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Model-Independent Distortion Analysis in SPICE Realized for Complex Modelled Bipolar and MOS Transistors

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Abstract

Small–signal distortion analysis requires the second–order and third–order derivatives of the model equations. The use of numerical differentiation schemes enables a fast and model–independent derivation. We propose here two efficient algorithms. Both use a central differential quotient. One algorithm is based on a Romberg scheme and finds iteratively an optimal step size. The other algorithm was developed especially for bipolar elements. Here, a *logarithmus naturalis* transforms the equations to the *ln*–plane, where they behave nearly linear. This makes a step–size control for the differentiation superfluous. The methods have been realized in SPICE for complex modelled bipolar and MOS transistors.

1. Introduction

Small–signal distortion analysis in SPICE uses the perturbation approach. This means, that the characteristic of each nonlinear circuit element is expanded about its dc operating point by a third–order Taylor series. The nonlinear components are lumped together and are represented by a nonlinear voltage–controlled current source, the distortion current source.

The Taylor–series representation of the nonlinear element requires the second–order and third–order derivatives of its characteristical equation. If they are evaluated symbolically, as it is done in SPICE, the following disadvantages occur: The evaluation of the derivatives is very time–consuming, even for a moderately complex model as the Gummel–Poon transistor model. Furthermore, the evaluation must be done for each nonlinear device, and after each modification of the equations.

A numerical approximation of the derivatives circumvents these disadvantages. We propose here two algorithms for approximation of the derivatives. Both solve the problem of finding an appropiate step size for a numerical differentiation. One algorithm uses a Romberg scheme [3] to find iteratively an optimal step size. It requires only topological information on the device because it uses the dc analysis routine. The other algorithm was developed especially for bipolar devices. A *logarithmus naturalis* of the equations has nearly linear characteristics. This makes a step–size control for the differentiation superfluous.

We have applied the methods in SPICE to complex modelled bipolar and MOS transistors. They can be adapted for other models of nonlinear devices with little effort.

2. Model–independent numerical approximation scheme

We use a Romberg scheme on the first-order derivatives of the model equations. They can be found as the Jacobi elements, calculated in the dc analysis routine. Fig. 1 shows a pseudo code of the method. The voltages are varied at the operating

0. Initialize: $i = 0$; h_0: starting step size
1. CALL DC($V_{OP} \pm h_0$) $\Rightarrow$ $f'(V_{OP} \pm h_0)$
2. $f''_{h_0} = (f'(V_{OP} + h_0) - f'(V_{OP} - h_0))/2h_0$
3. $i = i + 1$; $h_i = h_{i-1}/2$
4. CALL DC($V_{OP} \pm h_i$) $\Rightarrow$ $f'(V_{OP} \pm h_i)$
5. $f''_{h_i} = (f'(V_{OP} + h_i) - f'(V_{OP} - h_i))/2h_i$
6. Romberg linear combination(f''_{h_j}; $j = 0(1)i$) $\Rightarrow$ f''_i
7. Continue 3. ... 6. until $|f''_i - f''_{i-1}| < \varepsilon$ $\Rightarrow$ h_i is optimal step size h_{opt}
8. $f''' \approx (f'(V_{OP} + h_{opt}) - 2f'(V_{OP}) + f'(V_{OP} - h_{opt}))/h^2_{opt}$

Figure 1: Model–independent differentiation scheme

point (OP) to a step size h_i. The dc analysis routine returns the first–order derivatives, $f'(V \pm h_i)$. A differential quotient approximates the second–order derivatives, f''_{h_i}. A linear combination of the approximations of the second–order derivatives to different step sizes yields a second–order derivative with smaller truncation error, f''_i. Decreasing of the step size and linear combination of the approximations is continued, until the second–order derivatives reach convergency. The last step size is considered optimal. With this step size, we approximate the third–order derivatives, f'''.

3. Numerical approximation scheme for bipolar devices

The differentiation scheme for bipolar devices exploits the exponential characteristic of the device equations. A *logarithmus naturalis* of the device equations obtains linear characteristics. The *ln*–function is used on the equations at the operating point (OP) and at a variation to a constant step size around the OP. Then a differential quotient approximates the second–order and third–order derivatives in the *ln*–plane. The inverse transformation leads to the second–order derivatives in the original plane. Eq. 1 shows that a multiplication of the *logarithmical differential quotients* with the original equations corresponds to the inverse transformation to the original plane.

$$\begin{aligned} \frac{d\ln(f(V_{OP}))}{dV} &= f'(x) \cdot \frac{1}{f(V_{OP})} \Rightarrow \\ f'(V_{OP}) &\approx \frac{\Delta \ln(f(V_{OP}))}{\Delta V} \cdot f(V_{OP}). \end{aligned} \quad (1)$$

To increase the accuracy of the numerical differentiation, we split the first–order derivatives into summands of exponential functions. Fig. 2 illustrates the method.

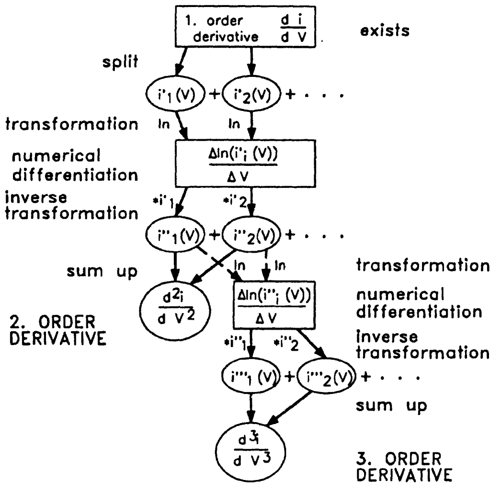

Figure 2: Differentiation scheme for bipolar devices

4. Examples

Fig. 3 shows the small signal distortion equivalent circuit of the MOS transistor model with emphasis on analog applications. The distortion current sources are indicated with an asterisk. Fig. 4 shows the harmonic and intermodulation products of a simple MOST circuit.

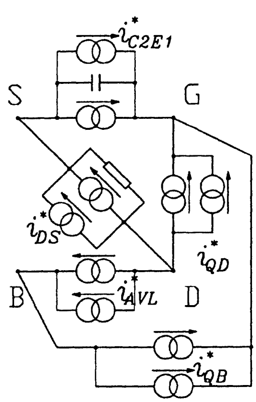

Figure 3: MOST equivalent circuit for distortion analysis

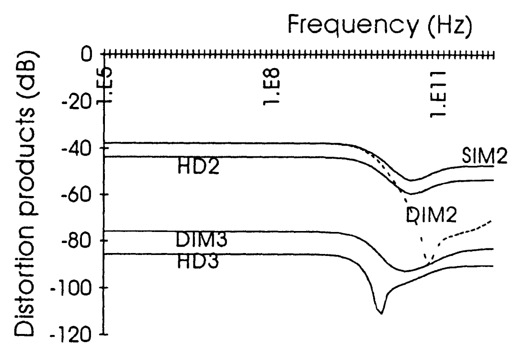

Figure 4: Distortion analysis of a MOST circuit

The "Most EXquisite TRAnsistor Model" (MEXTRAM) is described in [2]. Topologically, it is an extension of the Gummel–Poon type model with extra internal nodes. It incorporates many physical effects, for example the quasi–saturation is extensively modelled. Fig. 5 shows its equivalent circuit for distortion analysis. Fig. 6 shows the harmonic and intermodulation products of a MEXTRAM circuit.

For both devices, a comparison of the harmonic distortion products with the Fourier analysis following a transient excitation shows very good agreement for both the second–order and the third–order harmonics.

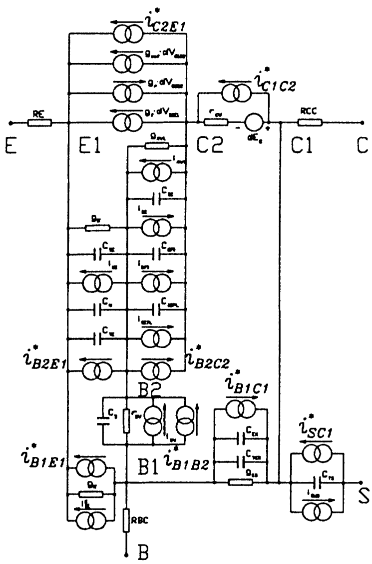

Figure 5: MEXTRAM equivalent circuit for distortion analysis

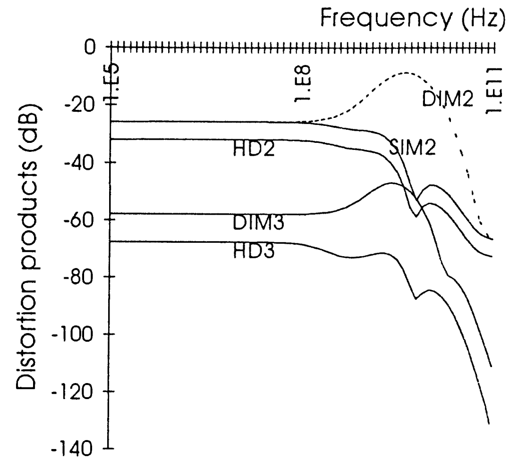

Figure 6: Distortion analysis of a MEXTRAM circuit

5. Discussion

The iterative Romberg scheme leads to an optimal step size for any operating point. It accesses the existing dc analysis routines. Therefore, updates or changes in the model's equations do not lead to any change in the distortion analysis routines. The scheme can be adapted for other nonlinear devices with little effort. It just requires the device's topology, and a simulation program for dc and ac analysis. This makes the proposed distortion analysis method model-independent. We have implemented this scheme into SPICE for the distortion analysis of a complex MOS transistor.
The differentiation scheme for bipolar devices is even faster, because it is exploits the exponential device characteristic. We use it with very good results for the distortion analysis of MEXTRAM.

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