Edited by H. Ryssel, P. Pichler - September 1995

Generalised Drift-Diffusion Model of Bipolar Transport in Semiconductors

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Abstract

A generalisation of the conventional relaxation-time approximation for bipolar transport with electron-hole scattering is presented. A simple phenomenological ansatz leads to Generalised Drift-Diffusion current equations, which contain both conventional Drift-Diffusion equations and matrix-form Drift-Diffusion equations with drag currents as special cases. The effect on carrier transport in semiconductor devices under low and high injection conditions is discussed analytically and compared with simulation results.

1. Introduction

The correct description of electron-hole scattering (EHS) influence on drift- and diffusion-dominated charge carrier transport is important for the simulation of all bipolar devices, especially for bipolar power devices, as it chiefly determines the voltage drop over the device in the forward biased high injection regime. Many authors [1-6] have investigated this subject, establishing two types of current equations, which can be characterised as conventional Drift-Diffusion equations (Van Roosbroeck-type) and matrix-form Drift-Diffusion equations with drag currents (Avakyants-type). However, once fitted to achieve agreement with ohmic mobility data, both approaches cannot describe other experiments properly. Avakyants-type equations predict an injection-independent ambipolar diffusion constant, in contradiction to experimental data [9]. Both types of equations fail to explain the asymmetric steady-state carrier distributions at high current densities [10]. In this paper, the analysis of the coupled Boltzmann Equations for electrons and holes leads to a natural generalisation of the usual relaxation-time approximation for the collision terms. A simple phenomenological ansatz yields analytic generalised Drift-Diffusion current expressions (GDD), which contain both Van Roosbroeck- and Avakyants-type models as limiting cases.

2. Derivation of Model Equations

The distribution functions for electrons and holes $f_e(\mathbf{k},\mathbf{x},t), f_h(\mathbf{k},\mathbf{x},t)$ are given by the solution of the coupled Boltzmann Transport Equations (BTEs)

$$D^e f_e = \left(\frac{\partial}{\partial t} + \frac{\hbar}{m_e} \mathbf{k} \cdot \nabla_{\mathbf{x}} - \frac{q}{\hbar} \mathbf{E} \cdot \nabla_{\mathbf{k}} \right) f_e(\mathbf{k},\mathbf{x},t) = \left(\frac{\partial f_e}{\partial t} \right)_{coll},$$
$$D^h f_h = \left(\frac{\partial}{\partial t} + \frac{\hbar}{m_h} \mathbf{k} \cdot \nabla_{\mathbf{x}} + \frac{q}{\hbar} \mathbf{E} \cdot \nabla_{\mathbf{k}} \right) f_h(\mathbf{k},\mathbf{x},t) = \left(\frac{\partial f_h}{\partial t} \right)_{coll}, \tag{1}$$

with the collision terms

$$\left(\frac{\partial f_e}{\partial t} \right)_{coll} = \left(\frac{\partial f_e}{\partial t} \right)_{coll}^{latt} + \left(\frac{\partial f_e}{\partial t} \right)_{coll}^{EHS} + \left(\frac{\partial f_e}{\partial t} \right)_{coll}^{EES},$$
$$\left(\frac{\partial f_h}{\partial t} \right)_{coll} = \left(\frac{\partial f_h}{\partial t} \right)_{coll}^{latt} + \left(\frac{\partial f_h}{\partial t} \right)_{coll}^{EHS} + \left(\frac{\partial f_h}{\partial t} \right)_{coll}^{HHS},$$

which can be separated into the terms caused by lattice scattering (mainly phonon and ionized impurities scattering), electron-hole scattering (EHS) and scattering of carriers of the same type (EES and HHS). At the moment, the EES and HHS terms will be neglected, since their influence on mobility is rather small and can be included by some phenomenological corrections to the end formulae [11,12].

For the linearised BTEs, the relaxation-time approximation proves to describe the collision terms for lattice scattering reasonably in most of the cases. The collision integrals due to EHS cannot be expressed by the non equilibrium parts of the distribution function of only one carrier type, since e.g. momentum transfer from non equilibrium holes can drag the electron distribution function out of initial equilibrium. The simplest physically consistent approximation for the EHS collision terms is a modified relaxation-time ansatz, which includes additional terms caused by carrier drag:

$$\left(\frac{\partial f_e}{\partial t} \right)_{coll}^{EHS} \approx -\frac{f_e - f_e^0}{\tau_{eh}(\mathbf{k})} + \beta_e(\mathbf{k}) \frac{f_h - f_h^0}{\tau_{he}(\mathbf{k})}, \quad \left(\frac{\partial f_h}{\partial t} \right)_{coll}^{EHS} \approx -\frac{f_h - f_h^0}{\tau_{he}(\mathbf{k})} + \beta_h(\mathbf{k}) \frac{f_e - f_e^0}{\tau_{eh}(\mathbf{k})}. \tag{2}$$

The superscript "0" in (2) denotes equilibrium distribution functions, τ_{eh}, τ_{he} are relaxation times of electron and hole systems due to EHS, and β_e, β_h are some phenomenological coupling functions. Equation (2) is far from being mathematically exact, but it gives a qualitative description of the two main physical processes caused by EHS : the relaxation of the non equilibrium part (first terms) as well as the drag out of equilibrium due to the existence of the non equilibrium part of the scattering partner (second terms). Using the approximation (2), the relaxation-time approximation for lattice scattering and neglecting the influence of same type carrier scattering, a formal solution of the coupled BTEs (1) can be derived. A further approximation step yields the Generalised Drift-Diffusion current equations

$$j_p = q\mu_p^p (pE - \frac{kT}{q} \nabla p) - q\mu_p^n (nE + \frac{kT}{q} \nabla n)$$
$$j_n = q\mu_n^n (nE + \frac{kT}{q} \nabla n) - q\mu_n^p (pE - \frac{kT}{q} \nabla p) \tag{3}$$

with the mobilities

$$\mu_p^p = \frac{\mu_p^0\left(1+\mu_n^0/\mu_{np}\right)}{\Delta}, \quad \mu_p^n = \frac{\beta\mu_n^0\mu_p^0}{\Delta\mu_{np}} \quad \mu_n^n = \frac{\mu_n^0\left(1+\mu_p^0/\mu_{pn}\right)}{\Delta}, \quad \mu_n^p = \frac{\beta\mu_p^0\mu_n^0}{\Delta\mu_{pn}}$$
$$\Delta = 1+\frac{\mu_n^0}{\mu_{np}}+\frac{\mu_p^0}{\mu_{pn}}+\left(1-\beta^2\right)\frac{\mu_n^0\mu_p^0}{\mu_{np}\mu_{pn}} . \tag{4}$$

μ_n^0, μ_p^0, are electron and hole lattice mobilities due to phonon and ionized impurity scattering, $\mu_{pn} = C/n, \mu_{np} = C/p$ are electron-hole mobilities (the constant C can weakly depend on $(n+p)$ because of screening effects) and β is a phenomenological drag parameter. A detailed motivation of (2) and (3) will be published in a longer paper [12]. It can be easily shown that for the special case β=0 the off-diagonal mobilities vanish and the equations (3,4) transform into the conventional Van Roosbroeck current equations. For the case β=1 the GDD becomes identical to the Avakyants-type equations in the form proposed by Mnatsakanov et al. [2,3,5].

3. Consequences for Bipolar Devices

For quasi neutral bulk regions, homogeneous doping implies $\nabla n \approx \nabla p$ and (3) becomes

$$j_p = qp\left(\mu_p^p - \mu_n^p\right)E - kT\left(\mu_p^p + \mu_p^n\right)\nabla p = q\mu_p^{Drift}pE - kT\mu_p^{Diff}\nabla p$$
$$j_n = qn\left(\mu_n^n - \mu_p^n\right)E + kT\left(\mu_n^n + \mu_n^p\right)\nabla n = q\mu_n^{Drift}nE + kT\mu_n^{Diff}\nabla n \tag{5}$$

Equation (5) differs from the conventional current equation due to the fact that for $\beta \neq 0$ the drift and diffusion mobilities are not equal. Using (4), one can show that for low injection conditions the *minority diffusion mobility* can be approximated as the usual Mathiessen combination of lattice and EHS mobilities and is practically independent on the choice of β. Therefore, as long as the current over a pn-junction can be described by the sum of minority diffusion currents (Shockley's approximation), the DC current-voltage characteristic should not depend on the choice of β and no difference should occur between diode characteristics calculated by Avakyants- or Van Roosbroeck current equations. Fig. 1 shows the IU-characteristics for a highly doped np-diode for $\beta = 0$ and $\beta = 1$. The resulting curves coincide over a long range of current values and only differ slightly in the high current region, where deviations from the exponential behavior due to ohmic losses occur. At high currents, the voltage drop is higher for β=1, since the drift mobility decreases with rising β.

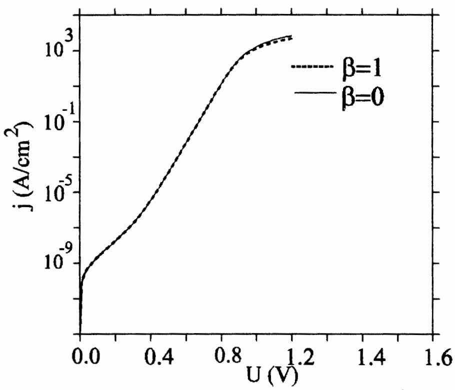

Fig. 1 : Calculated IU characteristics for a n^+p-diode and two values of β.

For high injection conditions, typically occurring in the low doped base of *psn*-diodes and thyristors, the voltage drop over the base region is determined by the sum of *drift mobilities*, which is therefore the quantity measured by voltage drop experiments like those performed by Dannhäuser and Krause [7,8]. Fig. 2 shows the numerically calculated current voltage characteristics of a wide base (440 µm) *psn*-diode for three different values of β. Obviously, the choice of the drag parameter β has significant influence on the forward voltage drop under high injection conditions.

The modeling of EHS also strongly affects the stationary spatial distribution of charge carriers. The simulated carrier distributions in the base are shown in Fig. 3 for a current density of 1000 A/cm^2 and three values of β: β=1 yields a distribution completely symmetrical around the minimum, while β=0 produces very strong asymmetries, moving the distribution's minimum towards the n-zone.

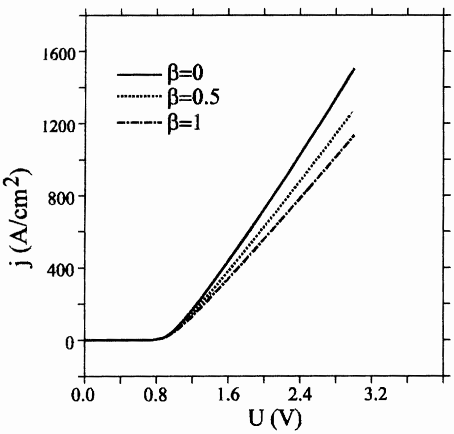

Fig. 2: Calculated current-voltage characteristics of a long base *psn*-diode

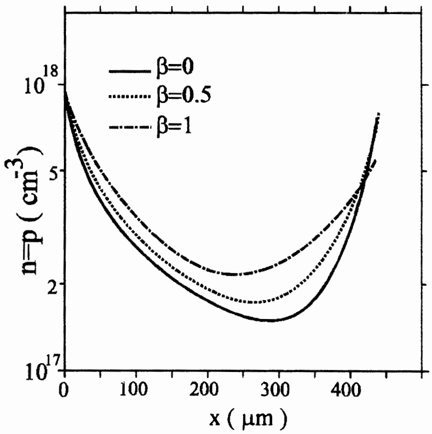

Fig. 3: Charge carrier distribution in the base of the diode from Fig. 2 at j=1000 A/cm^2

Acknowledgements

The author is indebted to R. Nürnberg and H. Gajewski, Weierstrass-Institute for Applied Analysis and Stochastics, for the implementation of the GDD into the 2D device simulator ToSCA.

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