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On the Integral Representations of Electrical Characteristics in Si Devices

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Abstract

It is shown that the 1D hydrodynamic model of the fundamental transport equations in differential form can be transformed into an equivalent integral representation. The advantage of this procedure lies in the fact that integrals are generally easier to evaluate than the corresponding differential equations. The technique of the integral representations is applied to two examples. In case of a MOSFET, a closed form analytical expression for the carrier concentration and the velocity is obtained. In case the electric field is a step-function with a strong discontinuity, the influence of the diffusion effect as well as the mobility model on the steady state velocity overshoot is analysed without the need for a dedicated numerical solver.

1. Introduction

Numerical modeling of transport in semiconductor devices plays a crucial role in their development. As MOSFET's are scaled down to the 0.1 μm range, the channel length approaches the mean free path of charge carriers and effects as non-stationary and quantum transport become apparent.

In conventional semiconductor devices, most quantum transport effects can be treated indirectly. For instance the effects of the rapidly varying crystal potential on electron transport can be modeled through the concepts of effective masses, energy gaps and the positively charged quasi-particle holes. On the macroscopic level, charge transport can then be modeled using the concepts of the semiclassical model.

In 1969, Rees [1] predicted an overshoot of the carrier mean velocity in semiconductors following rapid changes of the electric field by a self-consistent solution of the Boltzmann equation. In 1976, Shur [2] demonstrated that the full solution of the Boltzmann equation is not necessary. Using a coupled system of simplified particle-, momentum- and energy balance equations, called the hydrodynamic model, the basic physical mechanism underlying the velocity overshoot can be explained.

In principle, the Monte Carlo technique is superior to the solution of the moment equations, however, for the fundamental physical understanding of high field- and strong gradient effects, it is often satisfactory and preferable to go to analytic solutions. For instance, the saturation of carrier velocity in presence of constant high electric fields can be analytically assessed [3] using the three balance equations. It is shown that velocity saturation is caused by carrier heating (T_e>300°K). However, when strong gradients in the field are present, ensemble (diffusion) effects make the local models ($\mu(E)$) inaccurate, and non-local models ($\mu(Te)$ or $\mu(E_n)$) prevail.

The purpose of this report is to present the efficient technique of integral representations to calculate the physical quantities from the hydrodynamic model in a closed form expression. It is shown that the 1D hydrodynamic model in differential form can be reformulated in a set of integral type equations. There is a distinct advantage in using them as an analysis tool for devices because they can easily be evaluated. The integral expressions are written in a general way independent of the particular μ-model used. As such, it is a good basis to evaluate and compare different μ-models.

2. Model description

The hydrodynamic model (HD) together with the Poisson equation are well assessed [3]. In principle, these four equations are coupled which makes it extremely difficult to solve them, even numerically. However, by assuming no generation-recombination, by simplifying correctly the energy balance equation (e.g.[4]) and by solving the Poisson equation independently of the HD-model [5,6], it is possible to calculate the carrier density (n), the velocity (v), the current density (J) and the carrier temperature (T_e). The potential (ψ) is calculated from the Poisson, 'n' from the particle balance, 'J' from the momentum balance and 'T_e' from the energy balance equation.

As a first example, the integral representation technique was applied within the drift-diffusion approach (Te=300°K) in [7] to investigate the *short channel effect*. The details of the calculation can be found in [7] for a constant mobility (only dependent on the substrate concentration). In this report, we extend it to include high field effects ($\mu(E)$)

$$n(x) = \frac{n_i^2}{N_{sub}} exp\left(\frac{\psi(x)}{V_{th}}\right)\left[1-(1-exp(-\frac{V_{DS}}{V_{th}}))\frac{\int_0^x exp(\frac{\psi(u)}{V_{th}})\mu^{-1}(u)du}{\int_0^L exp(\frac{\psi(u)}{V_{th}})\mu^{-1}(u)du}\right] \tag{1}$$

with $\mu(x)$ any local high field mobility model (e.g. the Caughey-Thomas model [9]). The first factor gives the equilibrium concentration while the second one acts as a modulation factor. Fig.1 shows n(x) for both constant and field dependent mobilities together with MEDICI simulations. The field dependent mobility makes that n(x) increases to compensate for the loss in velocity. The velocity can be calculated by plugging (1) into the conventional Drift-Diffusion (DD) equation (i.e. T_e=300°K) and compare the expression with J=-qn(x)v(x). This results in

$$v(x)=\frac{n_i^2}{N_{sub}}\frac{V_{th}(1-exp(-\frac{V_{DS}}{V_{th}}))}{\int_0^L exp(\frac{\psi(u)}{V_{th}})\mu^{-1}(u)du}\frac{1}{n(x)} \tag{2}$$

In a second example, we look at the problem of the step-like field (see insert of Fig.2) mentioned in [10] including the energy balance equation to investigate the *velocity overshoot*. At first, the Slotboom [4] approximation of the energy balance equation for T_e,

$$T_e(x)=T_l+\frac{2}{5}\frac{q}{k}\int_0^x E(u)exp[\frac{u-x}{\lambda}]du \tag{3}$$

is used with T_m =T_e(L) and n_m=n(L) and λ=30 nm. The total set of integral equations can be straight-forwardly calculated as in [7]. We give here the result for 'n'

$$n(x)=n_o\frac{T_l}{T_e(x)}exp[\psi^*(x)][1-(1-\frac{T_m}{T_l}\frac{n_m}{n_o}exp[-\psi^*(x)])\frac{\int_0^x\mu^{-1}(u)exp[-\psi^*(u)]du}{\int_0^L\mu^{-1}(u)exp[-\psi^*(u)]du}] \tag{4}$$

$$with \qquad \psi^*(x)=\int_0^x E(u)\frac{q}{kT_e(u)}du$$

For this simplified case-study, all the integrals can be calculated analytically. The velocity is expressed in a similar way as (2). Fig.2 pictures T_e(x) showing an increased temperature for higher fields. Fig.3 and 4 show the results for n(x) and v(x) respectively for both the present HD-model as well as for the conventional DD-model (T_e=300°K).

3. Discussion

The carrier density 'n' (3) and the velocity 'v' within the HD-model depend on the type of mobility model used. Fig.3 and 4 show clearly the difference between local (only function of E) and non-local (function of T_e) μ-models. In case of a T_e dependent mobility, a velocity overshoot is observed while in case of an E dependent μ the opposite is true.
Another example, the calculation of a one carrier metal-semiconductor rectifier using an integral representation of J (expression (34)) was done by Stratton [8].
The evaluations of the integrals, done with the software package Mathematica running on a Macintosh Quadra 650, take about 20 s of time.

4. Conclusions

The 1D hydrodynamic model in differential form was transformed into an integral representation of the fundamental transport equations. The driving force for this procedure was the fact that integrals are generally easier to evaluate than the corresponding differential equations. The general applicability of the transport integrals to any transport problem was pointed out. Two examples were discussed. The integral representations are widely applicable to classic textbook examples (e.g.[9]) as well as to realistic devices (e.g.[7]).

References

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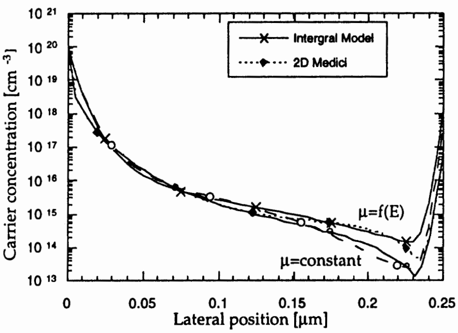

Fig.1 Comparison between MEDICI and integral model (1) for a constant and a field dependent mobility

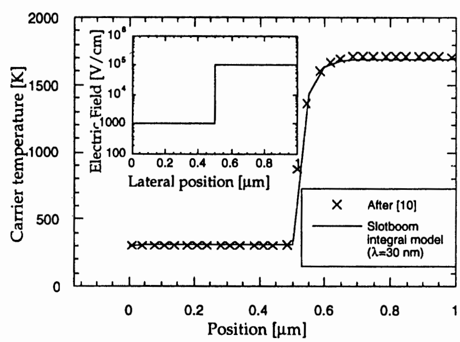

Fig.2 Carrier temperature for the given step-like electric field (insert)

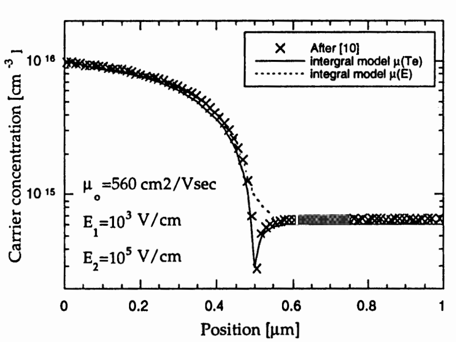

Fig.3 Comparison of the carrier concentration between the model of Baccarani [10] and the new integral model

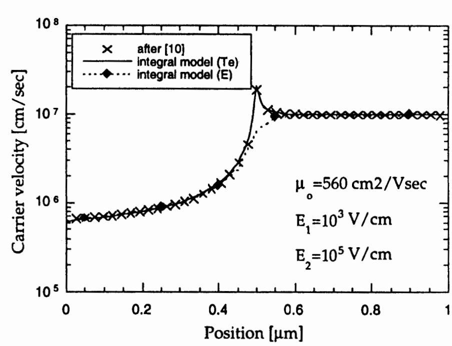

Fig.4 Comparison of the carrier velocity between the model of Baccarani and the new integral model