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Critical Assessment of Different Hydrodynamical Models for Avalanche Multiplication Calculation in Silicon Bipolar Transistors

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Abstract

Different hydrodynamical numerical and analytical models for avalance multiplication coefficient calculation in silicon BJTs are considered. A comparison with experimental data is made.

1. Introduction

Non-uniform electron energy distribution has a significant effect on impact ionization phenomena in silicon bipolar transistors. A number of different analytical and numerical hydrodynamic (HD) models were suggested and used by different authors during the last few years [1-4] to calculate ionization currents in silicon devices. The purpose of this report is to investigate the applicability of these models for avalanche multiplication coefficient M_e calculation in silicon n-p-n BJT.

2. Model descriptions

For the sake of simplicity we will consider a one-dimensional case and use the simplest HD model [3] which will be referred to as HDM1:

$$\frac{dJ}{dx} = q(R - G) \tag{1}$$

$$J = q\mu nF + \mu \frac{dnk_BT}{dx}, \tag{2}$$

$$\frac{dSn}{dx} = JF - \frac{3}{2}nk_B\frac{T - T_0}{\tau_w}, \tag{3}$$

$$Sn = -\frac{k_BT}{q}[C_1J + C_2\mu n\frac{dk_BT}{dx}], \tag{4}$$

where all notations have their usual meaning. In general, coefficients τ_w, C_1, C_2 are functions of electron energy W, but they are taken as constants in [3] and we will use

the values $\tau_w = 0.3$ ps, $C_1 = 1.7$, $C_2 = 1.2$ which were obtained by fitting simulation results with experimental and Monte-Carlo data [5]. We ignore the impact ionization rate term in (3) and therefore restrict ourselves to small M_e values.

To close this set of equations, we add Poisson's equation for electric potential, the continuity equation for hole concentration (for which we shall assume constant hole temperature distribution) and corresponding boundary conditions.

To calculate M_e values one should know the $W(x)$ distribution in the BJT because ionization coefficients depend on W but not on local field. The HD model calculates this distribution but takes much more time than the drift-diffusion (DD) model. That is why there were several attempts to find a simple formula for the $W(x)$ calculation using $F(x)$ obtained with the DD model [1,4]. Let us briefly trace the method of derivation of these forlmulas.

Neglecting the temperature diffusion term in (4) and assuming that $div(J) = 0$, equations (4) and (3) reduce to the first order differential equation

$$C_1 v \frac{k_B T}{dx} = -qvF - \frac{3}{2} k_B \frac{T - T_0}{\tau_w} \tag{5}$$

where $v = -J/qn$ is the electron velocity. We found out that the first assumption significantly influences the $v(x)$ distribution but much less the $T(x)$ distribution. The second assumption is correct because in the base-collector space-charge region (SCR) we have nearly one-dimensional electron flow without recombination. To proceed, one should make an approximation for $v(x)$. Using $2/3C_1 v(x)\tau_w = \lambda_w = const$, it is possible to integrate (5) analytically and obtain [1]:

$$T(x) - T_0 = -\frac{q}{k_B C_1} \int_0^x F(z) exp(\frac{x - z}{\lambda_w}) dz \tag{6}$$

This is obviously a poor approximation because v is position dependent and C_1 and τ_w in general depend on electron energy.

One can also use (2) to obtain v and, after substituting into (5), obtain the non-linear differential equation

$$\frac{dT}{dx} = -\frac{1}{2}[\frac{q}{k_B} F(1 + \frac{1}{C_1}) + T\frac{dln(n)}{dx}] - \sqrt{[\frac{q}{k_B} F(1 - \frac{1}{C_1}) + T\frac{dln(n)}{dx}]^2/4 + \frac{1.5q}{k_B}\frac{T - T_0}{C_1 \tau_w \mu}} \tag{7}$$

which should be solved by numerical integration. For $W = 3/2k_B T$, $C_1 = 5/2$ and $dln(n)/dx = 0$ (7) reduces to the equation, used in [4] for MOSFETs. However we cannot neglect the electron concentration gradient in the collector-base SCR and hence using the $n(x)$ distribution calculated by the DD model, will obviously be a poor approximation.

3. Results and discussion

A special numerical code has been designed for 1D BJT simulation using the DD model, the HDM1 and the more complicated HD model from [2] (which will be referred to as HDM2). For the DD calculations we used (6) or (7) to calculate the $T(x)$ distribution and in all cases for *effective field* calculation we use an equilibrium relationship

$$F_{eff} = \sqrt{V_T 1.5(T/T_0 - 1)/\mu/\tau_w} \tag{8}$$

which follows from (3) for $dSn/dx = 0$. The $F_{eff}(x)$ dependence was used for the ionization integral and, finally, for M_e calculation. Calculations using (6) were done with $\lambda_w = 34$ nm which corresponds to the assumption $v(x) = v_s = 10^7$ cm/s. Shown on Fig.1 are calculated $W(x)$ distributions for the BJT from [1] with $N_C = 6 \times 10^{17}$ cm^{-3}. A few remarks are appropriate:

1. Calculations using (6) and (7) give higher maximum W value and significantly broader $W(x)$ distribution near maximum value than do HD calculations. This is the consequence of a poor $v(x)$ distribution approximation in both methods. Accounting for the $n(x)$ gradient, calculated by the DD model, in (7) helps only a little.

2. The difference between $W(x)$ distributions calculated by HD and DD models depends on V_{CB} (compare Fig. 1a and 1b). That is why even treating C_1 and λ_w in (6) as fitting parameters, it is impossible to achieve good agreement for different V_{CB} values even for the same BJT.

3. Both HD models agree well in spite of the difference in their coefficients.

Finally, we calculated $M_e(V_{CB})$ dependencies using these models and $\alpha(F)$ dependence from [6] (see Fig. 2). We found out that (6) with $C_1 = 2.5$ and (7) with $C_1 = 2.4$ give maximum $W(x)$ values which are very close to those calculated by the HD models (but the shape of the $W(x)$ distributions remain different). Roughly speaking this corresponds to a λ_w value increased to 50 nm and partially explains why in previous work [1,5,7] significantly larger λ_w values were used in order to achieve agreement with experimental data. The other reason for the λ_w difference is that the real C_1 and τ_w values for the high-energy regime can be larger than those we used.

We see that agreement between all calculated and experimental curves is quite good and can be further improved by using other $\alpha(F)$ dependencies, available in the literature. Formula (7) seems to give a slightly better result than (6) because it uses a slightly better approximation for the $v(x)$ distribution.

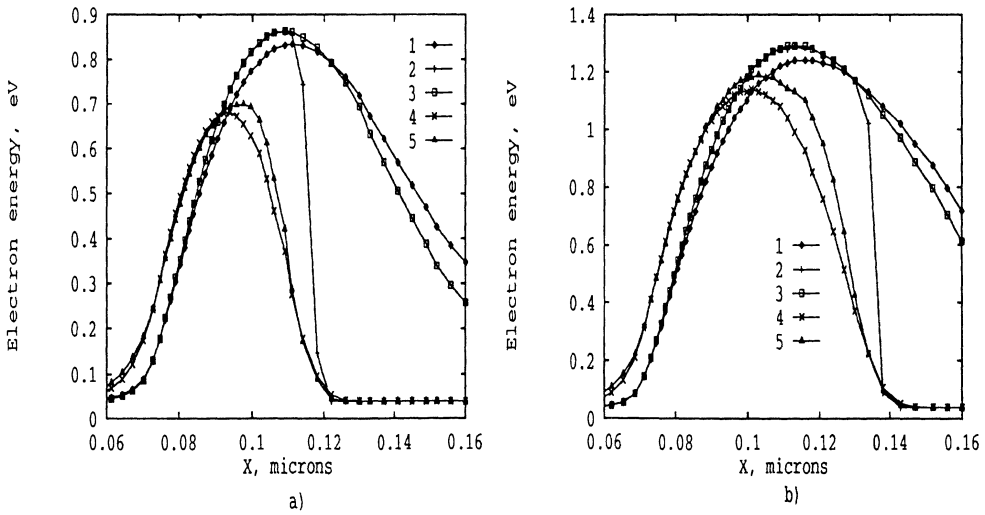

Fig.1. Calculated dependencies of electron energy for a) $V_{CB} = 1$V, b) $V_{CB} = 2.5$V, using: 1 - (6); 2 - (7); 3 - (7) with $dln(n)/dx = 0$; 4 - HDM1[3]; 5 - HDM2 [2].

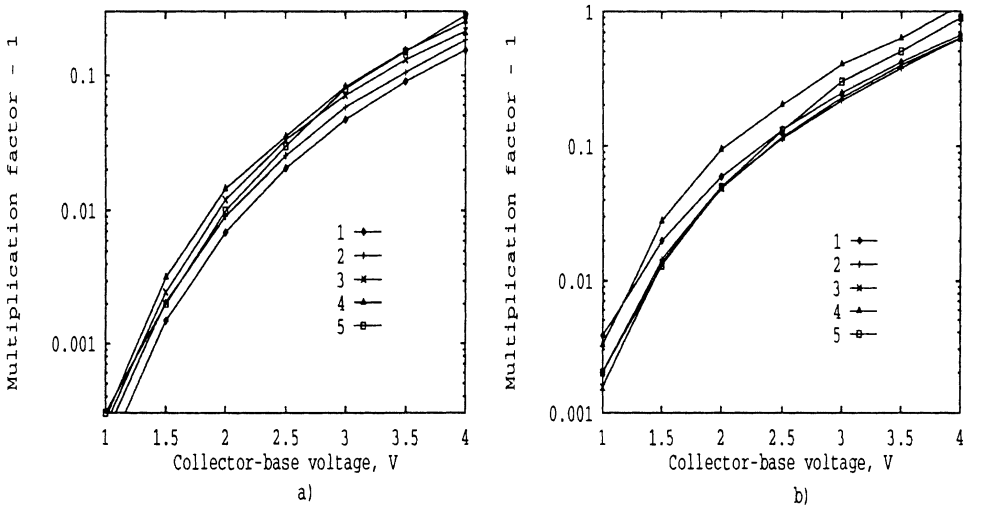

Fig. 2. Dependence of the avalanche multiplication factor M_e on the collector-base voltage V_{CB} for two BJTs with constant collector doping: a) $2 \cdot 10^{17}$ and b) $6 \cdot 10^{17} cm^{-3}$ calculated using: 1- (6); 2 - (7); 3 - HDM1 [3]; 4 - HDM2 [2]; 5 - experiment [1].

4. Conclusions

1. Simple analytical formulas (6) and (7) do not describe the $W(x)$ electron energy dependence correctly. However with the proper choice of coefficients they seems to be a good engineering tool for M_e value calculations in silicon BJTs.

2. HD models described in [2] and [3] provide quite accurate results for $M_e(V_{CB})$ dependence in spite of the large difference in their complexity. The HD model from [3] should be used with parameters which are quite different from their default values.

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