingly important role in their development. The mobility is a key quantity for the purpose of modeling. While the low- and high field behavior of the mobility in bulk Si is relatively well understood [2,3], the inversion channel mobility is more difficult to handle. It would require a fully self-consistent quantum-mechanical treatment of the carrier transport. However, because such a solution is still not within our reach, different analytical models have been proposed which are based on a careful inspection of the experimental material [4,5]. Several authors [6,7] have reported the experimentally measured effective (or average) mobility μeff as a function of the vertical field $E_\perp$ for a substantial range of doping, fixed interface charge and temperature.

The experimental material only provides an average mobility μeff which is related to the terminal current Id and the inversion layer density 'n' for small drain voltages Vd

$$\mu_{eff} = \lim_{Vd \to 0} \frac{I_d}{qnV_d} \tag{1}$$

The microscopic details are not available. Very useful models for the local mobility have been proposed by parameterizing the model by an effective field Eeff. However, they are in most cases using for the local model the one of the external average mobilty μeff. In 2D device simulators the current at each node in the MOS inversion layer is calculated using the local transverse field Eeff parameters for a chosen mobility model. It was noted in [8] that there should exists a relation between the measured μeff and the local mobility μl used within the device itself. However, their derivation is based on a simplified long channel analysis and neglected the variation of the vertical field along the Si/SiO2 interface. Using the technique of the integral representations [1], which has been successfully applied to investigate the short channel effect [9] and the velocity overshoot [1], it is possible to find a more general expression for the current depending on the internal local quantities E, Te and μl,

$$I_d = q\,W\,N_{sd}\,V_{th}\int_0^{\delta_n}\frac{(\exp(-\gamma(L,y))-1)}{\int_0^L \mu_l^{-1}(x)\exp(-\gamma(x,y))\,dx}dy \qquad with \quad \gamma(x)=\int_0^x \tfrac{-q}{kT_e(u)}E(u)\,du \tag{2}$$

To find the fundamental relation between the μeff and the local μl, this equation has to be compared with the conventional one,

$$I_d = \frac{W}{L}\mu_{eff}\,Q\,E_L \tag{3}$$

with Q the inversion sheet charge and EL the lateral electric field. At low Vds the electron temperature can be taken constant at 300°K. The fundamental equation becomes then

$$\mu_{eff}(E_{eff})QE_L = qN_{sd}\,V_{th}\left(\exp(-\tfrac{V_{ds}}{V_{th}})-1\right)\int_0^{\delta_n}\frac{dy}{\int_0^L \mu_l^{-1}(x)\exp(\frac{\psi(0,y)-\psi(x,y)}{V_{th}})\,dx} \tag{4}$$

Because the μeff is defined for Long channels, one can use a linear model for the potential ψ. Secondly, a transformation from the independent variable 'x' to the vertical field is possible through the relation

$$x = \left(\tfrac{\varepsilon_{si}}{\varepsilon_{ox}}t_{ox}E_\perp + V_{bi} - \psi_{gs}\right)\tfrac{1}{E_L} \tag{5}$$

which reduces (4) to

$$\mu_{eff}(E_{eff}) = \tfrac{C_{ox}}{\varepsilon_{si}}\exp\left(\tfrac{V_{bi}-\psi_{gs}}{V_{th}}\right)\frac{V_{th}(1-\exp(-\frac{V_{ds}}{V_{th}}))}{\int_{E_{\perp 0}}^{E_{\perp L}} \mu_l^{-1}(E_\perp)\,\exp(-\frac{\varepsilon_{ox}}{\varepsilon_{si}}\frac{t_{ox}E_\perp}{V_{th}})\,dE_\perp} \tag{6}$$

with $E_{\perp 0}$ and $E_{\perp L}$ the vertical fields at source and drain side. This is the fundamental relation between the experimentally measured μeff and the local mobility model. It can be seen that μeff is not just an average of μl but is modulated by an exponential function. If μl=μo is independent of the field, as is the case in the subthreshold regime, μeff=μo as expected.

## Discussion

To show the applicability of (6), we apply it to a particular mobility model often found in literature,

$$\frac{1}{\mu_l}=\frac{1}{\mu_o}+\sum_i \frac{E_\perp^{n_i}}{A_i}=\frac{1}{\mu_o}+\sum_i \frac{1}{\mu_o}\left(\frac{E_\perp}{E_{ci}}\right)^{n_i}=\frac{1+\sum_i\left(\frac{E_\perp}{E_{ci}}\right)^{n_i}}{\mu_o} \tag{7}$$

Plugging this model into (6) gives,

$$\mu_{eff}(\psi_{gs})=\frac{\mu_o}{1+\dfrac{\exp\left(\frac{V_{bi}-\psi_{gs}}{V_{th}}\right)}{(1-\exp(-\frac{V_{ds}}{V_{th}}))}\sum_i\left(\frac{E_{th}}{E_{ci}}\right)^{n_i}\left(\Gamma(n_i+1,\frac{E_{\perp L}}{E_{th}})-\Gamma(n_i+1,\frac{E_{\perp o}}{E_{th}})\right)} \tag{8}$$

where we have retained ψgs instead of Eeff. There is however a simple linear relation between ψgs and Eeff so and can go easily from one to the other. Fig.1 shows the measured mobility of devices fabricated in a standard 0.35 μm CMOS process together with the evaluation of the RHS of (8) for three sets of constants 'n' and 'Ec'. The first set is the one from fitting while the second and the third set are taken from [9] and [10]. The values for 'n' and 'Ec' are in Table 1.

It is apparent that the models proposed by most authors concentrate on reproducing directly the measured μeff, but that such a model cannot directly be used inside a 2D device simulator. Even though these models are a reasonable guess (as can be seen on Fig.1, they have nearly the right functional form but are off by a factor 2) for the real local mobility, one has to modify the fitting parameters according to (6) to give a good fit with experiment as shown in Fig.1.

## References


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| | n1 | Ec1 | n2 | Ec2 |
|---|---|---|---|---|
| best fit | 2.35 | 0.205 | - | - |
| after [7] | 0.657 | 0.0305 | - | - |
| after [8] | 0.33 | 0.0775 | 1 | 1.5625 |

Tabel 1 The model parameters used in Fig.1

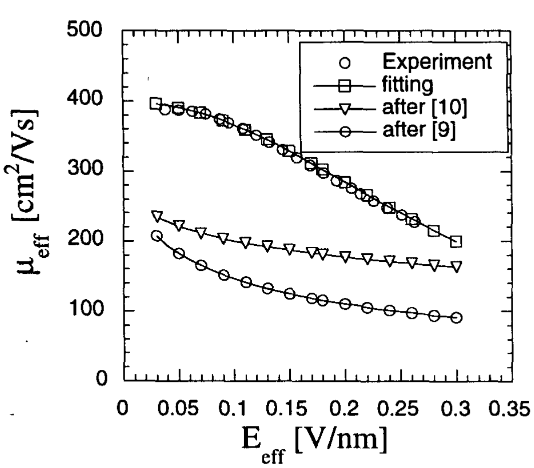


Fig.1 The measured effective mobility - experiment compared with three different models