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Dual Energy Transport Model with Coupled Lattice and Carrier Temperatures

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Abstract

A Dual Energy Transport (Dual ET) model was developed, that includes Poisson's equation, carrier continuity equations and the energy balance and thermal diffusion equation. Six variables (electric potential, electron and hole concentrations, electron and hole temperatures, and lattice temperature) can be obtained, describing all electro-thermal effects in the electrons, holes and lattice subsytems. Results for diode breakdown are shown.

1. Introduction

With progress in technology, progressively more electrical and thermal effects play a role. These effects also interact. To describe them accurately, an electro-thermal model including carrier and lattice energy transport is needed. Here, a dual energy transport (DUET) model is developed. The model considers the particle and energy transport within and between the three subsystems (lattice, electrons and holes) by solving six equations: Poisson's equation, the continuity equations for electrons and holes, the energy balance equations for electrons and holes, and the thermal diffusion equation for lattice. The electric potential ψ, the electron and hole concentrations n and p, the electron and hole temperatures T_n and T_p, and the lattice temperature T_L can be obtained. This model has been implemented in PISCES. Simulation results for a $n-p$ diode are presented, to show the advantages of the new model.

2. Equations

The following six equation are used in this model:

$$\nabla^2\psi = \frac{q}{\epsilon}(n - p + N_A - N_D) \tag{1}$$

$$q\frac{\partial n}{\partial t} - \nabla \cdot \mathbf{J}_n = -qU \ ; \tag{2}$$

$$q\frac{\partial n}{\partial t} + \nabla \cdot \mathbf{J}_p = -qU \ ; \tag{3}$$

$$\frac{\partial(n < E_n >)}{\partial t} + \nabla \cdot \mathbf{S}_n = \mathbf{F} \cdot \mathbf{J}_n - W_n \tag{4}$$

$$\frac{\partial(p < E_p >)}{\partial t} + \nabla \cdot \mathbf{S}_p = \mathbf{F} \cdot \mathbf{J}_p - W_p \tag{5}$$

$$C_L \cdot \frac{\partial T_L}{\partial t} - \nabla \cdot (\kappa \nabla T_L) = H \tag{6}$$

whith ψ, $\mathbf{F}$ the electrical potential and field; n, p the electron and hole concentrations; $\mathbf{J}_n$, $\mathbf{J}_p$ the electron and hole currents; E_n, E_p the electron and hole energies; $\mathbf{S}_n$, $\mathbf{S}_p$ the electron and hole energy flows; T_L the lattice temperature; C_L, κ the heat capacity and thermal conductivity for the lattice; U the net carrier recombination rate; W_n, W_p the net energy loss rate for electrons and holes; H the net heat source for lattice.

The currents and energy flows can be approximated by [1, 2]

$$\mathbf{J_n} = -q\mu_n(n\nabla\psi - V_n\nabla n - n(1+\gamma_n)\nabla V_n) \tag{7}$$

$$\mathbf{J_p} = -q\mu_p(p\nabla\psi + V_p\nabla p + p(1+\gamma_p)\nabla V_p) \tag{8}$$

$$\mathbf{S_n} = qC_{en}\mu_n V_n(n\nabla\psi - V_n\nabla n - n(2+\gamma_n)\nabla V_n) \tag{9}$$

$$\mathbf{S_p} = qC_{ep}\mu_p V_p(p\nabla\psi + V_p\nabla p + p(2+\gamma_p)\nabla V_p) \tag{10}$$

where μ_n and μ_p are the electron and hole mobilities, $V_n = k_B T_n/q$ and $V_p = k_B T_p/q$ with T_n and T_p being the electron and hole temperatures, and $C_{en} = 5/2 + \gamma_n$ and $C_{ep} = 5/2 + \gamma_p$ with $\gamma_n(T_n, T_L)$ and $\gamma_p(T_p, T_L)$ being defined by $\gamma_n = (T_n/\mu_n)\partial\mu_n/\partial T_n$ and $\gamma_p = (T_p/\mu_p)\partial\mu_p/\partial T_p$, respectively.

3. Exchanging Terms

The terms U, W_n, W_p and H need to be carefully evaluated, regarding all the important carrier and energy exchanging mechanisms. The net recombination rate can be written as

$$U = R_{srh} + R_n^{Aug} + R_p^{Aug} - G_n^{ii} - G_p^{ii} \tag{11}$$

where R_{srh} is the SRH recombination rate, R_n^{Aug} and R_p^{Aug} are the Auger recombination rate related to electrons and holes, and G_n^{ii} and G_p^{ii} are the I.I. rates due to electrons and holes, respectively. The conventional expressions for these terms can be found in [3]. The energy loss for electrons, W_n, includes the following terms:

$$\begin{aligned} W_{lat} &\simeq n(< E_n > - E_0)/\tau_{wn} \\ &\simeq \frac{3}{2} n k_B (T_n - T_L)/\tau_{wn} \end{aligned}$$

$$W_{shr} \simeq \frac{3}{2} k_B T_n R_{srh} \tag{12}$$

$$W_{imp} \simeq (E_{gap} + \delta_n K_B T_n) G_n^{ii} - \delta_p K_B T_p G_p^{ii} \tag{13}$$

It can be approximated by

$$W_{imp} \simeq E_{gap} G_n^{ii} \tag{14}$$

Auger recombination can be considered the reverse procedure of the impact ionizations:

$$W_{aug} \simeq E_{gap} R_n^{Aug} \tag{15}$$

The total energy loss for electrons and holes becomes

$$\begin{aligned} W_n &= W_{lat} + W_{shr} + W_{imp} - W_{aug} \\ &\simeq \frac{3}{2} n \frac{k_B(T_n - T_L)}{\tau_{wn}} + \frac{3}{2} k_B T_n R_{srh} + E_{gap}(G_n^{ii} - R_n^{Aug}) \end{aligned} \tag{16}$$

$$\begin{aligned} W_p &\simeq \frac{3}{2} p \frac{k_B(T_p - T_L)}{\tau_{wp}} + \frac{3}{2} k_B T_p R_{srh} \\ &\quad + E_{gap}(G_p^{ii} - R_p^{Aug}) \end{aligned} \tag{17}$$

Finally, we find the expression for the net heat source, H,

$$\begin{aligned} H &= W_n + W_p + E_{gap} U \\ &\simeq \frac{3n}{2} \frac{k_B(T_n - T_L)}{\tau_{wn}} + \frac{3p}{2} \frac{k_B(T_p - T_L)}{\tau_{wp}} + (\frac{3}{2} k_B(T_n + T_p) + E_{gap}) R_{srh} \end{aligned} \tag{18}$$

4. Simulations and Discussions

The DUET model can be approximated by simpler models for special applications. Assuming that T_L is constant, the DUET model defaults to the energy transport (ET) model [1, 2] (a hydrodynamic [4] -like model), which includes only the hot-carrier effects. Assuming that $T_n = T_p = T_L$, the thermodynamic (TD) model [5] can be derived from the DUET model; this choice would be appropriate for cases involving lattice heating and thermal diffusions. The Dual ET model has been implemented in PISCES. Excellent convergency behavior was observed in all cases.

Simulation results for a $n-p$ diode are shown below. Its doping profile is plotted in Fig. 1. In Figure 2 and 3, from DUET simulations, the distributions of T_n, T_p

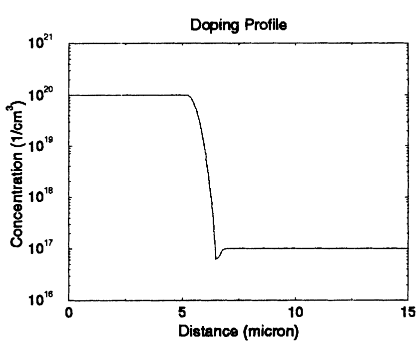

Fig. 1: Doping profile for a $n-p$ diode

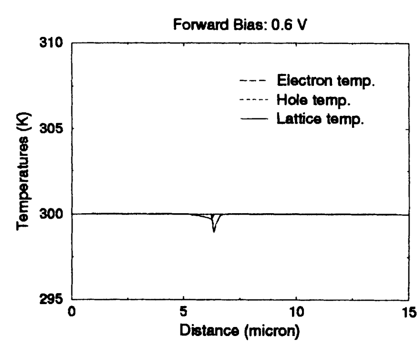

Fig. 2: Temperature distributions at forward bias: $V = 0.6V$

and T_L, for the forward biases $0.6V$ and $1V$, respectively, are plotted. As shown in these figures, $T_n \approx T_p \approx T_L$ under the forward biases, where the hot carrier effect

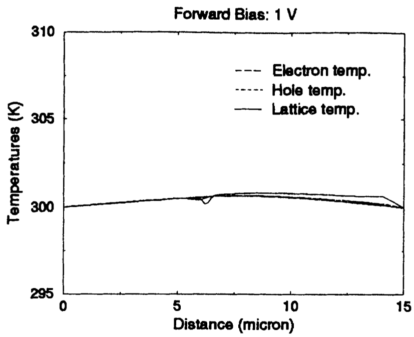

Fig. 3: Temperature distributions at forward bias: $V = 1V$

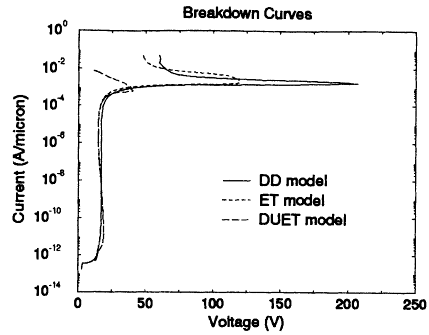

Fig. 4: I-V breakdown curves from DD, ET and Dual ET simulations.

is negligible, so that both TD and DUET models are valid. The reversed biased breakdown curves for this diode, using Drift-Diffusion (DD) model, ET model and Dual ET model, respectively, are included in Fig. 4, where the field-dependent impact ionization model was used for DD, and the carrier temperature-dependent I.I. model used for ET and DUET.

For these models breakdowns due to impact ionization happen at reverse biases around $18 - 20V$. A sharp snap-back (burn-out) occurs at high current levels for all the models. For different models, the burn-out voltage, V_{bo} shows a big difference. The DD model predicts V_{bo} at $208V$. The ET model, predicts V_{bo} at $148V$. For the Duel ET model, taking energy transport for both the carriers and lattice into account, V_{bo} is only about $40V$.

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