SIMULATION OF SEMICONDUCTOR DEVICES AND PROCESSES Vol. 5
Edited by S. Selberherr, H. Stippel, E. Strasser - September 1993

Modeling of Electron-Hole Scattering in Semiconductor Power Device Simulation

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Abstract

This paper gives an overview of different approaches of including electron-hole scattering (EHS) in physical semiconductor simulation models and compare the influence on the simulation results.

In the majority of semiconductor simulation codes, the physical model is based on the numerical solution of the *Van Roosbroeck* equations [1], consisting of the *Poisson* equation, the continuity equations, and the drift-diffusion transport equations. The latter originate from the solution of the *Boltzmann* kinetic equation in case of negligible electron-hole scattering (abbreviated as EHS in the following). Thus, the carrier transport in semiconductors is conventionally described by two equations in the form of

$$j_n = q\mu_n nE + qD_n\frac{\partial n}{\partial x}, \qquad j_p = q\mu_p pE - qD_p\frac{\partial p}{\partial x} \tag{1}$$

The conventional way to account for the EHS is to modify the carrier mobilities in Eq.(1) using the following reciprocal mobility summation rule (*Mathiessen*'s rule),

$$\frac{1}{\mu_n} = \frac{1}{\mu_{no}} + \frac{1}{\mu_{np}}, \qquad \frac{1}{\mu_p} = \frac{1}{\mu_{po}} + \frac{1}{\mu_{pn}} \tag{2}$$

where μ_{np} is the mobility component due to electron scattering on holes, μ_{pn} is the mobility component due to hole scattering on electrons, and the subscript o denotes the mobility components independent of EHS. This approach was proposed by *Fletcher* [2] and since then it has widely been used. However, by a conventional use of Eq.(2) it is assumed, that $\mu_{np}=\mu_{pn}$, which is correct only in a particular case of equal concentrations $n=p$.

For practical device analysis, the EHS related mobility components had to be evaluated. The first theoretical expression for μ_{np}, when $n=p$, was proposed by *Fletcher* and the first measurements were made by *Davies* [3] on a Ge P^+–L–N^+ structure. By extraction of μ_{np} from a measured voltage drop on the device *Davies* made the assumption, that

$$\frac{1}{\mu_n+\mu_p} = \frac{1}{\mu_{no}+\mu_{po}} + \frac{1}{\mu_{np}} \tag{3}$$

This summation rule differs principally from the *Fletcher* summation of Eq.(2), which turns out to be incorrect in case of any momentum exchange between the electron and hole subsystems. *Davies* also presented a theoretical expression for the calculation of μ_{np}.

Models from [4,5,6] and [7,8,9] are all of *Fletcher*- or *Davies*-type, respectively. Models from [10,11] are empirical and are mainly based on experimental data by *Dannhäuser* [12] and *Krausse* [13]. The *Davies*-type models give somewhat lower mobilities in comparison to the *Fletcher*-type models. The *Dorkel–Leturcq* model [14], employed in the commercially available simulator MEDICI [29], is using *Choo*'s [4] μ_{np} expression and gives at the case $n=p$ lower mobility values than the model used in the program DYNAMIT (developed at the Institute of Electronics, Tallinn Technical University). This is the main reason of elevated forward voltage drops.

Another way for accounting EHS was proposed by the Armenian scientist

Avakyants and his co-workers in 1963 [15], starting from phenomenological plasma theory. From the *Avakyants*-type carrier motion equations the following current equations can be derived [16]

$$j_n = q\mu_{n1} nE + k_B T\mu_{n2}\frac{\partial n}{\partial x} + k_B T\mu_{n3}\frac{\partial p}{\partial x} \tag{4}$$

$$j_p = q\mu_{p1} pE - k_B T\mu_{p2}\frac{\partial p}{\partial x} - k_B T\mu_{p3}\frac{\partial n}{\partial x} \tag{5}$$

where $\mu_{n1} = \mu_{n2} - \mu_{p3}$, $\mu_{p1} = \mu_{p2} - \mu_{n3}$ and $n \cdot \mu_{p3} = p \cdot \mu_{n3}$ (6)

These current equations differ from the *Van Roosbroeck* equations by two principal features. To begin with, there are extra cross terms representing the carrier drag effect [17–19]. Thereafter, the *Einstein* relationship does not hold for the electron and hole drift mobilities μ_{n1}, μ_{p1} and for the relevant diffusion coefficients in the second and the third terms. Later on it was shown [20], that the validity of *Einstein*'s relationship can be restored by writing the current equations in a matrix form.

The *Avakyants*-type current equations with explicitly written drag terms appeared probably for the first time in 1972 [8] and in 1976 [9]. However, the same equations can be obtained by solving the *Boltzmann* kinetic equation using *Kohler*'s variational principle [21 - 24]. These more accurate, modified current equations, Eqs.(4-6), have been introduced into the numerical simulation practice in 1983 [16, 25] by the simulation code called DYNAMIT and since then have been used for power device simulation, eg. [26–28].

The purpose of this paper is to show how these different approaches influence the simulation results in case of high current densities (J>100 A/cm2). We have chosen a semiconductor power diode and have made simulations for the following cases:

Case #1: Full EHS model with the modified transport equations, Eqs.(4-6) These simulations were made by using the code DYNAMIT. The mobilities are specified as follows:

$$\mu_{n1} = \mu_{n2} - \mu_{n3}, \quad \mu_{n2} = \mu_{nB} + \mu_{n3}, \quad \mu_{p1} = \mu_{p2} - \mu_{p3}, \quad \mu_{p2} = \mu_{pB} + \mu_{p3} \tag{7}$$

$$\mu_{n3} = \mu_{nB}\mu_{po} n \cdot J^{eh}, \quad \mu_{p3} = \mu_{pB}\mu_{no} p \cdot J^{eh}, \quad \mu_{nB} = B \cdot \mu_{no}, \quad \mu_{pB} = B \cdot \mu_{po} \tag{8}$$

$$B = \frac{1}{1 + \mu_{no} p \cdot J^{eh} + \mu_{po} n \cdot J^{eh}} \tag{9}$$

where J^{eh} is the EHS function [30]. In our DYNAMIT simulations, the following empirical formula [10] has been used

$$J^{eh} = 1.3513 \cdot 10^{-20} \frac{1 + 3.591 \cdot 10^{-18} \frac{n+p}{2} [cm^{-3}]}{1 + 2.857 \cdot 10^{-17} \frac{n+p}{2} [cm^{-3}]} \quad [V \cdot sec \cdot cm] \tag{10}$$

The function J^{eh} can be represented using the mobilities μ_{np} or μ_{pn} as follows[3]

$$J^{eh} = \frac{1}{p \cdot \mu_{np}} = \frac{1}{n \cdot \mu_{pn}} \tag{11}$$

Case #2: Like in case #1, but neglecting the cross-terms in the modified transport equations, Eqs.(4-5). The mobilities are specified in the followings:

$$\mu_{n3} = \mu_{p3} = 0, \quad \mu_{n1} = \mu_{n2} = \mu_{nB}, \quad \mu_{p1} = \mu_{p2} = \mu_{pB} \tag{12}$$

Case #3: Like in case #2, but using the *Fletcher*-type summation of mobilities. Electrons and holes as scattering centers are treated as non-drifting impurities. As a consequence, cross-terms disappear ($\mu_{n3} = \mu_{p3} = 0$) and the following formulas are valid:

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[3] This relationship was originally established by *Avakyants* and *Lazarev* [31], who used the notation α instead of our J^{eh}.

$$\mu_{n1} = \mu_{n2} = \mu_{nB} \Big|_{\mu_{po}=0} = \frac{\mu_{no}}{1+\mu_{no}p\cdot J^{eh}} = \frac{1}{\frac{1}{\mu_{no}}+\frac{1}{\mu_{np}}} \tag{13}$$

$$\mu_{p1} = \mu_{p2} = \mu_{pB} \Big|_{\mu_{no}=0} = \frac{\mu_{po}}{1+\mu_{po}n\cdot J^{eh}} = \frac{1}{\frac{1}{\mu_{po}}+\frac{1}{\mu_{pn}}} \tag{14}$$

Case #4 : With no EHS taken into account at all, calculated by DYNAMIT. The scattering function $J^{eh} = 0$, and

$$\mu_{n3} = \mu_{p3} = 0\,, \quad \mu_{n1} = \mu_{n2} = \mu_{no}\,, \quad \mu_{p1} = \mu_{p2} = \mu_{po} \tag{15}$$

Case #5: EHS effect included only in the mobilities. These calculations were made by MEDICI [29], using it's *Dorkel-Leturcq*-type mobility model. All other physical models and their parameters were identical to those used in the DYNAMIT simulations, also

$$\mu_{n3} = \mu_{p3} = 0\,, \quad \mu_{n1} = \mu_{n2} = \mu_{n}\,, \quad \mu_{p1} = \mu_{p2} = \mu_{p} \tag{16}$$

Next, we demonstrate the differences between these 5 cases in 3 figures.

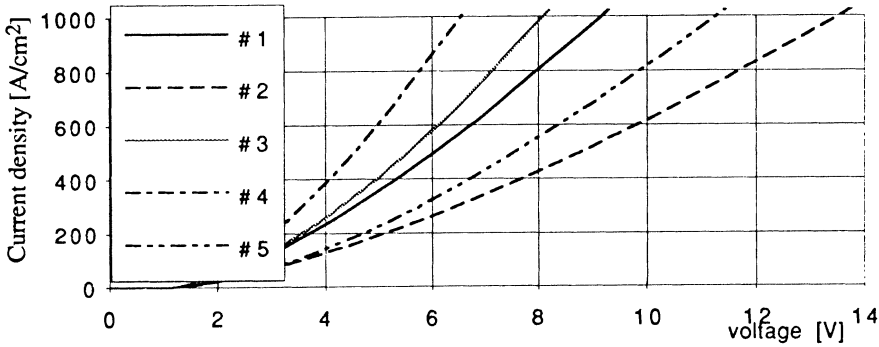

Fig.1. Calculated IV characteristics in the mentioned 5 different cases

The simulation results show a rather big difference in I-V characteristics and the structure internal variable distributions depending on the EHS accounting way. At higher current densities the forward voltage drop is predominantly determined by the electric field integral over the thick base region, where $n \approx p$ and the drift transport of carriers is dominating. The local electric field in the base is then approximately given by

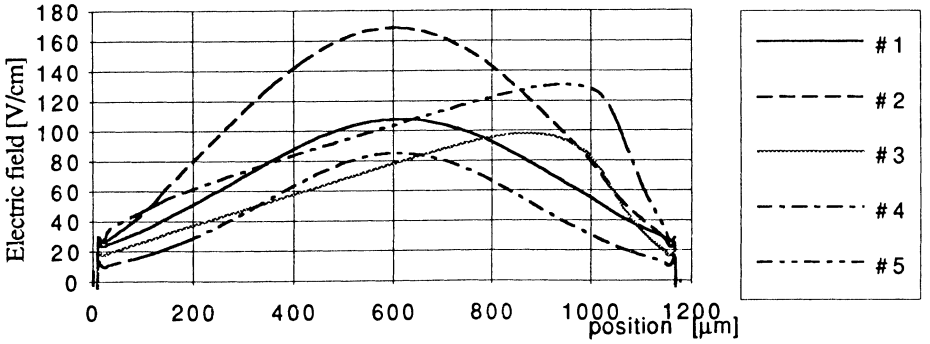

Fig.2. Calculated electric field distributions in the mentioned 5 different cases, J=1000 A/cm2

$$E = \frac{j}{qn(\mu_{n1}+\mu_{p1})} \tag{17}$$

where $j=j_n+j_p$. Thus, the structure voltage drop at the given total current density j is directly depending how the selected EHS accounting way is affecting both the carrier distribution $n(x)$ and the sum of the drift mobilities $\mu_{n1}+\mu_{p1}=f[n(x)]$.

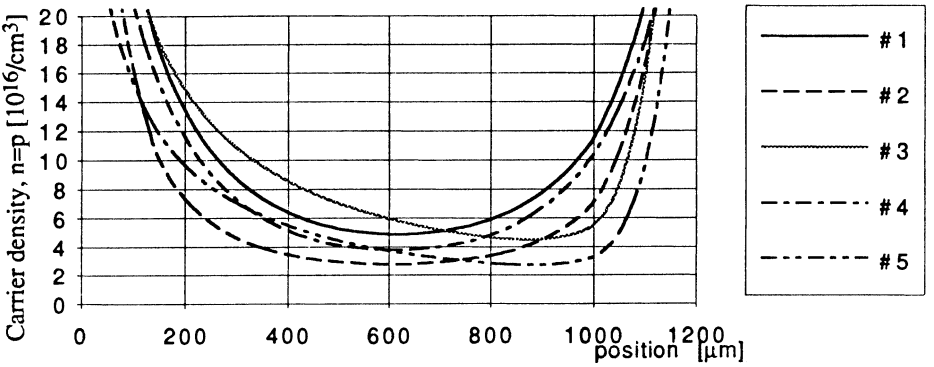

Fig.3. Calculated carrier distributions in the mentioned 5 different cases, J=1000 A/cm^2

Here we will point out only two important conclusions. First, neglecting cross-terms in Eq.(4-5) and simultaneously keeping $J^{eh}\neq 0$ (our case #2) is physically inadequate. Second, the *Fletcher*-type mobility summation in Eq.(2) results in a distortion of the carrier distribution shape, turning it strongly unsymmetrical at higher current densities. This is caused by a strong decrease of the ambipolar diffusion coefficient D_a at higher injection levels, which is not consistent with the *Kohler*'s variational principle solution, as noticed in [22-24,30]. A more detailed consideration of various aspects of the EHS influence will be published elsewhere.

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