STOCHASTIC GEOMETRY EFFECTS IN POLYSILICON INTERCONNECTS

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Summary

Stochastic geometry effects such as fluctuations of the oxide thickness give rise to statistical components of the device parameters. In this paper a perturbation method will be outlined in order to construct the potential equations. It will then be possible to evaluate the mean square deviation of the device current starting from the statistical parameters of the geometry.

1) Introduction

Stochastic geometry effects become more and more important in present semiconductor device design. Due to the reduction of the device dimensions it is necessary to include the stochastic properties of the domain. This requires the solution of stochastic differential systems giving the expectation value or the mean square deviation of a potential or a current. Because the general equations are quite complicated a perturbation approach is used. The stochastic properties are treated as small perturbations of a well defined geometry. This approach will lead to analytical formulae which can be interpreted physically.

In the recent literature, some articles deal with stochastic effects in semiconductor devices such as MOS capacitances and MOS transistors [1][2]. For the MOS capacitances a very good agreement between theoretical and experimental results were found [3]. This paper deals with polysilicon interconnects where the square resistance $R_{\square}$ is no longer negligible. The equations are analoguous to those used for modelling distributed RC filters in hybrid microelectronics [4][5].

In this contribution we shall limit ourselves to a two dimensional problem. However, the techniques used to model stochastic variations can still be applied to the three dimensional case.

2) Fundamental equations

We assume the polysilicon interconnect being flat and isolated from the underlying perfect conductor by a dielectric layer having a capacitance C per unit area (fig.1).

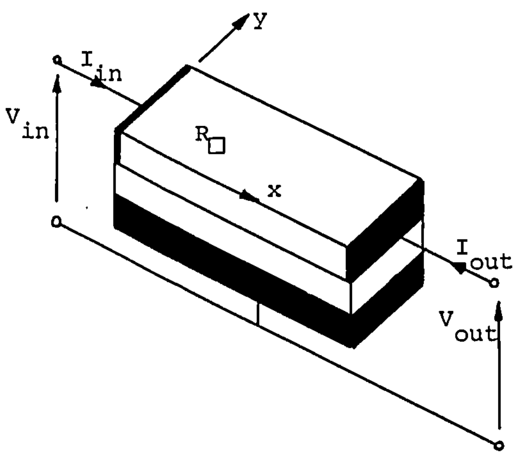

Fig.1: Isometric view of the model for a polysilicon interconnect. The conducting top layer has a capacitance C per unit area with respect to the bottom layer.

If the polysilicon has a square resistance $R_{\square}$, the potential equation using phasor notation reads |4|:

$$\nabla^2\phi - j\omega R_{\square}C\phi = 0 \qquad (1)$$

Defining the characteristic distance L by:

$$L = \sqrt{2/(\omega R_{\square}C)} \qquad (2)$$

(1) is reduced to:

$$\nabla^2\phi - \frac{2j}{L^2}\phi = 0 \qquad (3)$$

In practical cases the geometry is not well defined. Due to stochastic thichness varations the values of $R_{\square}$ and C vary from point to point. The easiest way to model similar phenomena is to write the characteristic length (2) as:

$$L = L_0 + L_1(\bar{r}) \qquad (4)$$

where L_0 is a constant denoting the mean value of L and L_1 is a two dimensional stochastic process describing the stochastic part superimposed on L_0. It is reasonable to assume the amplitude of L_1 being much smaller than L_0 so that a perturbational approach can be set up. The potential ϕ is then written in a similar way as (4):

$$\phi = \phi_0 + \phi_1(\bar{r}) \tag{5}$$

The equations for ϕ_0 and ϕ_1 are found to be:

$$\nabla^2\phi_0 - \frac{2j}{L_0^2}\phi_0 = 0 \tag{6}$$

$$\nabla^2\phi_1 - \frac{2j}{L_0^2}\phi_1 = -\frac{4jL_1}{L_0^3}\phi_0 \tag{7}$$

Other date such as the currents I_{in} and I_{out} can be expanded in a similar way as (4).

3) Solution of the zeroth order equation

A formal solution can be obtained by using the Green's function $G(\bar{r}|\bar{r}')$ which is a solution of:

$$\nabla^2 G(\bar{r}|\bar{r}') - \frac{2j}{L_0^2} G(\bar{r}|\bar{r}') = \delta(\bar{r}-\bar{r}') \tag{8}$$

A solution of (2) in the two dimensional infinite plane can be found in the literature [6]. In our case we use a Green's function satisfying the boundary conditions shown on fig.2.

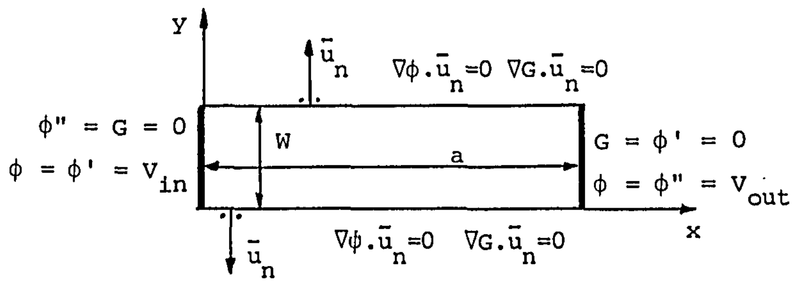

Fig.2 : Boundary conditions for the potentials and the Green's function. $\bar{u}_n$ is the normal unity vector.

Applying Green's theorem yields:

$$\phi_0(\bar{r}') = \oint_{\partial S} |\phi_0(\bar{r})\nabla G(\bar{r}|\bar{r}').\bar{u}_n - G(\bar{r}|\bar{r}')\nabla\phi_0(\bar{r}).\bar{u}_n|\, dC \tag{9}$$

The externally applied voltages are V_{in} and V_{out}. However, as will be shown in the next section, it is more convenient to write $\phi_0 = \phi_0' + \phi_0''$ where ϕ_0' corresponds to $v_{out}=0$ and ϕ_0'' to $V_{in}=0$. Inserting the boundary conditions for ϕ_0', ϕ_0'' and G in (9) one obtains:

$$\phi_0'(\bar{r}') = V_{in} \int_{A'A} \nabla G(\bar{r}|\bar{r}').\bar{u}_n \, dC \qquad (10)$$

and

$$\phi_0''(\bar{r}') = V_{out} \int_{BB'} \nabla G(\bar{r}|\bar{r}').\bar{u}_n \, dC \qquad (11)$$

These formal solutions will be very usefull to get closed form expressions for the first order currents $I_{1,in}$ and $I_{1,out}$.

4) Solution of the first order equation

The first order potential can also be written as $\phi_1 = \phi_1' + \phi_1''$. Using Green's theorem the potentials are easily found to be:

$$\phi_1'(\bar{r}') = -\frac{4j}{L_0^3} \iint_S L_1(\bar{r}) \, \phi'(\bar{r}) \, G(\bar{r}|\bar{r}') \, dS \qquad (12)$$

$$\phi_1''(\bar{r}') = -\frac{4j}{L_0^3} \iint_S L_1(\bar{r}) \, \phi_0''(\bar{r}) \, G(\bar{r}|\bar{r}') \, dS \qquad (13)$$

For the calculation of the currents, we shall limit ourselves to $I_{1,in}'$ (i.e. the input current for a short circuited output $V_{out}=0$):

$$I_{1,in}' = \frac{1}{R_\square} \int_{A'A} \nabla\phi_1'.\bar{u}_n \, dl' \qquad (14)$$

Filling (12) in (14) and changing the order of integration gives:

$$I_{1,in}' = \frac{4j}{R_\square L_0^3} \iint_S dS \, L_1(\bar{r}) \, \phi_0'(\bar{r}) \int_{A'A} \nabla_{\bar{r}'} G(\bar{r}|\bar{r}').\bar{u}_n' \, dl' \qquad (15)$$

Remark that the last integral appearing in (15) is nothing else than the relation (10) for the potential ϕ_0' (using the reciprocity properties of the Green's function). One gets finally:

$$I'_{1,in} = \frac{4j}{R_\square L_0^3} \iint_S dS\, L_1(\bar{r})\, \phi'(\bar{r})\, \frac{\phi'_0(\bar{r})}{V_{in}} \tag{16}$$

Similar expressions can be obtained for the other current components $I'_{1,out}$, $I''_{1,in}$ and $I''_{1,out}$. Note that (16) gives the current $I'_{1,in}$ as a function of the stochastic process $L_1(\bar{r})$ and the zeroth order potentials. It is interesting to remark that the Green's function no longer appears in (16).

The function $L_1(\bar{r})$ is a stochastic process and therefore not known as an explicit function of x and y. Only statistical parameters such as the mean value $\langle L_1 \rangle = 0$ or the correlation function $\langle L_1(\bar{r}_1) L_1(\bar{r}_2) \rangle$ are given. The mean square deviation of the current $I'_{1,in}$ can be evaluated by:

$$\langle I'_{1,in} I'^{\star}_{1,in} \rangle = \frac{16}{R_\square^2 L_0^6 V_{in}^2} \iint_S dS_1 \iint_S dS_2 \; \langle L_1(\bar{r}_1) L_1(\bar{r}_2) \rangle \; \phi'_0(\bar{r}_1)\phi'_0(\bar{r}_1) \;\; \phi'_0(\bar{r}_2)^{\star} \phi'_0(\bar{r}_2)^{\star} \tag{17}$$

Starting from the correlation function and the zeroth order potential the mean square deviation of the current $I'_{1,in}$ can be evaluated. Similar formulae can be written down for the other current components.

5) Application to a short circuited interconnect

If the second terminal is short circuited $V_{out}=0$ and the zeroth order solution is then easily found to be:

$$\phi_0(x) = \phi'_0(x) = -V_{in} \frac{\mathrm{sh}\,(1+j)\,\frac{x-a}{L_0}}{\mathrm{sh}\,(1+j)\,\frac{a}{L_0}} \tag{18}$$

The input current $I'_{0,in}$ is then:

$$I'_{0,in} = -\frac{1}{R_\square} W \left(\frac{\partial \phi_0}{\partial x}\right)_{x=0} = \frac{1}{R_\square} W V_{in} \frac{1+j}{L_0} \frac{\mathrm{ch}\frac{(1+j)a}{L_0}}{\mathrm{sh}\frac{(1+j)a}{L_0}} \tag{19}$$

The mean square deviation of the input current is then:

$$\langle I_{1,in} I^{\star}_{1,in} \rangle = \frac{16}{R_{\square}^2 L_0^6 V_{in}^2} \iint dS_1 \iint dS_2 \; \langle L_1(\bar{r}_1) L_1(\bar{r}_2) \rangle \; \phi_0(x_1)^2 \; \phi^{\star}_0(x_2)^2 \tag{20}$$

For the correlation function the following relation is proposed |2|:

$$\langle L_1(\bar{r}_1) L_1(\bar{r}_2) \rangle = A^2 e^{-|\bar{r}_1 - \bar{r}_2|^2 / 2\rho^2} \tag{21}$$

Although there is no physical or mathematical proof for (21), the function (21) decreases with the distance $|\bar{r}_1 - \bar{r}_2|$ making (21) acceptable. The distance ρ can be defined as the correlation distance: if two points are separated by a distance greater than ρ the L-values in both points become statistically independent.

Inserting (18) and (21) into (19) and normalising the mean square deviation to $I_{0,in} I^{\star}_{0,in}$ one gets:

$$\frac{\langle I_{1,in} I^{\star}_{1,in} \rangle}{I_{0,in} I^{\star}_{0,in}} = \frac{8}{L_0^4 W^2} \; \frac{A^2}{ch^2\left(\frac{2a}{L_0}\right) - \cos^2\left(\frac{2a}{L_0}\right)} \int_0^W dy_1 \int_0^W dy_2 \; e^{-(y_1 - y_2)^2 / 2\rho^2} \int_0^a dx_1 \int_0^a dx_2 \; e^{-(x_1 - x_2)^2 / 2\rho^2} \left[sh \frac{(1+j)(x_1 - a)}{L_0} \; sh \frac{(1-j)(x_2 - a)}{L_0} \right]^2 \tag{22}$$

After some calculations using $\rho \ll a$ (not $\rho \ll W$) one obtains finally:

$$\frac{\langle I_{1,in} I^{\star}_{1,in} \rangle}{I_{0,in} I^{\star}_{0,in}} = \frac{2\sqrt{2}\,\pi \rho^3 A^2}{L_0 W^2} \; f\left(\frac{W}{\rho}\right) \; g\left(\frac{a}{L_0}\right) \tag{23}$$

where:

$$f(\frac{W}{\rho}) = \frac{W}{\sqrt{2}\rho} \operatorname{erf}(\frac{W}{\sqrt{2}\rho}) + \frac{1}{\sqrt{\pi}} e^{-W^2/2\rho^2} - \frac{1}{\sqrt{\pi}} \quad (24)$$

$$g(\frac{a}{L_0}) = \frac{sh\frac{2a}{L_0}ch\frac{2a}{L_0} + sin\frac{2a}{L_0}cos\frac{2a}{L_0} + \frac{4a}{L_0} - sin\frac{2a}{L_0}ch\frac{2a}{L_0} - cos\frac{2a}{L_0}sh\frac{2a}{L_0}}{ch^2(\frac{2a}{L_0}) - cos^2(\frac{2a}{L_0})} \quad (25)$$

The functions f and g are displayed on fig.3. For $W \gg \rho$, (24) can be approximated by $f(W/\rho) \sim W/\sqrt{2}$. The mean square deviation (23) is then proportional to 1/W. This means that if the width W of the interconnect is reduced by scaling factor α , the mean square deviation will increase with $1/\alpha$ if the same fabrication technology is used (i.e. ρ remains unchanged).

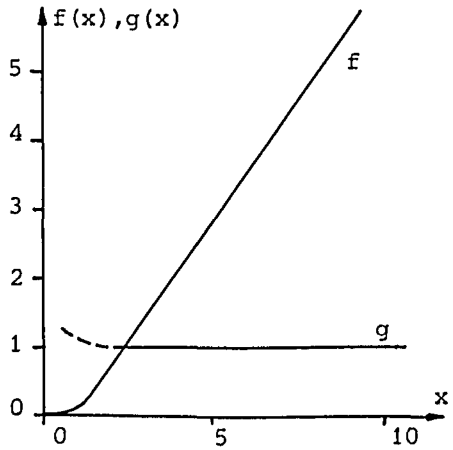

Fig.3: Plot of the functions f and g. Note that $g \cong 1$ in the interesting interval $a \geqslant L_0$.

6) conclusion

In this paper a technique has been outlined to model stochastic phenomena of a polysilicon interconnect. A simplified two dimensional example was used to illustrate the method. For the case of a short circuited line it was possible to obtain analytical results yielding the mean square deviation of the input current as a function of the geometry and the statistical parameters such as the correlation distance ρ. The method gives essentially a stochastic component as a function of the zeroth order potential distribution. It is not essential whether ϕ_0 in (17) is known analytically or numerically. Although the geometry shown on fig.1 is not a realistic picture of an actual polysilicon interconnect, the same perturbation method can still be applied to three dimensional problems. The method can also be extended to include other effects such as random edge phenomena.

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